Reference Guide and Exercise Problems

Intensive Course for Graduate Students at Rikkyo University, fall, 2012

1 Basic Things on Supersymmetric Unification Theories

Reference

- review articles on SUSY phenomenology (section 1.1)
  - H. P. Nilles, Phys.Rept. 110 (1984) 1-162
  - S. P. Martin, hep-ph/9709356
- In order to sniff more flavor of model building in SUSY phenomenology, one might be interested, e.g., in a review article
  - G. Giudice and R. Rattazzi, hep-ph/9801271
- spontaneous R-parity violation scenario and unbroken U(1) symmetry scenario in their connection to string theory (section 1.1)
  - M. Kuriyama, H. Nakajima and T. Watari, arXiv:0802.2584
  - R. Tatar and T. Watari, hep-th/0602238
- GUT model building (section 1.2–1.4): no particular reference to recommend. This is because all the model builders claim that their own model is the best. Be aware of the impact of large mixing angles in neutrino oscillation in this field.
- Wilson line breaking and line-bundle breaking of GUT symmetry group (section 1.4)
  - E. Witten, Nucl.Phys. B258 (1985) 75
  - E. Witten, Nucl.Phys. B268 (1986) 79

Exercise

- One might try to reproduce the hierarchy among Yukawa eigenvalues in the up-, down- and charged lepton-sectors by assigning charges of a weakly broken U(1) flavor symmetry. In the flipped SU(5) model, how would the charge assignment be for $\bar{\mathbf{5}}_{i=1,2,3}$, $\mathbf{10}_{i=1,2,3}$ and $\mathbf{1}_{i=1,2,3}$? Under that assignment, how are the mixing angles like in the quark sector and lepton sector? Are the results realistic?

2 Compactification of Superstring Theory and String Duality

References

- basic things on Kähler manifold (section 2.2):
  - Wess–Bagger textbook “Supersymmetry and Supergravity” appendix
  - textook by K. Kodaira “Complex Manifolds”
- CFT condition on non-linear sigma model beyond leading order in α' (section 2.2)
  - Nemeschansky and Sen, Phys.Lett. B178 (1986) 365
- section 2.3
  - on T-duality, K. Hori and C. Vafa, hep-th/0002222
  - on Heterotic–Heterotic duality, Polchinski’s textbook vol II
  - on K3, P. Aspinwall, hep-th/9611137
- D6-branes and O6-planes in M-theory language (section 2.4)
  - A. Sen, hep-th/9707123

Exercise

- In the multi-centered Taub-NUT metric, the first term in

$$U(\vec{x}) = 1 + \sum_{I=1}^{N} \frac{m}{|\vec{x} - \vec{x}_I|} \tag{1}$$

  can be ignored if $|\vec{x}_I| \ll m$, and if one focuses on a region $|\vec{x}| \ll m$.
  - Now, verify that the metric obtained in this way, with $\vec{x}_{I=1\cdots N} = \vec{0}$, is that of $\mathbb{C}^2/\mathbb{Z}_N$. [hint: $\mathbb{C}^2$ can be regarded as the (radius $\times S^3$), and S^3 can be regarded as S^1 fibration over S^2; take the fibre coordinate so that the $\mathbb{Z}_N$ action is the translation in the fibre. Find the metric of $\mathbb{C}$ 2 in this coordinate, first, and then determine the metric of the quotient.]
  - Why is multi-centered Taub-NUT space non-singular, if all $\vec{x}_I$’s are distinct from one another?
- Consider a compactification of M-theory on $S^1 \times S^1$ with the radii R_A and R_B. Verify, by following the chain of S^1 compactification to IIA string, and then another S^1 compactification that leads to T-duality with IIB string theory, that the resulting dilaton vacuumm expectation value in IIB is $1/g_s = R_B/R_A$.

3 Math Supplements (mainly about A-D-E singularity)

References

- resolution of singularity (blow up, proper transform): any textbooks on algebraic geometry
- description of geometry in terms of ring of functions: any textbooks on algebraic geometry
- "Singularity and Root System" by J. Matsuzawa (Asakura Publ. (in Japanese))
- D-brane probe: M. Douglas and G. Moore, hep-th/9603167
- quiver construction of ALE space: Kronheimer, J.Diff.Geom. 29 (1989) 665-683

Exercise

- Resolution of singularity by blow ups.
  - Resolve a singularity in a curve given by $y^2 = x^N$ by repeating blow up of the ambient space $[N/2]$ times. Draw a picture of the proper transform of the curve for the $N = 2, 3, 4$ case in the helical stairs (and in their open charts $\mathcal{U}_{1,2}$), the ambient space obtained after the first blow up of $\mathbb{C}^2 = \{(x, y)\}$, in order to understand intuitively that blow up indeed contributes in softening singularities.
  - Pick one of D_n, E_6, E_7 and E_8 and resolve the singularity by blowing up the ambient space multiple times. Verify that the exceptional curves in this resolution indeed form the Dynkin diagram of the corresponding type.
- The $E_{r=6,7,8}$ type Lie algebra: consider, first, a lattice $\mathrm{Span}_{\mathbb{Z}}\,\{H, E_{1,\cdots,r}\}$ with the intersection form given by $H \cdot H = 3$, $E_i \cdot E_j = -\delta_{ij}$ and $H \cdot E_i = 0$. The root lattice of E_r Lie algebra is its sublattice given by the orthogonal complement of an element $[x] = 3H - (E_1 + \cdots + E_r)$ in this lattice. Let us take $a_i = E_i - E_{i+1}$ $(i = 1, \cdots, r-1)$ and $\alpha_r = H - (E_1 + E_2 + E_3)$ as a set of simple roots. Confirm that the intersection of these simple roots reproduce the Cartan matrix. Furthermore, work out all the positive roots, and determine the maximal root θ. [hint: appendix of hep-th/0602238 (Tatar Watari)]
- To each node of the extended Dynkin diagram, an integer n_i is assigned. Verify that $\sum_{i=0}^{r} n_i = T_G$, the Dynkin index of adjoint representation in the given A-D-E type, or also known as the dual Coxeter number; $T_G = 2n - 2$ for D_n, $T_G = 12$ for E_6, $T_G = 18$ for E_7 and $T_G = 30$ for E_8. Confirm also that, if the defining equations of the D-type and E-type singularity are regarded as homogeneous functions of X, Y, Z, and if the weights assigned to the coordinates X, Y, Z are all integers and minimum, then the homogeneous functions are of degree T_G. Furthermore, $\sum_{i=0}^{r} (n_i)^2$ is the twice the cadinality of dihedral group $(\mathcal{D}_{n-2})$, tetrahedral, octahedral and icosahedral groups.

4 F-theory (incl. Heterotic-F duality)

References

- section 4.1
  - (p, q)-string: M. Gaberdiel and B. Zwiebach, hep-th/9709013
  - review on the relation between IIB and F-theory: F. Denef, arXiv:0803.1194
- line bundle, available sections (linear system) etc. (section 4.2–4.4): any textbooks in algebraic geometry
- Heterotic–F theory duality: easier to get started with
  - D. Morrison and C. Vafa, hep-th/9603161
  - Bershadsky et.al. (6 authors), hep-th/9605200
  - Bershadsky, Johansen, Pantev and Sadov, hep-th/9701165
- those with more immunity to math might also be interested in
  - Friedman, Morgan and Witten, hep-th/9701162
  - Hayashi et.al. (5 authors), arXiv:0805.1057
- for the moduli map in this duality, see
  - S. Katz, R. Mayr and C. Vafa, hep-th/9706110
  - Hayashi et.al. section 6 of the article above
  - Hayashi et.al. (4 authors), arXiv:0901.4941
  - Hayashi et.al. (4 authors), appendix B of arXiv:1004.3870
- Field theory description of "open-string-like" degrees of freedom: S. Katz and C. Vafa, hep-th/9606086

Exercise

- Study monodromy of 1-cycles of the elliptic fibre [Seiberg–Witten theory] in such geometries as $y^2 = x^3 + x^2 - z$, $y^2 = x^3 + x^2 - z^N$, $\cdots$. [more info, see arXiv:1004.3870 above]
- Study spectrum of U(N) Yang–Mills–Higgs theory with 16 SUSY charges with linear Higgs background, and compare the spectrum with those of bifundamental open string.

5 Yukawa Couplings

References

- Chiral charged matter multiplets in various string compactification (section 5.1):
  - (Heterotic) textbook Green Schwarz Witten,
  - (Heterotic) J. Distler and B. Greene, Nucl.Phys. B304 (1988) 1
  - (IIA,IIB) Ooguri, Oz and Yin, hep-th/9606112
  - (IIA) Berkooz, Douglas and Leigh, hep-th/9606139
  - (M/G_2) B. Acharya and E. Witten, hep-th/0109152
  - (G_2 holonomy) E. Witten, hep-th/0108165
  - (IIB) S. Katz and E. Sharpe, hep-th/0208104
  - (IIB D-brane charge) Y.K. Cheung and Z. Yin, hep-th/9710206
  - (IIB local chirality formula) T. Watari and T. Yanagida, hep-ph/0402160
- effective acton (section 5.2)
  - (4D effective action) E. Witten, Phys.Lett. B155 (1985) 151
  - (higher-dim action) N. Arkani-Hamed, T. Gregoire and J. Wacker, hep-th/0101233
- instanton in string theory: E. Witten, hep-th/9604030
- Yukawa coupling using E_r-type algebra: R. Tatar and T.Watari, hep-th/0602238
- Yukawa coupling using instanton
  - M.Dine, N.Seiberg, E.Witten and X.G.Wen, Nucl.Phys. B278 (1986) 769, Nucl.Phys. B289 (1987) 319
  - R.Blumenhagen, M.Cvetic, S.Kachru and T.Weigand, arXiv:0902.3251

Exercise

- Using the result of the 2nd exercise problem for section 3, work out the irreducible decomposition of the E_6 adjoint representation under the subgroup $\mathrm{U}(1) \times \mathrm{SU}(2) \times \mathrm{SU}(5)$.
- Have fun by following various trilinear couplings in terms of string junction in Gaberdiel–Zwiebach hep-th/9709013 of E_6 and E_7 super Yang–Mills theory.

6 Low-energy Physics of F-theory Compactification

References

- elliptic function: any textbooks. e.g., A. Hurwitz and R. Courant, "Elliptic Functions" (Springer)
- on right-handed neutrino mass and dinension-4 proton decay problem, see R. Tatar, Y. Tsuchiya and T. Watari, arXiv:0905.2289
- Ideas of how to realize semi-realistic flavor structure in F-theory compactification have been discussed, e.g., in
  - J. Heckman and C. Vafa, arXiv:0811.2417
  - S. Cecotti, M. Cheng, J. Heckman, C. Vafa, arXiv:0910.0477
  - H. Hayashi, T. Kawano, Y. Tsuchiya, T. Watari, arXiv:0910.2762

Disclaimer:

This reference guide for section 1–6 is only meant to be supplementary material for the intensive course lectures I gave at Rikkyo University in fall 2012. It is not meant to be a complete reference list in any other contexts.