§ 6. Low-energy Physics of F-theory Compactification.

§ 6.1 chiral charged matter multiplets

☆ In IIB : $X_3 = CY_3$; $S' \subset X_3$.

chiral adj. : $H^0(S'; N_{S|X_3}) = H^0(S'; K_{S'})$.

In F-theory $X_4 \to B_3'$.

$\{\Delta = 0\}$ containing S'

Is it $H^0(S'; N_{S|B_3'})$? or $H^0(S'; K_{S'})$?

How do we know that ?

A1: use Het-F duality $\Rightarrow K_S$ is correct.

A2: consider field-theory local model on a local patch of S'.

require supersymmetry.

$\Rightarrow A_m \ (m = 0, 1, \cdots, 7) + \underline{(\varphi_{\alpha\beta} dz^\alpha \wedge dz^\beta)}$ on S'.

$\bar{\varphi} \in H^0(S'; T^{*2}S')$ not the φ^i in $N_{S|B_3'}$

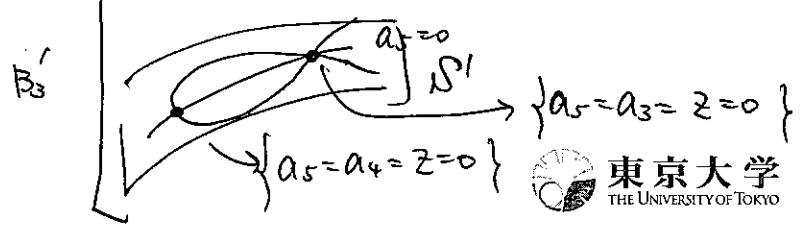
☆ behavior of chiral matter @ codim-3 pts? (K_S)

$SU(5)_{GUT}$ model.

$$y^2 = x^3 + f_0 z^4 x + g_0 z^6$$
$$+ a_5 xy + a_4 z x^2 + a_3 z^2 y + a_2 z^3 x + a_0 z^5$$

$$\Delta = (4f^3 + 27g^2) \propto z^5 [a_5^4 \cdot P_{(5)} + z \times \cdots]$$

common 0 of $(a_5 = 0)$ & $(P_{(5)} = 0)$ $\quad a_0 \underline{a_5^2} - a_2 \underline{a_5 a_3} + a_4 \underline{a_3^2} = 0$



use Het dual

in sugra approx

dual to Wilson line. given by

$$\underline{a_5 x y + a_4 x^2 + a_3 y + a_2 x + a_0 = 0}$$

in $\pi_Z : Z_3 \to B_2$.

spectral surface.

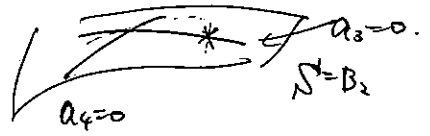
In a given fiber. : $\underline{5 \text{ zero's}}$. $\longrightarrow$ 5 Wilson lines.

$H^1(Z_3; \rho_{R_i}(V))$ $\rightsquigarrow$. sheaf cohomology on the matter curves.

SO(10) models.

$$y^2 = x^3 + f_0 z^4 x + g_0 z^6 + a_4 z x^2 + a_3 z^2 y + a_2 z^3 x + a_0 z^5$$

B_3'



$$\Delta \propto z^7 . \left[a_3^2 a_4^3 + O(z) \right]$$

$SO(10)$-$\underline{16}$ $\Leftrightarrow$ $H^0(\{a_4=0\}; \mathcal{L})$

$SO(10)$-$\underline{\underline{\text{vect}}}$ $\Leftrightarrow$ $H^0(\{a_3=0\}; \pi_{D*}(\mathcal{L}_D))$

$= H^0(D; \mathcal{L}_D)$.

$$\begin{array}{c} D \\ \pi_D \downarrow \\ \{a_3=0\} \end{array}$$

ramified over. $\{a_3=0\}$ at pts w/. $(a_2^2 - 4a_4a_0) = 0$

§ 6.2 field theory local model

In ... E_6 model

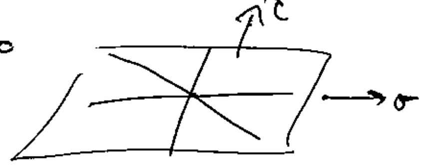
$$y^2 = x^3 + f_0 z^4 x + g_0 z^6$$
$$+ a_3 z^2 y + a_2 z^3 x + a_0 z^5$$

at $a_3 = a_2 = 0$. ($y^2 \simeq x^3 + z^5 a_0$ E_8 singul)

$a_3 = 0$ ($y^2 \simeq x^3 + z^3 x a_2$ E_7 singl.)

$\Rightarrow \langle \varphi \rangle \sim \left(\begin{array}{cc|c} 0 & & \\ & \tau & 0 \\ \hline & 0 & -(\sigma+\tau) \end{array}\right)$ in $\langle SU(3) \rangle \subset E_8$?

$\Rightarrow$ on $z=0$



triple intersection ?

in reality .. $a_3^4 z^8 + O(z^9)$

$\{a_3 = 0\}$: smooth.

Katz–Morrison '93

deformation parameters of ADE $\Longleftrightarrow$ Cartan parameter of ADE / Weyl.

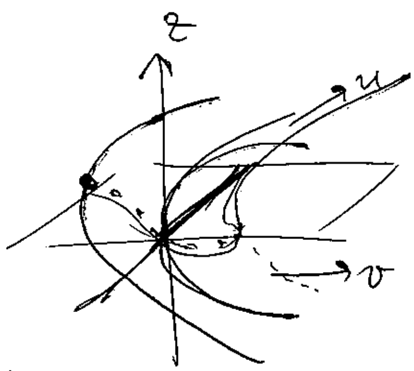
eg. A_{N-1}

$$y^2 = x^2 + z^N + (a_2 z^{N-2} + \cdots + a_N)$$

$$\langle \varphi \rangle = \begin{pmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \ddots & \\ & & & \sigma_N \end{pmatrix}$$ dep. on $(u$ traceless

$$\det \left(z + \langle \varphi \rangle_{(u,v)} \right)$$

$a_2 = \sum_{i<j} \sigma_i \sigma_j$ .... $a_N = \prod_i (\sigma_i)$



$z^2 + uz + v = 0$.

$\ne (z-\sigma_1)(z-\sigma_2)$

ramified spectral cover.

pheno for compactification to 4D.

$E_6 \to \langle U(2) \rangle \times SU(5)$

$E_7 \to \langle U(2) \rangle \times SO(10)$

2-fold ramified spectral cover.

$H^0(S ; \underline{\underline{K_S}} \otimes \mathcal{U}(2)) \ni \langle \varphi \rangle.$

$\det(\xi_{u\bar{v}} \varphi_{uv}) = 0$: spectral surface.

$\boxed{\mathbb{K}_S \supset \underline{\underline{C_{R_i}}}}$

$\underline{\mathrm{Res}^{G}_{\langle G_{S(V)} \rangle \times H} \, \mathrm{adj}. \supset \bigoplus_i (R_i, U_i)}$

not in $\underline{\underline{Z_3^{Het} \supset C_{R_i}}}$

not just bifundamental.

$\underline{\text{not in } B_3'}$.

$$\begin{cases} a_5 xy + a_4 x^2 + a_3 y + a_2 x + a_0 = 0 \subset Z_3^{Het} \\ a_5 + a_4 \xi + a_3 \xi^2 + a_2 \xi^3 + a_0 \xi^5 \subset \mathbb{K}_S. \end{cases}$$

$$\left[\begin{array}{l} \left(y \sim \frac{1}{\xi^3} \;\&\; x \sim \frac{1}{\xi^2} \right) \\ \text{if } \quad a_r \sim \mathcal{O}(\varepsilon^r) \end{array} \right]$$

Het-F duality : $\boxed{C_{R_i}^{Het} \cong C_{R_i}^{F}}$

$$\begin{pmatrix} A_{\bar\alpha} d\bar z^{\bar\alpha} + h.c. \\ + A_{\bar 3} d\bar z^{\bar 3} + h.c. \end{pmatrix} \Leftrightarrow \begin{pmatrix} A_{\bar\alpha} d\bar z^{\bar\alpha} + h.c. \\ \varphi + h.c. \end{pmatrix}$$

(duality map)

<u>fine details</u>

$y^2 = x^2 + (z^2 + 2uz + v)$

$$\Rightarrow \langle \varphi_{uv} \rangle \sim \begin{pmatrix} -u + \sqrt{u^2 - v} & 0 \\ 0 & -u - \sqrt{u^2 - v} \end{pmatrix}$$

but...

$$\begin{array}{ccc} \mathbb{K}_S & \overset{\pi_{K_S}}{\Leftarrow} & C \\ \downarrow & & \\ S & & \end{array} \qquad \pi_C : C \to S$$

$(\xi_{uv})^2 + 2u(\xi_{uv}) + v = 0$

$\xi_{uv}^{\times}$ on $\mathcal{O}_{C*}$

$\downarrow$

$(\xi_{uv}) \times$ on $\pi_{C*}(\mathcal{O}_C)$

$\hookrightarrow \begin{pmatrix} 0 & -v \\ 1 & -2u \end{pmatrix}$

BPS condition

$$\begin{cases} \omega \wedge F^{(1,1)} - \frac{1}{2}[\varphi, \bar{\varphi}] = 0 \\ F^{(0,2)} = 0 \\ D''\varphi = 0 \end{cases}$$

$$\begin{cases} \delta A^{(0,1)} = \psi \\ \delta \varphi^{(2,0)} = \chi \end{cases} \Rightarrow \begin{cases} \omega \wedge D'\psi + \frac{1}{2}\rho_{R^*}(\langle\bar{\varphi}\rangle)\chi = 0 \\ D''\psi = 0 \\ D''\chi - i\rho_R(\langle\varphi\rangle)\psi = 0 \end{cases}$$

holomorphic frame

$$\bar{\partial}\tilde{\psi} = 0 \quad \rightarrow \quad \tilde{\psi} = \bar{\partial}\tilde{\Lambda}.$$

$$\bar{\partial}\tilde{\chi} - i\rho_R(\langle\tilde{\varphi}\rangle)\tilde{\psi} = 0 \quad \rightarrow \quad \tilde{\chi} = i\rho_R(\langle\tilde{\varphi}\rangle)\tilde{\Lambda} + \tilde{f}$$

$\tilde{\Lambda}, \tilde{\chi}, \tilde{f}$: rank(R) components.

$$(\tilde{\Lambda}, \tilde{f}) \sim (\tilde{\Lambda} + \tilde{k}, \tilde{f} - i\rho_R(\langle\tilde{\varphi}\rangle)\tilde{k})$$

as $\Gamma(C_R ; \mathcal{O}_{C_R}) \ni \tilde{f}$

$$\tilde{f} \sim \tilde{f} - i\xi\tilde{k}$$

$\Rightarrow \Gamma(C_R|_{\xi=0} ; \mathcal{O}_{C_R}|_{\xi=0}) \Leftrightarrow \tilde{f}$ on the matter curve.

field theory local model.

w/ $\rho_U(\langle\tilde{\varphi}\rangle) = \pi_{C*}(\xi\times)$. Het dual calculation

$$\boxed{H^0(\text{irr. curve} ; \text{line bdle})}$$