☆ $\boxed{\mathrm{Het}/T^2 \;-\; F/\text{ell.}\,(K3 = CY_2) \quad @\, 8D}$

$\hookrightarrow \; y^2 = x^3 + fx + g$

$\pi_X : X_2 \longrightarrow (B_1 = \mathbb{P}^1)$

$f \in \Gamma(B_1 ; \mathcal{O}_{\mathbb{P}^1}(8) = T^*B_1^{\otimes 4})$
$g \in \Gamma(B_1 ; \mathcal{O}_{\mathbb{P}^1}(12) = (T^*B_1)^{\otimes 6}$
$\Updownarrow$
$X_2 = CY_2$

☆ $\boxed{\mathrm{Het}/\text{ell. } K3 \;-\; F/\text{ell. } CY_3 \text{ also } K3\text{-fib.} \quad @\, 6D}$

$\pi_Z : Z_2 \longrightarrow (B_1 = \mathbb{P}^1)$ ell. fibr.

$\pi_X : X_3 \longrightarrow B_2'$

$\pi' : B_2' \longrightarrow B_1 = \mathbb{P}^1.$

$\pi' \circ \pi_X : X_3 \longrightarrow B_1$ (ell. K3)-fibration.

$f \in \Gamma(B_2' ; (\wedge^2 TB_2')^{\otimes 4})$
$g \in \Gamma(B_2' ; (\wedge^2 TB_2')^{\otimes 6})$
$\Updownarrow$
$X_3 = CY_3$

$\boxed{12\, c_1(TB_1) = (12-n)\,\mathrm{pts} + (12+n)\,\mathrm{pts}.} \iff \boxed{B_2' = \mathbb{P}\left[\mathcal{O}_{B_1}(D_{vis}) \oplus \mathcal{O}_{B_1}(D_{vis} + \pi'^*(n\,\mathrm{pt}))\right]}$

$= F_n. \quad (D_{vis} \cdot D_{vis} = -n)$

☆ $\boxed{\mathrm{Het}/\text{ell. } CY_3 \;-\; F/\text{ell. } CY_4 \text{ also } K3\text{-fib} \quad @\, 4D}$

$\pi_Z : Z_3 \to B_2$ (ell.-fib) $\quad \pi_X : X_4 \longrightarrow B_3'$ (ell.-fib)

$\pi' : B_3' \longrightarrow B_2$ ($\mathbb{P}^1$-fib)

$\pi' \circ \pi_X : X_4 \longrightarrow B_2$ (ell. K3)-fibr.

$f \in \Gamma(B_3' ; (\wedge^3 TB_3')^{\otimes 4})$
$g \in \Gamma(B_3' ; (\wedge^3 TB_3')^{\otimes 6})$
$\Updownarrow$
$X_4 = CY_4$

$$c_2(TZ_3) = \sum_a - \frac{ch_{2,\rho}(V_\rho)}{(2T_\rho)}$$

$\Updownarrow$

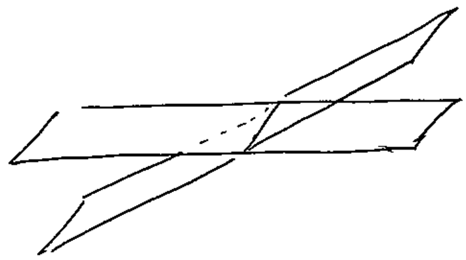
$$\begin{cases} c_2(TZ_3) = \sigma \cdot \pi_Z^*(12\, c_1(TB_2)) + \pi_Z^*(\mathrm{pts}) \\ \mathrm{RHS} \equiv \sum_a \sigma \cdot \pi_Z^*(\eta_a) + \pi_Z^*(\mathrm{pts}_a) \end{cases}$$

$\boxed{12\, c_1(TB_2) = \eta_{vis} + \eta_{hid}} \iff \boxed{B_3' = \mathbb{P}\left[\mathcal{O}_{B_2}(D_{vis}) \oplus \mathcal{O}_{B_2}(D_{vis} - (6K_{B_2} + \eta_h\right.}$

§ 4.5 Katz-Vafa's field theory local model

Katz-Vafa "Matter from geometry" '96.

consider D-brane configuration



Dp-brane × 1

U(1) vector A_μ $\mu = 0, 1, \cdots, p$.

scalar ϕ^m $m = p+1, \cdots, 9$.

+ fermionic. on $d^{p+1}\xi$

Dp-brane × 2

$\Rightarrow U(2)$ on $d^{p+1}\xi$.

Dp-branes intersecting with angles.

$\langle \phi^m \rangle$: Dp-brane location in the transverse direction.

(quantizing string / eff. action. bifundamental <massive>)

$\Rightarrow$ ξ-dependent $\langle \phi^m(\xi) \rangle$ for intersecting configuration. ?

adiabatic.

one transverse direction.

$\langle \phi(\xi) \rangle$ $\Rightarrow$ effectively just one direction in $d^{p+1}\xi$ is relevant.

$$\bar{\psi}\,(\partial_\xi + \underbrace{F\xi}_{\langle \phi(\xi) \rangle})\,\psi \quad \rightarrow \quad (\partial_\xi + F\xi)\psi = 0$$

$$\rightarrow \propto e^{\left(-\frac{F}{2}\xi^2\right)}$$

approximately Gaussian profile. at the intersection.

$(9-p)$ transverse direction

$\Rightarrow$ localize matter in (real) - codimension $(9-p)$ locus.

$\Bigg\{$
IIA D6–D6 $\Rightarrow$ real codim-3 in D6

IIB D7–D7 $\Rightarrow$ real codim-2 in D7.

lift from IIB to F-theory

e.g. $\left(y^2 = x^3 + x\cdot\left(f_4 z^4 + f_3(u) z^3\right) + \left(g_5(u) z^5 + (g_4(u))^2 z^4\right)\right)$

on (x, y, z, u) $\Rightarrow$ 3-fold $\downarrow$ cpt. to 6-dim.

$\left\{ \begin{array}{l} E_6 \text{ singularity for generic } u. \\ E_6 \to E_7 \text{ singl. in } (x,y,z) \text{ for } g_4(u_*) = 0. \\ \qquad u_* \text{ w/} \end{array} \right\}$

$\Longrightarrow$ field-theory local model on $\mathbb{R}^{5,1} \times (\text{cpx } u)$ (7-brane world vol)

gauge group E_7. $\left\{ \begin{array}{ll} A_\mu & \mu = 0, \sim, 7 \\ \phi & \text{cpx } (m = 8, 9). \end{array} \right.$

$\langle \phi \rangle \neq 0$ dep. on u.

$\langle \phi \rangle = 0$ at u_* for $g_4(u_*) = 0$.

$(E_7\text{ adj} = 133) \Rightarrow 1 + 78 + 27 + \overline{27}$ under E_6

E_6-$(27 + \overline{27})$ hyper multiplet localized @ $u = u_*$.

✓ approximate description of physics associated w/ local geometry.

✓ captures D.O.F. associated w/ "branes"

not (necessarily) all the D.O.F. including gravity.