§5. Yukawa Coupling

§5.1 chiral matter multiplets

- Het/$CY_3 = Z_3$ w/ vector bdles. V. (0.2)-compactification.
(or Type I)

eg. $E_8 \times E_8$.

$$\mathrm{Res}^{E_8}_{G_{str} \times H}(E_8\text{-adj}) = (\text{adj.}, 1) + (1, \text{adj.}) + \bigoplus_i (R_i, U_i).$$

$\Rightarrow$ chiral multiplets H-neutral $\Leftrightarrow H^1(Z_3; \rho_{adj}(V))$

" H- U_i repr $\Leftrightarrow H^1(Z_3; \rho_{R_i}(V))$

bosonic part: $(R_i \otimes U_i)$-valued $(A_{\bar{\alpha}} d\bar{z}^{\bar{\alpha}}) \phi(x)$
fermionic part:

(0.1)-form
$\updownarrow$
$SO(6) - 4 \to \underline{3} \oplus 1$

" $h^{1,1}(Z_3)$ Kählermoduli $T_\alpha = (J_\alpha + iB_\alpha) \omega$

" $h^{2,1}(Z_3)$ cpx str. moduli

" S dilaton.

" embedding of spin connection"

$G_{str} = SU(3) \subset E_8^{vis.}$ $H = E_6^{vis} \times E_8$.

$$C_2(TZ_3) = \frac{-ch_{2.\,fnd}(V^{vis})}{(2 \times T_{fund})} = C_2(V^{vis}) \quad \text{automatically.}$$

$E_6^{vis} - \underline{27} \Leftrightarrow H^1(Z_3; TZ_3) = H^1(Z_3; \wedge^2 T^* Z_3) = H^{2,1}(Z_3; \mathbb{C})$

$E_6^{vis} - \overline{\underline{27}} \Leftrightarrow H^1(Z_3; T^*Z_3) = \quad = H^{1,1}(Z_3; \mathbb{C})$

[also known as (2.2) compactification.]

- IIA / $CY_3 = X_3$ $I : X_3 \to X_3$ antiholomorphic ; $I^2 = id_{X_3}$.

fixed locus of I : O6-plane.

D6-branes wrapped on special Lag. 3-cycles L of $CY_3 = X_3$.

(supersym)

$CY_3 = X_3$. Kähler. (X_3, ω) ; $d\omega = 0$.

$$c_1(TX_3) = 0 \to H^{3,0}(X_3; \mathbb{C}) = H^0(X_3; \wedge^3 T^* X_3)$$

$$\Omega^{(3,0)} \propto \varepsilon_{abc}\, e^a_\alpha\, e^b_\beta\, e^c_\gamma\, dz^\alpha \wedge dz^\beta \wedge dz^\gamma.$$

$$\omega = h_{\alpha\bar\beta}\, dz^\alpha \wedge d\bar z^{\bar\beta} \ ; \ h_{\alpha\bar\beta} = (e_\alpha{}^a)(\bar e_{\bar\beta}{}^a)$$

sLag $L \subset X_3$:

- $\omega|_L = 0$,
- $^\exists (e^{i\theta})\, \Omega|_L$ is pure real.

$N_i \times$ D6-branes on $L_i \Rightarrow U(N_i)$ gauge group.

- $\left[\text{chiral in } (N_i, \bar N_j) \text{ repr} - \text{anti-chiral in } (N_i, \bar N_j)\right] = L_i \cdot L_j$ intersect#

anti-chiral in $(N_i, \bar N_j)$ = chiral in $(\bar N_i, N_j)$

- $b_1(L_i) \Leftarrow$ adj. (wilson line & deform)

- M-theory / G_2-hol. mfd.

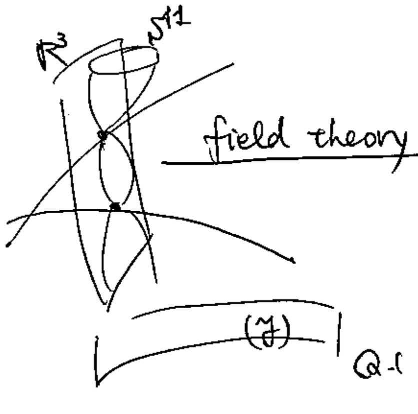
↳ real 7-dim.

locally ... <u>ALE space fibred over a 3-cycle. Q</u>

$SU(2) \times \langle SU(2) \rangle \times \langle SO(3) \rangle$

spinor $(2, 1, 2) \Rightarrow 2 \oplus 2$
$\oplus$
$(1, 2, 2) \Rightarrow 1 \oplus 3$

$\langle$ diag $\rangle$.



const. spinor. 1 in 8.

<u>field theory local model</u> $\Rightarrow$ 4 in 32 SUSY charge

$\vec\xi(y)$ vary over Q

orientation.

$U(N+M) \to U(N) \times U(M)$ etc.

for ADE and their deformation

generically rk 1 enhance

· IIB $/ CY_3 = X_3$. $\quad I : X_3 \to X_3$. hol. & $I^2 = id_{X_3}$.

fixed locus of I: ~~O7~~ orientifold plane.

e.g. fixed locus of I: 4-cycle $\begin{pmatrix} \text{hol-2 cycle}). \\ \text{cpx codim-1.} \end{pmatrix}$

D7–D7 intersection.

hol. 2-cycle (surface) $(S_i, \mathcal{L}_i)$ ← line bdle on S_i.

$\Rightarrow$ chiral multiplets $H^0\left(S_i \cdot S_j ; \mathcal{L}_i^* \otimes \mathcal{L}_j \otimes (N_{S_i \cdot S_j}|S_i \otimes N_{S_i \cdot S_j}|S_j)^{1/2}\right)$

in $(\overline{N_i}, N_j)$ repr

$H^0(S_i \cdot S_j ; \mathcal{L}_j^* \otimes \mathcal{L}_i \otimes (\quad , \quad)^{1/2})$

$\wr\wr$

$[H^1(S_i \cdot S_j ; \mathcal{L}_i \otimes \mathcal{L}_j^* \otimes (N \quad \otimes \quad)^{1/2}]$

net chirality $\underline{\chi\left(S_i \cdot S_j ; \mathcal{L}_i^* \otimes \mathcal{L}_j \otimes (N_{S_i \cdot S_j}|S_i \otimes N_{S_i \cdot S_j}|S_j)^{1/2}\right)}$

chiral adj: $H^0(S_i ; N_{S_i|X_3}) = H^0(S_i ; K_{S_i})$

Transv. on D7 $\quad = [H^2(S_i ; \mathcal{O}_{S_i})]^*$

$\oplus$ $H^1(S_i ; \mathbb{C})$. gauge field on D7

Het / T^3-fibration over $Q^{(3\text{-cycle})}$ — M / K3-fibration over $Q^{(3\text{-cycle})}$

if $\mathrm{vol}(Q_3) \gg \mathrm{vol}(T^3)$.

$\Rightarrow$ gauge field bg. for Het ...

($\vec{A}$ on T^3) vary over Q — Wilson line. ($\vec{\xi}$ for ALE) vary over Q

(part of $\Gamma^{3,19}$ moduli) $\Leftrightarrow$ (K3 metric moduli)

matter zero mode.

under varying $\vec{A}$

$\Longrightarrow$ localized @ $\vec{A}=\vec{0}$ on Q $\Longleftrightarrow$ localized on Q.

Het / T^2-fibration over B_2 — F / (ell. K3)-fibration over B_2

if $[\mathrm{vol}(T^2)]^2 \ll \mathrm{vol}(B_2)$

$\Rightarrow$ gauge field bg. for Het ...

· gauge field on B_2

· ($\vec{A}$ on T^2) varying over B_2. $\Longleftrightarrow$ (ell. K3) cpx str. varying over B_2.

$\hookrightarrow$ (part of $\Gamma^{2,18}$ moduli) $\longleftarrow$

charged matter

localized in 2-directions in B_2 $\Longleftrightarrow$ cpx curve in B_2. in B_3'.

§5.2. Mechanism generating Yukawa couplings

In Het

· perturbative.
$$W^{Het} \propto \int_{Z_3} \Omega^{(3,0)} \wedge \left[dB - \frac{\alpha'}{4} \mathrm{Tr}_{E_8 \times E_8} \left(AdA + \frac{2}{3} AAA \right) - \omega_{cs.\,grav.} \right]$$

In (2.2) compactification.

$$\left.\begin{array}{l} E_6^{vis} \; \underline{\overline{27}} \Leftrightarrow H^{1,1}(Z_3;\mathbb{C}) \\ \qquad\qquad = H^1(Z_3; T^*Z_3) \end{array}\right\} \Rightarrow \overline{27}\cdot\overline{27}\cdot\overline{27} \text{ Yukawa} \leftrightarrow \text{intersection } \#.$$

otherwise.... $\left\{ \begin{array}{l} CY_3 = Z_3 \text{ cpx str. moduli} \\ \text{vector bdle moduli} \end{array} \right\}$ dependent.

$$H^1(Z_3; V_5) \times H^1(Z_3; \wedge^2 V_5) \times H^1(Z_3; \wedge^2 V_5) \longrightarrow \mathbb{C}.$$

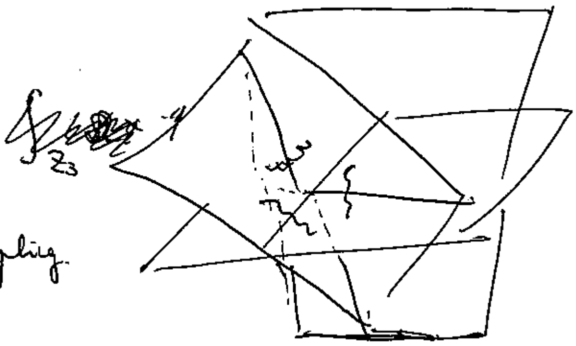
worldsheet instanton

wrap. worldsheet on cpx curves. in Z_3.

< insert vertex op. check fermionic 0-modes absent. >.

suppressed by e^{-T_α}.

In IIB



· $(\text{bifund})^3$ coupling.

$\propto$ (product of wavefcns. at that point.)

localized picture: quite simple.

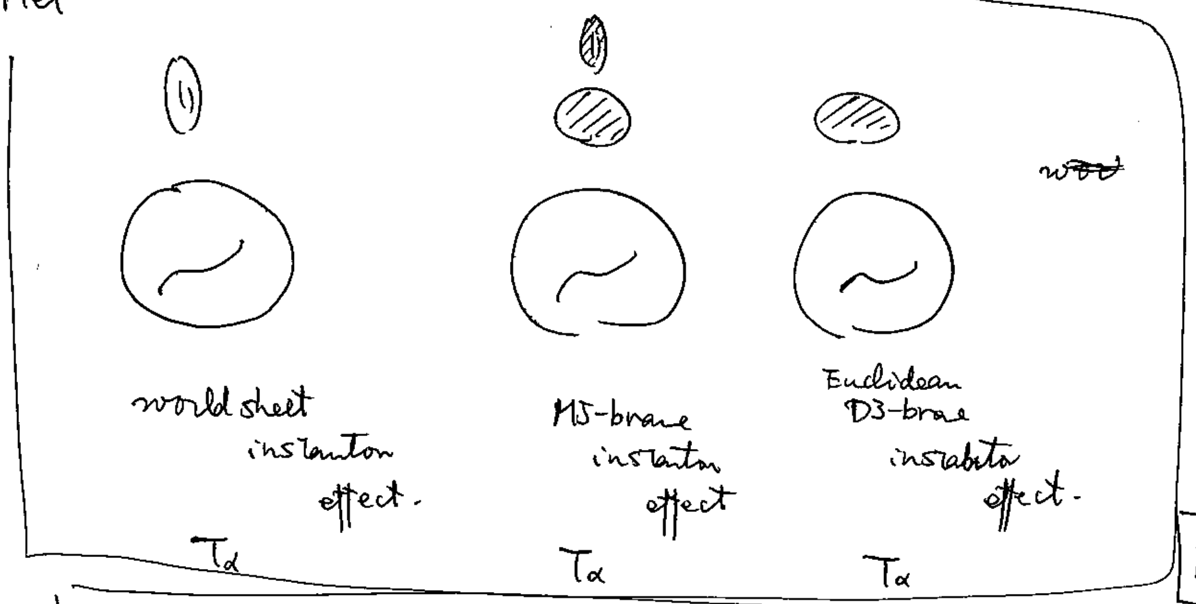
· adj/ field theory
$$W^{IIB} \propto \int_{S} \Omega_{ijk}\, \mathrm{tr}\left[\Phi^k (dA + AA)_{\bar{i}\bar{j}} \right] dz^i \wedge dz^j \wedge d\bar{z}^{\bar{i}} \wedge d\bar{z}^{\bar{j}}$$

Loc. model for intersecting system.

$$+ \int_{X_3} \Omega \wedge G^{(3)}$$

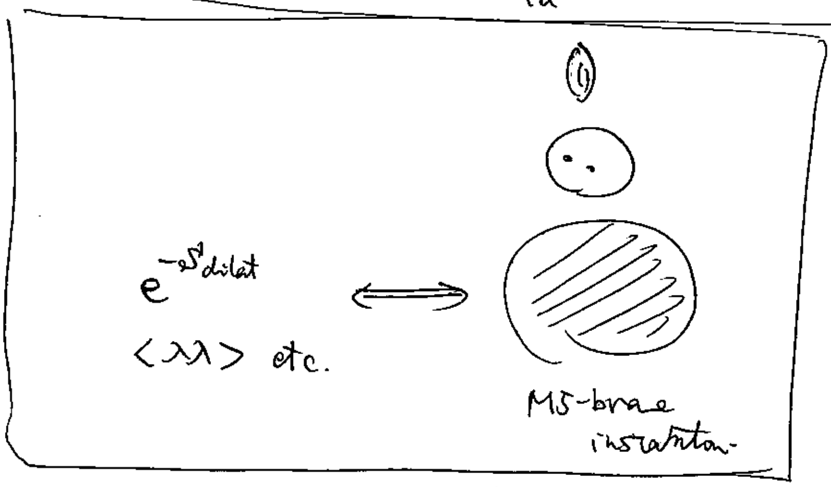
$G^{(3)} \ni (F^{(3)} - \tau H^{(3)})$

Het —— F —— IIB



$h^{1,1}(B_2)$

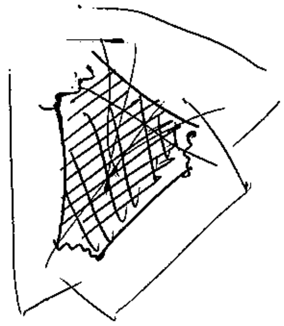
e^{-T_α}.



IIA / M-theory

- worldsheet or. M2-brane.

$$e^{-\left(\frac{area}{\alpha'}\right)} \Leftrightarrow e^{-\left(\frac{vol}{l_{11}^3}\right)}$$



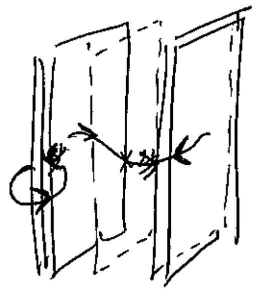
- D-brane instanton & M5-instanton

§5.3 up-type Yukawa coupling & E-type algebra

focus on the mechanism w/o ($\cancel{e^{-T} \text{ or } e^{-N}}$ ...)
instanton effect.

Yukawa from gauge theory. → algebra.

$$\Delta W = \lambda^{(u)} \bar{U} Q H_u \Longleftarrow \lambda^{(u)}_{ij} 10_i^{ab} 10_j^{cd} H^e(5) \underline{\epsilon_{abcde}}.$$



$(\ )^a_d (\ )^a_{ab} (\ )^{bd}$ not the type of ϵ_{abcde} in A_N, D_N-type alg.

$E_6 \supset U(2)_{str} \times SU(5)$.

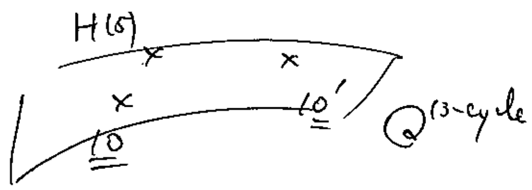
$$78 \to (3,1) + (1,24) + (1,1) + [(2,10) + (\wedge^2 \bar{2}, 5)]$$

$$\mathrm{tr}\left[(2,10), [(2,10), (\wedge^2 \bar{2}, \bar{5})]\right] \neq 0 \ ? \quad + \text{ h.c.}$$

E-type : Het. M/G_2 , F/CY_4

Het : difficult to analyze.

M/G_2-hol mfd.



$\underline{10} \cdot \underline{10}' \cdot H$ can be generated w/ e^{-A} but not $\underline{10}$-$\underline{10}$-H w/o bigger suppress.