§ 3. Perturbation : Loop Expansion.

§ 3.1. Feynman rule.

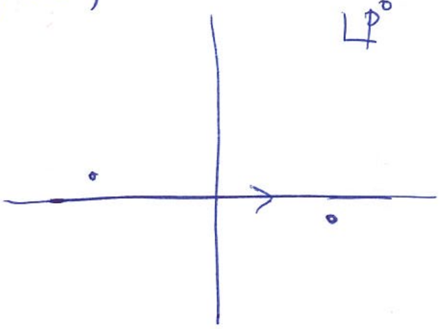
eg. 1. cpx scalar field.

$$\begin{cases} \phi_I(x) = \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \left(e^{-ip\cdot x} a_{\vec{p}} + e^{ip\cdot x} b_{\vec{p}}^\dagger \right) \\ \phi_I^*(x) = \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \left(e^{-ip\cdot x} b_{\vec{p}} + e^{ip\cdot x} a_{\vec{p}}^\dagger \right) \end{cases} \quad p^0 = +E_{\vec{p}}$$

$$\langle 0 | T\{ \phi_I(x) , \phi_I^*(y) \} | 0 \rangle = \begin{cases} \text{use } a_{\vec{p}} \ \& \ a_{\vec{p}'}^\dagger \text{ when } x^0 > y^0 . \\ \text{use } b_{\vec{p}'} \ \& \ b_{\vec{p}}^\dagger \text{ when } x^0 < y^0 . \end{cases}$$

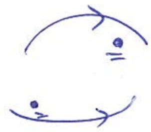
$$= \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{1}{(2E_{\vec{p}})} \begin{cases} e^{-ip\cdot(x^0-y^0)} & (x^0 > y^0) \\ e^{-ip\cdot(y^0-x^0)} & (y^0 > x^0) \end{cases}$$

$$\boxed{= \int \frac{d^4p}{(2\pi)^4} \frac{i \, e^{-ip\cdot(x-y)}}{(p^2 - m^2 + i\varepsilon)}}$$



$x^0 > y^0 \Rightarrow$

$y^0 > x^0 \Rightarrow$



e.g 2. photon

$$\langle 0 | T\{ A_\mu(x) A_\nu(y) \} | 0 \rangle = \int \frac{d^4p}{(2\pi)^4} \frac{-i \eta_{\mu\nu} e^{-ip\cdot(x-y)}}{(p^2 + i\varepsilon)}$$

(Feynman gauge)

eg. 3 Dirac fermion

$$\langle 0 | T\{ \Psi(x) \bar{\Psi}(y) \} | 0 \rangle = \int \frac{d^4p}{(2\pi)^4} \frac{i(\not{p} + m) e^{-ip\cdot(x-y)}}{(p^2 - m^2 + i\varepsilon)}$$

$\langle 0| a^\dagger a^\dagger |0\rangle = 0$ ann. op. to the right $\to |0\rangle$

$\langle 0| a\, a |0\rangle = 0$ creat. op. to the left $\to \langle 0|$

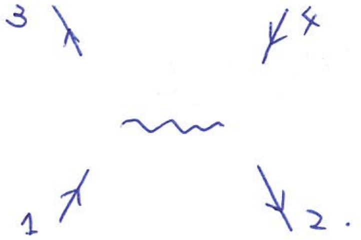
$\langle 0| a^\dagger a |0\rangle = 0$.

remnant = scattering amplitude / correlation fcn.

$a a^\dagger = \pm a^\dagger a + [a, a^\dagger\}$

eg. 4

(*) — $\langle 0| T\{ \phi_I(x_3)\, \phi_I^*(x_4)\, \phi_I^*(x_1)\, \phi_I(x_2) \exp\left(-i\int dt' V_I(t')\right) \} |0\rangle$



$\mathcal{L} = (D^\mu\phi)^\dagger (D_\mu \phi)$; $D_\mu = (\partial_\mu - ie A_\mu)$ on ϕ.

pull out

$-i\int V_I = -e\int A_\mu \left(\phi_I^*(\partial^\mu\phi_I) - (\partial^\mu\phi_I^*)\phi_I\right)$ twice.

$= (\phi_I^* \overleftrightarrow{\partial}^\mu \phi_I)$

$\Downarrow$

$\langle 0| T\{ \phi_I(x_3)\, \phi_I^*(x_4) \left(-e\int A_\mu(y_L)(\phi^* \overleftrightarrow{\partial}^\mu \phi)(y_L)\right)\left(-e\int A_\nu(y_R)(\phi^* \overleftrightarrow{\partial}^\nu \phi)(y_R)\right) \phi^*(x_1)\, \phi(x_2) \}|0\rangle$

$\phi_I \sim (a e^{-ip\cdot x} + b^\dagger e^{ip\cdot x})$.

$\phi_I^* \sim (a^\dagger e^{ip\cdot x} + b e^{-ip\cdot x})$

$$\int (*)(x_1, x_2, x_3, x_4)\, e^{-ip_1\cdot x_1} e^{-ip_2\cdot x_2} e^{ip_3\cdot x_3} e^{ip_4\cdot x_4} \prod_i d^4x_i$$

$$= \prod_i \left[\int d^4x_i \int \frac{d^4k_i}{(2\pi)^4} \frac{i}{(k_i^2 - m^2 + i\varepsilon)}\right] e^{-ip_1\cdot x_1} e^{-ik_1\cdot(y_L - x_1)} e^{-ip_2\cdot x_2} e^{-ik_2\cdot(y_R - x_2)} e^{ip_3\cdot x_3} e^{-ik_3\cdot(x_3 - y_L)} e^{ip_4\cdot x_4} e^{-ik_4\cdot(x_4 - y_R)}$$

$$\int d^4y_L \int d^4y_R \int \frac{d^4q}{(2\pi)^4} \frac{-i\eta_{\mu\nu}}{(q^2 + i\varepsilon)} \left[ei(k_1 + k_3)_\mu\right]\left[-ei(k_4 + k_2)_\nu\right]$$

§ 3.2 momentum loop

- external lines $\Rightarrow \dfrac{i}{(p_i^2 - m^2 + i\varepsilon)}$ $\left(\begin{array}{l}\Rightarrow \text{forget it!} \\ \text{need } \underline{\text{residue}}.\end{array}\right)$

- internal lines (propagators) $\Rightarrow \dfrac{i\,[P]_{\alpha\beta}}{(k_a^2 - m^2 + i\varepsilon)} \dfrac{d^4k_a}{(2\pi)^4}$

- vertex $\Rightarrow \underbrace{\int d^4y_i \; e^{-iy_i\cdot\left(\sum_a \varepsilon_{ia} k_a\right)}} \times (-i) \times (V_{a_1 a_2 \cdots a_N})$

$$\| \quad (2\pi)^4 \delta^4\left(\sum_a \varepsilon_{ia} k_a\right)$$

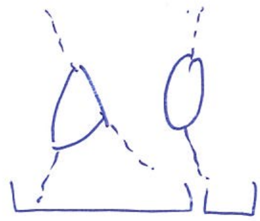
$\varepsilon_{ia} = \pm 1$ (momentum orientation) $\left(\begin{array}{l} + \text{ if } k_a^\mu \text{ in} \\ - \text{ if } k_a^\mu \text{ out}\end{array}\right)$

$\varepsilon = +1$, $\varepsilon = -1$

<u>remaining momentum integration</u>

$= \#[\text{propagator}] - \#[\text{vertex}]$

$\quad + \#\left[\begin{array}{l}\text{momentum} \\ \text{consv. of ext. lines}\end{array}\right]$



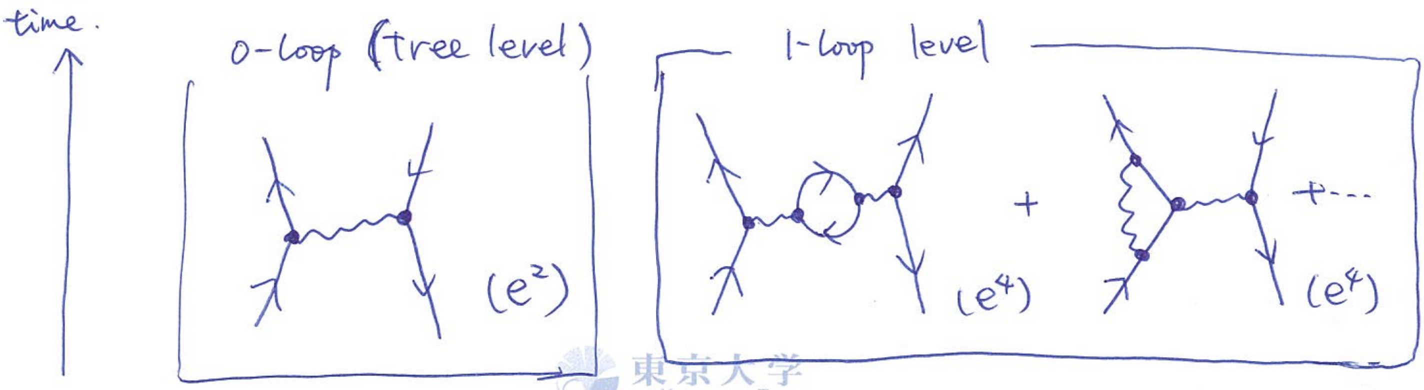
$\#[\text{1-dim. objects}] - \#[\text{0-dim. objects}]$

$= \#[\text{1-dim. cycle}] - \#[\text{connected components}]$

(cycle → no boundary) (topology)

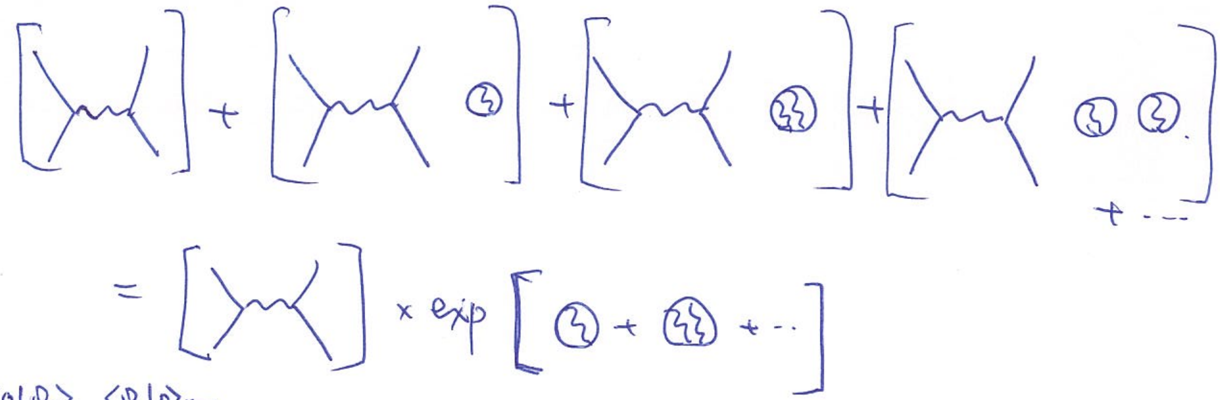
<u>momentum loop integration</u>

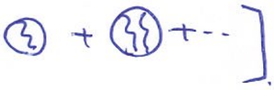
<u>eg. in QED</u>



Remark

- vacuum bubble



- $\langle 0|\Omega\rangle_{(T+)}\langle\Omega|0\rangle_{(T-)}$ ~~$|\langle 0|\Omega\rangle|^2$~~ $= \langle 0|T\{\exp(-i\int dt' V_I(t'))\}|0\rangle = \exp\Big[$  $\Big]$

$$\Rightarrow \frac{\langle 0|T\{\phi_I\cdots\phi_I \exp(-i\int V_I)\}|0\rangle}{\langle 0|T\{\exp(-i\int V_I)\}|0\rangle} = \langle 0|T\{\phi_I\cdots\phi_I \exp(-i\int V_I)\}|0\rangle_{\text{conn}}$$

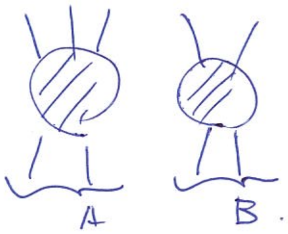
$$\parallel$$

$$\langle\Omega|T\{\phi\cdots\phi\}|\Omega\rangle$$

(Peskin-Schröder §4.4)

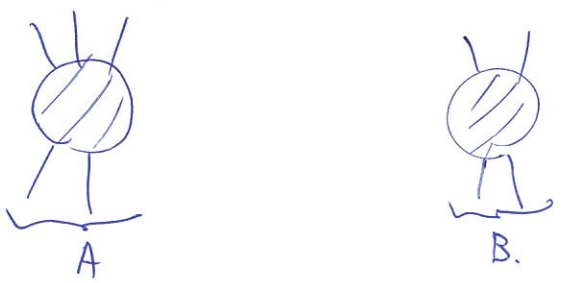
- cluster decomposition

group of momentum conservation.



$$\Delta G(\{x_i\text{'s}\}_A, \{x_j\text{'s}\}_B) = \Delta G(\{x_i\text{'s}\}_A, \{x_j^\mu\text{'s} + x_*^\mu\}_B)$$

$$\parallel$$



local physics.   ‖   fully connected diagrams.

→ #[conn. comp] ∨ 1

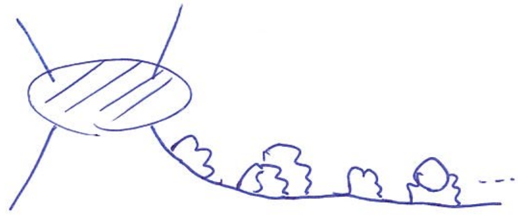
(Weinberg vol I §4.3)

§ 3.3 Self Energy

✓ need "Z" in LSZ reduction formula.

$$\langle \Omega | T\{\phi(x)\phi(y)\} | \Omega \rangle = \int \frac{d^4p}{(2\pi)^4} \frac{iZ\, e^{-ip\cdot(x-y)}}{p^2-m^2+i\varepsilon} + \cdots$$

✓ always have



lasts forever...

what's this ?

the same question

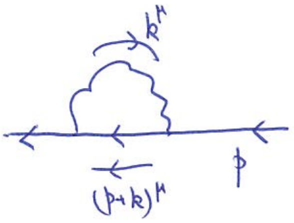
eg. QED.

$$\int \langle \Omega | T\{\psi(x)\bar\psi(y)\}|\Omega\rangle e^{ip\cdot(x-y)} d^4(x-y) = \int d^4(x-y) \langle 0 | T\{\psi_I(x)\bar\psi_I(y) \exp(-i\int V_I)\}|0\rangle_{\text{conn.}} e^{ip\cdot(x-y)}$$

$$= \frac{i(\not p+m)}{(p^2-m^2+i\varepsilon)} + \frac{i(\not p+m)}{(p^2-m^2+i\varepsilon)}(-i\Sigma)\frac{i(\not p+m)}{(p^2-m^2+i\varepsilon)} + \frac{i(\not p+m)}{p^2-m^2+i\varepsilon}\left(-i\Sigma\frac{i(\not p+m)}{p^2-m^2}\right)^2 + \cdots$$

$(-i\Sigma) =$ 

$$\left(-i\Sigma^{\text{1-loop}}(p^\mu)\right) = (ie)^2 \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^\mu\, i[(\not p+\not k)+m]\gamma^\nu}{[(p+k)^2-m^2+i\varepsilon]} \frac{-i\eta_{\mu\nu}}{[k^2+i\varepsilon]}$$



$$\langle 0|\ \psi(x)\ (\bar\psi\gamma^\mu\psi)A_\mu\ (\bar\psi\gamma^\nu\psi)A_\nu\ \bar\psi(y)\ |0\rangle$$

[A little formal treatment]

$$-i\Sigma(p) = -i\left[\not p\, A(p^2, m^2) + B(p^2, m^2)\right]$$

$$\Rightarrow \int \langle \Omega | T\{\psi(x)\bar\psi(y)\} | \Omega\rangle\, e^{ip\cdot(x-y)}\, d^4(x-y)$$

$$= \frac{i}{\not p - m} + \frac{i}{\not p - m}(A\not p + B)\frac{1}{\not p - m} + \frac{i}{\not p - m}\left((A\not p + B)\frac{1}{\not p - m}\right)^n \quad n = 2, 3, 4 \ldots$$

$$= \frac{i}{\not p - m - (A\not p + B) + i\epsilon} = \frac{i}{(1-A)\not p - (m+B) + i\epsilon}.$$

$$= \frac{i\left[(1-A)\not p + (m+B)\right]}{(1-A)^2 p^2 - (m+B)^2 + i\epsilon}$$

☆ find a ~~pole~~ zero in $\left[(1 - A(p^2, m^2))\, p^2 - (m + B(p^2, m^2))^2\right]$

$\Rightarrow$ ~~real~~ true (mass) $= m_{true}$.

☆ residue at $(p^2 = m_{true}^2)$

$$Z \simeq \frac{1}{(1-A)|_{p^2 = m_{true}^2 \simeq m^2}}$$

mass & normalization

shift (change) by wearing "cloud"