QFT II

"Category E" homework problems

(prepared by TW and Nozomu Kobayashi (TA))

- **At the head of your report, please write your name, student ID number and a list of problems that you worked on in a report (like "II-1, II-3, IV-2").**
- Homework problems in category [E] are reading materials. Find one(s) that you are interested in, and read; any kinds of record that you have done so can be submitted as a report to get a credit in this course.
  - "Any kinds of records" include summary written in your own language, and a photocopy of back-of-the-envelope calculations during the process of reading through these articles.
  - Excerpts from these articles (or their translation) are NOT recognized as the record, however. It is important to try to crystalize your own understanding in your own language, in your own brain. The process of understanding should be something more than swallowing statements in a review article. I believe students at Tokyo University do not need such a reminder, but just in case.
- It is not necessary to read one article from the beginning all through to the end. If you manage to have understood/digested some part of an article / some subject, I must say that is already a great achievement! I just encourage you to eat as much as you like while you have appetite!!
- This PDF file provides links to the articles referred to here.
- References listed below are only meant to be suggestions, to save your time. If you find a better reading material on your own, you do not have to stick to the references listed below.

[**E-1**] **Recursion relations on scattering amplitudes (using spinor helicity formalism)**: It is certainly the standard and orthodox method in computing scattering amplitedes to use Feynman rules. There is an alternative, however, which uses recursion relations among scattering amplitudes; using amplitudes of scattering m particles with $m < n$ and the recursion relations, one can work out amplitudes of scattering n particles. This alternative method has turned out to be much more powerful and fast, when n is large, than the standard method using Feynman diagram. This alternative method has been a subject of intensive study in particle theory in recent years; the practical benefit referred to above is one of the reasons, but theorists also hope that the alternative method may be further developed into a reformulation of building blocks of scattering amplitudes, just like Lagrangian / Hamiltonian

formalism did for Newton dynamics.
A review article [arXiv:1111.5759] "*An Introduction to On-shell Recursion Relations,*" by Bo Feng and Mingxing Luo allows you to catch a glimpse of this "alternative method."
The homework problem D-2 will serve as a side reader to this review article.

[**E-2**] **In-In/Schwinger–Keldysh formalism**: We has had little time in the class to explore formulations known under the name of "In-in formalism," "Shwinger–Keldysh formalism," or any others of that sort. This formalism is suited for solving time evolution of operator correlation functions when an initial state (or density matrix) is prepared, and time-dependent field background is applied to the system. By reading textbooks, review articles or any other suitable materials, you will be able to see what can be done with this formalism. Since we (TW and NK) are not experts on this subject, we cannot recommend with confidence which reference to look at. While we refer to the two following references (in addition to a few textbooks referred to in the course web page), we do not necessarily mean that they are our top recommendation.

- [Rev. Mod. Phys. **58**, (1986) 323] "*Quantum field-theoretical methods in transport theory of metals,*" by J. Rammer and H. Smith,
- [hep-th/0506236] "*Quantum contributions to cosmological correlations,*" by S. Weinberg.

[**E-3**] **Out-of-time-ordered correlation functions**: We discussed mainly time-ordered product correlation functions in the class, and we will (hope to) cover also the in-in formalism correlation functions later in the course (some time in December–January). But they are not all the classes of observables that can be computed in QFT and can also be measured experimentally. Imagine

$$F(t) := \langle \text{state}| \left([A(t), B(0)]\right)^\dagger [A(t), B(0)] |\text{state}\rangle, \tag{1}$$

or more generally,

$$F(t) = \text{Tr}\left[\left([A(t), B(0)]\right)^\dagger [A(t), B(0)]\, \rho\right], \tag{2}$$

where ρ is a density matrix. This class of quantity is interesting, for example, when the operators A and B are associated with different points in space (= differnt lattice sites). At $t = 0$, operators placed at different points commute, and hence $F(t = 0)$ vanishes. As a given quantum system evolves in time, however, $A(t) = e^{iHt}A(0)e^{-iHt}$ ceases to commute with $B(0)$. The quantity $F(t)$ captures how fast this non-commutativity develops between two separate points in space. $F(t)$ written down above is an example of observables called out-of-time-ordered (or out-of-time-order) correlation functions.

- As a homework to the QFT II course, we suggest you to explore literatures proposing (or carrying out) experiments that measure such out-of-time-ordered correlation functions. To find articles, you can Google with such a combination of key words as "out of time order" measure.
- If you do not want to Google on you own, an alternative may be to look at [arXiv:1602.06271] "*Measuring the scrambling of quantum information,*" by B. Swingle et.al., [arXiv:1607.01801] "*Interferometic approach to fast scrambling,*" by N. Yao et.al., and [arXiv:1608.08938] "*Measuring out-of-time-order correlation functions and multiple quantum spectra in a trapped ion quantum magnet,*" by M. Gärttner et.al..

[**E-4**] **Chern–Simons theory / Quantum Hall effects**: Chern–Simons theory on 2+1 dimensional space-time has various interesting properties that relativisitic/non-relativisitic QFTs in 3+1-dimensions do not have. It is important also because it is motivated as a low-energy effective theory of some condensed matter systems.

- [A Les Houches lecture note] "*Aspects of Chern–Simons Theory,*" G. Dunne
- The two following articles may be too tough for some of undergraduate students, but you might still be interested in working on.
  * [Comm.Math.Phys.**121**(1989) 351-399] "*Quantum field theory and the Jones polynomial,*" by E. Witten (mathematical physics),
  * [A lecture note] "*Lectures on the Quantum Hall Effect.*" by David Tong (condensed matter physics).

[**E-5**] **Entanglement entropy**: Correlation functions / matrix elements of local operators are not all the observables we can define theoretically, or measure experimentally. Entanglement entropy and its extensions are such observables, though we did not have enough time to discuss those observables in the QFT II course. The following articles explain how those observables might be useful.

- [cond-mat/0510613] "*Detecting topological order in a ground state wave function,*" by M. Levin and X.G. Wen,
- [arXiv:0704.3906] "*Area laws in quantum systems: Mutual Information and Correlators,*" by M. Wolf et.al.,
- you might also be interested in the following articles, though the first one involves the notion of renormalization (which the QFT II course did not cover), and the latter two require knowledge on quantum hall effects and Chern-Simons theory.
  * [hep-th/9303048] "*Entropy and area,*" by M. Srednicki,

* [hep-th/0510092] "*Topological entanglement entropy*," by A. Kitaev and J. Preskill,
* [arXiv:0805.0332] "*Entanglement spectrum as a generalization of entanglement entropy: identification of topological order in non-Abelian fractional quantum hall effect states*," by H. Li and F. Haldane.

[**E-6**] **Any other subjects/topics** that you are interested in, so far as that is remotely relevant to quantum field theory.