§3.2 More on ADE singularity

D-brane probe and quiver gauge theory.

consider

$$\mathbb{C}^2 = \{(X, Y)\} \curvearrowleft \Gamma$$

discrete group.

$$(\mathbb{C}^2/\Gamma) \quad \text{orbifold.}$$

as a target of CFT.
world sheet

$$(\times \mathbb{R}^{3,1} \times \mathbb{C})$$

eg. $\Gamma = \mathbb{Z}/N\mathbb{Z}$. ※

generated by σ

$$\sigma : \mathbb{C}^2 \longrightarrow \mathbb{C}^2$$
$$(X, Y) \longmapsto (e^{2\pi i/N} X, e^{-2\pi i/N} Y)$$

σ : in $SU(2)$ on $\mathbb{C}^2$ (not $SO(4)$)

Within $\mathbb{C}[X, Y]$. σ-inv. polynomials

$$\mathbb{C}[X, Y]^{\sigma} = \mathbb{C}[X, Y]^{\Gamma}$$
$$= \mathbb{C}[X^N, Y^N, XY] / ((X^N)(Y^N) - (XY)^N)$$
$$= \mathbb{C}[a, b, c] / (ab = c^N)$$

coordinate ring of $\mathbb{C}^2/(\Gamma = \mathbb{Z}/N\mathbb{Z})$

※ $M : \{(z_1, \cdots, z_n) \in \mathbb{C}^n \mid f(z_i) = 0\}$

$\Longleftrightarrow \mathbb{C}[z_1, \cdots, z_n]/(f)$

$f_1 = f_2 = \cdots = f_k = 0 \Longleftrightarrow \mathbb{C}[z_1, \cdots, z_n] / \{f_1 \times (\cdots) + f_2 \times (\cdots) + \cdots + f_k \times (\cdots)\}$

$f_1 = 0$ or $f_2 = 0$ or $\cdots \Longleftrightarrow \mathbb{C}[z_1, \cdots, z_n] / (f_1 \cdot f_2 \cdots f_k \times (\cdots))$

alg. of hol. fcns. know the geometry.

Consider $|\Gamma| \times$ D3-branes on $\mathrm{IIB}/\mathbb{C}\times\mathbb{C}^2/\Gamma \times \mathbb{R}^{3,1}$

$\updownarrow$ T-dual.

D0-brane on $\mathrm{IIA}/\mathbb{C}\times\mathbb{C}^2/\Gamma \times \mathbb{R}^{3,1}$

<u>orbifold projection.</u>

(CFT $\longrightarrow$ new CFT. by proj. out some states.)

massless D3–D3 open string.

$\rightarrow$ $U(|\Gamma|)$ SYM. $(A_\mu,\ Z,\bar{Z},\ X,\bar{X},\ Y,\bar{Y})$ $|\Gamma|\times|\Gamma|$

new spectrum

$\forall \sigma \in \Gamma$ $\quad \rho_\Gamma(\sigma)$. finite group – regular repr.

$$\left(\begin{array}{l} V = \sum_{\sigma\in\Gamma} \mathbb{C}\, e_\sigma \\ \sigma \cdot e_\tau = e_{(\sigma\tau)} \end{array}\right.$$

require.

$$\begin{cases} \left(\rho_\Gamma(\sigma)\right)\left(A_\mu \text{ or } Z,\bar{Z}\right)\left(\rho_\Gamma(\sigma)\right)^{-1} = \left(A_\mu \text{ or } Z,\bar{Z}\right) \\ \left(\rho_\Gamma(\sigma)\right)\begin{pmatrix} X \\ Y \end{pmatrix}\left(\rho_\Gamma(\sigma)\right) = \underset{SU(2)}{\underset{\Uparrow}{\sigma}} \cdot \begin{pmatrix} X \\ Y \end{pmatrix} \end{cases}$$

16-SUSY charge $\xrightarrow[1/2]{}$ 8-SUSY charge.

(D=4 N=2 SUSY).

$$\begin{cases} \text{Coulomb branch: } \langle Z\rangle \neq 0 \quad \langle X\rangle, \langle Y\rangle = 0 \\ \text{Higgs branch } \quad \langle X\rangle \neq 0, \langle Y\rangle \neq 0 \quad \langle Z\rangle = 0. \end{cases}$$

$$W = \sqrt{2}\, \mathrm{tr}_{|\Gamma|\times|\Gamma|}\left(X, [Y, Z]\right)$$

<u>Higgs branch</u>

e.g. $\Gamma = \mathbb{Z}/N\mathbb{Z} \Rightarrow |\Gamma| = N.$

$$\rho_\Gamma(\sigma) = \cancel{\begin{pmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{pmatrix}} \begin{pmatrix} 1 & & 1 \\ & \ddots & \\ & & 1 \end{pmatrix}$$

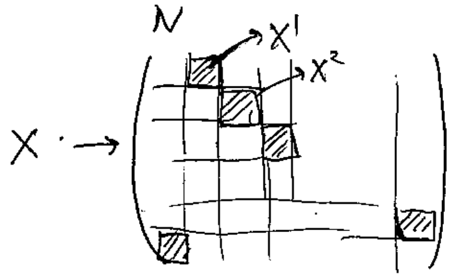
basis. $(e, \sigma, \sigma^2, \dots, \sigma^{N-1})$

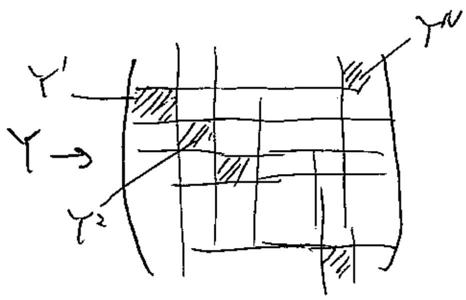
change basis $\Rightarrow$
$$\begin{pmatrix} 1 & & & & \\ & e^{2\pi i/N} & & & \\ & & e^{2\pi i/N \cdot 2} & & 0 \\ & & & \ddots & \\ & 0 & & & e^{(N-1)2\pi i/N} \end{pmatrix}$$

$U(N)$ $\mathcal{N} = \cancel{4}$ SYM

$$\Downarrow$$

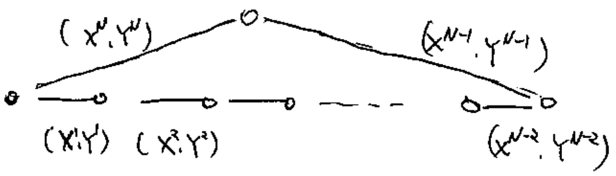
$\underbrace{U(1) \times U(1) \times \cdots \times U(1)}_{N}$ $\mathcal{N}=2$ vector mult.





chiral multiplets.
(Chyper)

$$W = Z^1(X^1Y^1 - X^NY^N) + Z^2(X^2Y^2 - X^1Y^1) + \cdots$$
$$\Rightarrow X^1Y^1 = X^2Y^2 = \cdots = X^NY^N$$



$2N$ chiral multiplets

$$\begin{cases} (N-1) \times \text{F-term cond.} \\ (N-1) \times \text{vector } \mathcal{N}=1 \end{cases}$$

$\Rightarrow$ $\boxed{\text{2-dim moduli space}}$

$$(X^1X^2\cdots X^N)\cdot(Y^1Y^2\cdots Y^N) = (X^1Y^1)(X^2Y^2)\cdots(X^NY^N) \Rightarrow$$

$U(1)$-invariants
$2N - (N-1) = (N+1)$ indep ($\mathbb{C}^{N+2}$ w/ 1 rel.)

w/ $(N-1)$ more (F-term) relations $\Rightarrow$ $\boxed{ab = c^N}$

D_N, $E_{6,7,8}$. also $\Gamma \subset SU(2)$.

$$\underbrace{\left(V = \sum_{\sigma\in\Gamma} \mathbb{C}\, e_\sigma\right)}_{\substack{\Uparrow \\ \text{regular repr. of} \\ \text{finite groups.}}} \xrightarrow[\text{irr. decomp.}]{} \bigoplus_{i=0}^{r} \mathbb{C}^{n_i} \otimes \underline{\underline{V_{\rho_i}}}$$

$\uparrow$ irr. repr. of Γ

$\dim \rho_i = n_i$

$$|\Gamma| = \sum_{i=0}^{r} (n_i)^2$$

After orbifold

$U(n_0) \times U(n_1) \times \cdots \times U(n_r)$ gauge theory $\mathcal{N}=2$ SUSY

$\Downarrow$

<u>nodes</u>. of extended Dynkin diagram.

hyper multiplets

edge between nodes.

Exercise

What is the dim. of moduli space.
associated w/ (e.g) E_6 extended Dynkin diagram?

Higgs branch

$\Longleftrightarrow$ those fractional D3-branes form a bound state $=$ D3-brane.
and move in the geometry $(\mathbb{C}^2/\Gamma)$

D-brane probe.

Deformation preserving $N=2$ SUSY

$\Longleftrightarrow$ $SU(2)_R$-triplet $(\vec{\xi}_i)$. $i = 0, 1, \ldots, r$. FI parameter.

$$\begin{cases} \text{FI - F-term} \Rightarrow & ab = \prod_i (c - \mu_i) \quad \text{def. parameters.} \\ \text{FI - D-term} \Rightarrow & ab = c^N \text{ remain; resolved.} \end{cases}$$

center of mass $U(1)$

lin. comb of all $U(1)$'s as $\mathrm{diag}(1,1,\ldots,1)$ in $U(n_0) \times U(n_1) \cdots$ $\Rightarrow$ no charged matter.

$$\Rightarrow \boxed{\sum_{i=0}^{r} n_i \vec{\xi}_i = 0}$$

$3 \times \left(\begin{matrix}\#\text{ of} \\ \text{2-cycles} = \text{rank}\end{matrix}\right)$ D.O.F.