Theory of Elementary Particles

homework VII (May 23)

- **At the head of your report, please write your name, student ID number and a list of problems that you worked on in a report (like "II-1, II-3, IV-2").**
- Pick up any problems that are suitable for your study. **You are not expected to work on all of them!**
- Format: Reports do not have to be written neatly; hand-writing is perfectly O.K. Do not waste your time!
- Keep your own copy, if you need one. Reports will not be returned.

1. **Dimensional Regularization 1 (1-Loop Calculation V)** [B]

   (a) Photon vacuum polarization in QED is given at 1-loop by

   $$\Pi^{\mu\nu}_{(1)} = -e^2 \int \frac{d^n k}{(2\pi)^n} \mu^{4-n} \frac{\mathrm{Tr}\left[\gamma^\mu (\not{k} + m)\gamma^\nu ((\not{k} + \not{q}) + m)\right]}{(k^2 - m^2 + i\epsilon)((k+q)^2 - m^2 + i\epsilon)},$$

   if we use dimensional regularization. Introducing the Feynman parameter and shifting the origin of the loop momentum integration as usual, the expression above turns into

   $$\Pi^{\mu\nu}_{(1)} = -4e^2 \int_0^1 dx \int \frac{d^n k}{(2\pi)^n} \mu^{4-n} \frac{2(k^\mu k^\nu - x(1-x)q^\mu q^\nu) + \eta^{\mu\nu}(m^2 - k^2 + x(1-x)q^2)}{[k^2 - m^2 + x(1-x)q^2]^2}.$$

   In order to proceed further with dimensional regularization, replace $k^\mu k^\nu$ with $k^2\eta^{\mu\nu}/n$, instead of $k^2\eta^{\mu\nu}/4$.

   i. Carry out the Wick rotation and integration of (Euclidean) loop momentum, to show that there is no pole at $n = 2$; although a factor $\Gamma(1 - n/2)$ arises after the loop momentum integration, this term has a coefficient $(1 - n/2)$, and hence there is no pole at $n = 2$. [This phenomenon corresponds to the absence of quadratic divergence in the vacuum polarization in QED.]

   ii. Using the relation $(1 - n/2)\Gamma(1 - n/2) = \Gamma(2 - n/2)$, show that

   $$\Pi^{\mu\nu}_{(1)} = -i(q^2\eta^{\mu\nu} - q^\mu q^\nu)\frac{e^2}{2\pi^2}\int_0^1 dx x(1-x)\Gamma\left(2 - \frac{n}{2}\right)\left(\frac{4\pi\mu^2}{m^2 - x(1-x)q^2}\right)^{2-\frac{n}{2}}. \quad (1)$$

   Miraculously, the vacuum polarization is proportional to $(q^2\eta^{\mu\nu} - q^\mu q^\nu)$, a combination expected from the gauge invariance of QED.

   iii. Confirm that the vacuum polarization in dimensional regularization and Pauli–Villars regularization have the same regulator dependence, if we adopt the following interpretation:

   $$\frac{1}{2 - \frac{n}{2}} - \gamma + \ln[(4\pi)\mu^2] \longrightarrow \ln(M^2_{\rm reg}). \quad (2)$$

(b) Consider a Yukawa theory whose Lagrangian is given in the homework problem III-2.

i. Compute the scalar self-energy $i\mathcal{M} = -i\Sigma$ at 1-loop, using the dimensional regularization. The result should be

$$i\mathcal{M} = i\frac{2|\lambda|^2}{16\pi^2}\int_0^1 dx \left(\frac{4\pi\mu^2}{m^2 - x(1-x)q^2}\right)^{2-\frac{n}{2}} \left[x(1-x)q^2\Gamma\left(2-\frac{n}{2}\right) + (m^2 - x(1-x)q^2)\Gamma\left(1-\frac{n}{2}\right)\right]. \tag{3}$$

The pole at $n = 2$ indicates the existence of quadratic divergence.

ii. The homework problem III-2 asked you to compute the same quantity by using the Pauli–Villars regularization. The result should have been

$$i\mathcal{M}^{\text{P.V.}} = i\frac{2|\lambda|^2}{16\pi^2}\int_0^1 dx \sum_{i=0}^{3}\left[\,\gamma_i(2m_i^2 - 3x(1-x)q^2)\ln[m_i^2 - x(1-x)q^2]\,\right], \tag{4}$$

where $m_0 = m$ is the fermion mass, and $m_{i=1,2,3}$ are the regulator (Pauli–Villars field) masses, and $\gamma_{i=0,3} = +1$, $\gamma_{i=1,2} = -1$; for the reason explained in the problem III-2, those mass parameters have to satisfy a relation,

$$m_0^2 + m_3^2 = m_1^2 + m_2^2. \tag{5}$$

Parametrize the regulator masses by

$$m_0^2 = m^2 \qquad m_3^2 = M^2, \qquad m_1^2 = \alpha M^2, \qquad m_2^2 = (1-\alpha)M^2 + m^2, \tag{6}$$

and rewrite the Pauli–Villars result $i\mathcal{M}^{\text{P.V.}}$ by keeping M^2, $\ln(M^2)$ and finite terms, while ignoring terms that are suppressed by powers of M.

iii. Ignore the pole at $n = 2$ (ignore the quadratic divergence) in the result of dimensional regularization, focus on the $n \to 4$ limit, and adopt the interpretation (2). Confirm that the result obtained in this way has the same coefficient for the $\ln(M_{\text{reg}}^2)$ term as in the result obtained in the Pauli–Villars calculation above.

2. **Dimensional Regularization 2 (1-Loop Calculation VI)** [B]

Consider a scalar QED, where a complex scalar field ϕ has (-1) unit of electric charge (just like an electron field Ψ):

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^\dagger(D^\mu\phi) - m^2|\phi|^2; \qquad D_\mu\phi = (\partial_\mu - ieA_\mu)\phi. \tag{7}$$

The interaction part of this Lagrangian, therefore, is given by

$$\mathcal{L}_{\text{int}} = ieA_\mu\left(\phi^*(\partial^\mu\phi) - (\partial^\mu\phi^*)\phi\right) + e^2A_\mu A^\mu|\phi|^2. \tag{8}$$

Let us use dimensional regularization in order to calculate the photon vacuum polarization in scalar QED.

(a) There are 2 Feynman diagrams contributing to the photon vacuum polarization at 1-loop level: Figure 1 (a) using the 3-point coupling (the 1st term in (8)), and Figure 1 (b) using the 4-point coupling (the 2nd term in (8)). Those two contributions are given by

$$\begin{aligned}\Pi^{\mu\nu}_{(1);3}(q) &= \int \frac{d^n k}{(2\pi)^n}\mu^{4-n}\frac{i}{k^2-m^2+i\epsilon}(ie(2k+q)^\mu)\frac{i}{(k+q)^2-m^2+i\epsilon}(ie(2k+q)^\nu),\\ \Pi^{\mu\nu}_{(1);4}(q) &= 2ie^2\eta^{\mu\nu}\int \frac{d^n k}{(2\pi)^n}\mu^{4-n}\frac{i}{k^2-m^2+i\epsilon}.\end{aligned}$$

Show that they are

$$\begin{aligned}\Pi^{\mu\nu}_{(1);3} &= \frac{ie^2}{16\pi^2}\int_0^1 dx\left(\frac{4\pi\mu^2}{m^2-x(1-x)q^2}\right)^{2-\frac{n}{2}} \qquad (9)\\ &\qquad \left[-2\eta^{\mu\nu}(m^2-x(1-x)q^2)\Gamma\left(1-\frac{n}{2}\right)+(1-2x)^2q^\mu q^\nu\Gamma\left(2-\frac{n}{2}\right)\right],\\ \Pi^{\mu\nu}_{(1);4} &= \frac{ie^2}{16\pi^2}\int_0^1 dx\left(\frac{4\pi\mu^2}{m^2}\right)^{2-\frac{n}{2}} m^2\, 2\eta^{\mu\nu}\,\Gamma\left(1-\frac{n}{2}\right). \qquad (10)\end{aligned}$$

(b) Both $\Pi^{\mu\nu}_{(1);3}$ and $\Pi^{\mu\nu}_{(1);4}$ have a pole at $n=2$, but confirm that the residue cancels when they are added up.

(c) Focus on the $n \longrightarrow 4$ limit, and show that the divergent part is

$$\Pi^{\mu\nu}_{(1);3+4} \simeq -i(q^2\eta^{\mu\nu}-q^\mu q^\nu)\frac{e^2}{16\pi^2}\times\frac{1}{3}\times\frac{1}{2-\frac{n}{2}}, \qquad (11)$$

where

$$\int_0^1 dx\; 2x(1-x) = \int_0^1 dx(1-2x)^2 = \frac{1}{3}. \qquad (12)$$

[Note that this divergent part for a complex scalar loop is 1/4 of a Dirac fermion loop (1). This means that a complex scalar contributes a quarter of a Dirac fermion (a half of a Weyl fermion) to 1-loop beta-function of QED.]

(d) Determine the complex scalar contribution to the photon vacuum polarization (at 1-loop) in the $\overline{\text{MS}}$ scheme.

(e) (if you have enough time) show that the expression above satisfies Ward identity ($\Pi^{\mu\nu}_{(1);3+4;\overline{\text{MS}}}\times q_\mu = 0$).

3. **1-Loop Calculation VII, Renormalizability and SUSY** [B]

Consider a Yukawa theory whose action is given in the homework problem III-2.

(a) There are six Feynman diagrams (like Figure 2 (a)) contributing to the scattering amplitude (matrix element) $i\mathcal{M}(\varphi+\varphi^* \longrightarrow \varphi+\varphi^*)$. Show that two of them are logarithmically divergent, while the remaining four are finite. Simple momentum cut-off

(a) (b)

Figure 1: Scalar loop contributions to the photon vacuum polarization.

(after introducing Feynman parameters and shifting the origin of the loop momentum for integration) is enough for this purpose. [The Yukawa theory without a $|\varphi|^4$ term is therefore not a renormalizable theory. If $\Delta\mathcal{L} = \lambda|\varphi|^2/4$ term (and $\Delta\mathcal{L} = g\varphi^2\varphi^* + \text{h.c.}$) is included, however, then the Yukawa + $|\varphi|^4$ (and $\varphi^2\varphi^* + \text{h.c.}$) theory becomes renormalizable. Pure Yukawa theory is not the same (at least theoretically) as the Yukawa + $|\varphi|^4$ theory where the renormalized coupling λ_r just happens to be zero.]

(b) Introduce a pair of complex scalar fields ϕ and ϕ^c, and write down the following Lagrangian, in addition to what we already have for the Yukawa + $|\varphi|^4$ theory:

$$\Delta\mathcal{L} = |\partial\phi|^2 - M_\phi^2|\phi|^2 + |\partial\phi^c|^2 - M_\phi^2|\phi^c|^2 - |y|^2|\phi|^2|\varphi|^2 - |y|^2|\phi^c|^2|\varphi|^2. \qquad (13)$$

Here, the coefficient of the last two terms, $|y|^2$, is set to be the same as those in the Yukawa interaction in the problem III-2. Compute the 1-loop contribution with the ϕ-loop and ϕ^c-loop (as in Figure 2 (b)) to the scattering amplitude $i\mathcal{M}(\varphi+\varphi^* \longrightarrow \varphi+\varphi^*)$. Simple momentum cut-off is enough.

(c) (not a problem) One will find that the 1-loop contributions from the fermion loop and scalar loop are both logarithmically divergent, and have precisely the same coefficient for the log divergence with opposite sign. Therefore, the logarithmic divergence cancels (at least at 1-loop). This phenomenon is actually due to supersymmetry [φ: Higgs scalar, Ψ: quark and ϕ and ϕ^c: scalar quark]. The logarithmic divergence is made finite at (and above) the mass scale M_ϕ of the new scalar fields ϕ and ϕ^c, because of the supersymmetric modification of the Yukawa + $|\varphi|^4$ theory at the energy scale M_ϕ. Although we often employ such regularizations as Pauli–Villars, higher covariant derivatives and dimensional regularization, true mechanism for rendering scattering amplitudes finite in QFT may be something like this. [$\mathcal{N} = 1$ supersymmetric extension of the Standard Model is known not to be sufficient in rendering all the Standard Model amplitudes UV finite, however.]

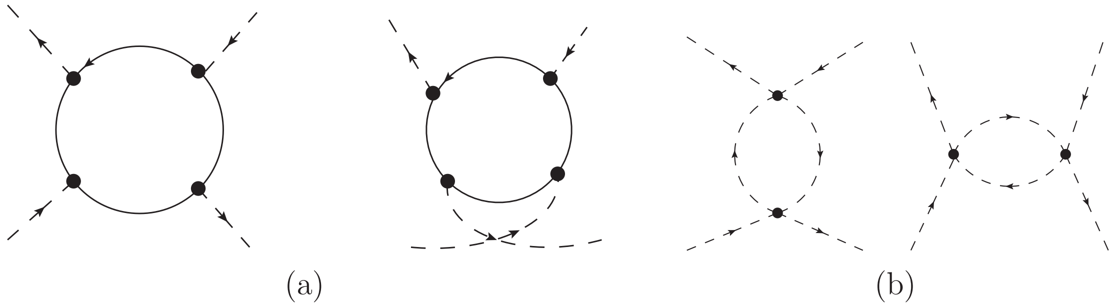


Figure 2: 1-loop contributions to the $\varphi + \varphi^* \to \varphi + \varphi^*$ scattering.

4. **Renormalized Perturbation for a Non-Renormalizable Theory** [C]

Consider a theory with a complex scalar field φ (with mass m_φ) and a Dirac fermion field Ψ (with mass m_ψ), and let their interactions be given by[1]

$$\mathcal{L}_{\text{int}} = -\frac{\lambda}{4}|\varphi|^4 + A\,\overline{\Psi}\Psi|\varphi|^2, \tag{14}$$

for now; λ and A are parameters. The naive mass dimension of the first operator is +4 (and hence marginally renormalizable), but the second operator is of dimension +5, and hence this theory is not expected to be "renormalizable". In the following, let us carry out the renormalized perturbation of this theory.

Let us first set the renormalization condition for the coupling constants λ and A, by requiring the matrix element of $\varphi+\varphi^* \to \varphi+\varphi^*$ scattering and $\varphi+\varphi^* \to \Psi+\Psi^\dagger$ scattering to be given by $-i\lambda_{(E)}$ and $iA_{(E)}(\bar{u}v)$, respectively, at certain kinematical configuration (like $s = t = -E^2$).

(a) Compute 1-loop contribution to the $\varphi + \varphi^* \to \varphi + \varphi^*$ scattering amplitude coming from the diagrams in Figure 2 (b) (in simple momentum cut-off). The result may be something like this:

$$i\mathcal{M}_{(1);\varphi} = i\frac{\lambda^2_{(E)}}{16\pi^2}\int_0^1 dx\left[\ln\left(\frac{m_\varphi^2 - x(1-x)s + \Lambda^2}{m_\varphi^2 - x(1-x)s}\right) - 1\right] + [(s \to t)]\,. \tag{15}$$

(b) Yet another group of diagrams (Figure 3) also contributes to the same scattering amplitude. Compute them in simple momentum cut-off. The result may be something like this:

$$\begin{aligned} i\mathcal{M}_{(1);\psi} &= i\frac{4A^2_{(E)}}{16\pi^2}\int_0^1 dx\left[\Lambda^2 - 3(m_\psi^2 - x(1-x)s)\ln\left(\frac{m_\psi^2 - x(1-x)s + \Lambda^2}{m_\psi^2 - x(1-x)s}\right)\right. \\ &\qquad \left. +2(m_\psi^2 - x(1-x)s)\right] \qquad + [(s \longrightarrow t)]\,. \end{aligned} \tag{16}$$

[1]This model is a simplified version of Standard Model interactions; φ for Higgs doublet, Ψ for lepton, the 1st term of (14) for Higgs quartic coupling and the 2nd term for dimension-5 neutrino mass term. This model is modified from the Standard Model interactions, in order to avoid Feynman rules for Weyl fermion.

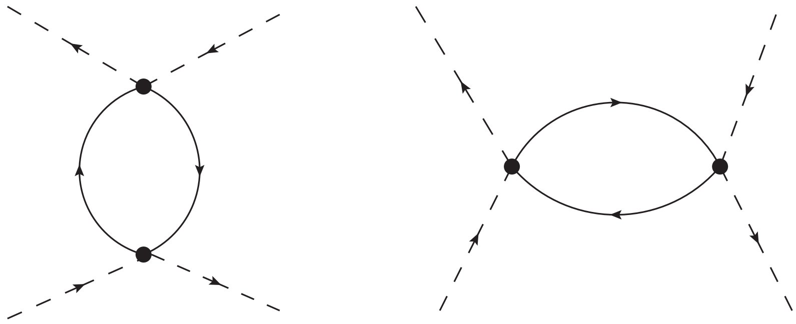


Figure 3: 1-loop contribution to the $\varphi + \varphi^* \to \varphi + \varphi^*$ scattering involving dim.-5 operators.

(c) The counter term associated with the $-(\lambda/4)|\varphi|^4$ term also contributes to this scattering amplitude. If we are to denote the normalization of the φ field by $\varphi = \sqrt{Z_2^{(E)}}\varphi_{r(E)}$, then

$$\lambda|\varphi|^4 = \lambda(Z_2^{(E)})^2|\varphi_{r(E)}|^4 \equiv (\lambda_{(E)} + \delta\lambda)|\varphi_{r(E)}|^4, \tag{17}$$

and the contribution from this counter term is given by

$$i\mathcal{M}_{(1);\text{c.t.0}} = -i(\delta\lambda). \tag{18}$$

If we are to determine the value of $(\delta\lambda)$ in terms of $\lambda_{(E)}$, $A_{(E)}$ and Λ, by enforcing the renormalization condition,

$$\left[i\mathcal{M}_{\text{tree}} = -i\lambda_{(E)}\right] + \left[i\mathcal{M}_{(1);\varphi} + i\mathcal{M}_{(1);\psi} + i\mathcal{M}_{(1);\text{c.t.0}}\right]|_{s=t=-E^2} = -i\lambda_{(E)}, \tag{19}$$

what would $\delta\lambda$ be? Confirm further that $i\mathcal{M}_{\text{tree}} + i\mathcal{M}_{(1);\varphi} + i\mathcal{M}_{(1);\psi} + i\mathcal{M}_{(1);\text{c.t.0}}$ is finite for the special kinematics $s = t = -E^2$ for the renormalization condition, but is divergent for other kinematics, if we are to set $(\delta\lambda)$ in the way specified above.

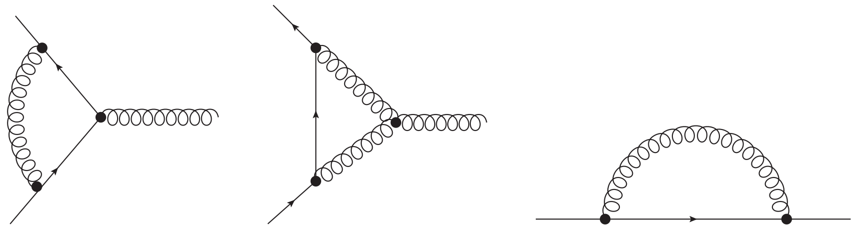
(d) A theory with Lagragian (14) is therefore not renormalizable, but the divergence we encountered above can be removed (absorbed by the redefinition of coupling constants), if we are to add one more term in the Lagrangian:

$$\mathcal{L}_{\text{int}} = -\frac{\lambda}{4}|\varphi|^4 + A\,\overline{\Psi}\Psi|\varphi|^2 + \frac{B}{2}\,|\varphi|^2(-\partial^2|\varphi|^2). \tag{20}$$

There is an extra contribution

$$i\mathcal{M}_{(1);B} = i(B_{(E)} + \delta B)(s + t). \tag{21}$$

Determine $\delta\lambda$ and δB so that the amplitude at this level remains finite for arbitrary value of kinematical variables (s, t). [There is a remaining ambiguity in changing $B_{(E)}$ and δB by finite amount, so that the combination $B_{(E)} + \delta B$ does not change. This scheme dependence still remains.] [Such a procedure can be carried out order by order; in this sense, this "non-renormalizable theory" in the historical sense is renormalizable.]



(a) (b) (c)

Figure 4: (a)+(b) vertex corrections in non-Abelian gauge theories, and (c) wavefunction renormalization of a fermion.

(e) Confirm that the quantum corrections to the $\varphi + \varphi^* \to \varphi + \varphi^*$ scattering amplitude from non-renormalizable operators (terms involving $A_{(E)}$ and $B_{(E)}$) are much smaller than the corrections coming from renormalizable operators (terms involving $\lambda^2_{(E)}$), when $A_{(E)} \sim 1/M$ and $B_{(E)} \sim 1/M^2$ for some mass scale M much higher than m_ψ, m_φ, $\sqrt{s}$ and $\sqrt{|t|}$. In this sense, quantum corrections from non-renormalizable operators to processes that exist already within the renormalizable part of a theory is quite *irrelevant*.

(f) Derive renormalization group equation for $\lambda_{(E)}$ and $B_{(E)}$, by requiring that the renormalized scattering amplitude does not depend on the choice of renormalization scale E (for a given choice of kinematical variable s, t).

5. **β-function in Non-Abelian Gauge Theories** [B]

Let us derive the β-function of gauge coupling constant of non-Abelian gauge theories, and show that non-Abelian gauge theories can be asymptotically free.

In non-Abelian gauge theories, there are two diagrams contributing to the 1-loop vertex correction, as in Figure 4 (a), (b). Its divergent part can be calculated easily in dimensional regularization, and the result is as follows:

$$(-igt^a)\Gamma^\mu_{(1);(a)} \sim (-igt^a)\gamma^\mu \frac{g^2}{(4\pi)^2}\Gamma\left(2-\frac{n}{2}\right)\left[C_2(R)-\frac{1}{2}T_G\right][1+(\xi-1)], \qquad (22)$$

$$(-igt^a)\Gamma^\mu_{(1);(b)} \sim (-igt^a)\gamma^\mu \frac{g^2}{(4\pi)^2}\Gamma\left(2-\frac{n}{2}\right)\left[\frac{1}{2}T_G\right]\left[3+\frac{3}{2}(\xi-1)\right]; \qquad (23)$$

here, g is the gauge coupling constant, and ξ is the gauge parameter of vector boson propagator ($\xi = 1$ corresponds to the Feynman gauge). T_G is a constant that depends on the choice of gauge group G; $T_G = N$ for $G = \mathrm{SU}(N)$ gauge theories, while $T_G = 0$ in Abelian gauge theories (like QED). $C_2(R)$ is a constant that depends on the particle species we consider in the vertex correction in Figure 4 (a, b). It corresponds to $(Q_f)^2$, electric charge squared in the case of QED. Before taking account of energy scale dependent field renormalization, the

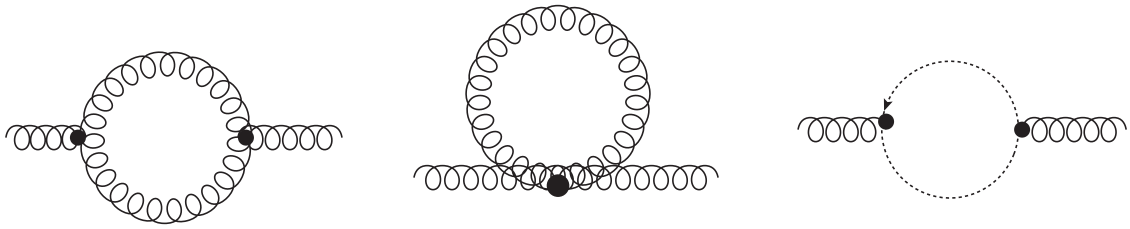


Figure 5: Vacuum polarization for a non-Abelian vector field.

coupling constant at energy scale μ would be given by

$$(-igt^a)\left[\gamma^\mu + \Gamma^\mu_{(1);(a+b)}|_{(2-n/2)^{-1}\to\ln(\Lambda^2/q^2)} - \text{c.t.}\right], \tag{24}$$

where c.t stands for the counter terms $\propto \ln(\Lambda^2/\mu^2)$, which just replace $\ln(\Lambda^2)$ by $\ln(\mu^2)$ in the end.

The divergent part of Fermion self-energy in Figure 4 (c) is given by

$$i\mathcal{M}_{(1);(c)} = -i\Sigma \sim i\not{p}\,\frac{g^2}{(4\pi)^2}\Gamma\left(2-\frac{n}{2}\right)C_2(R)\left[1+(\xi-1)\right] \sim -i\not{p}\,A_{(1)}. \tag{25}$$

This means

$$Z_2 \sim 1 + A_{(1)}|_{(2-n/2)^{-1}\to\ln(\Lambda^2/p^2)}, \qquad \frac{Z_2}{Z_2^{(\mu)}} \sim \frac{1+A_{(1)}|_{(2-n/2)^{-1}\to\ln(\Lambda^2/p^2)}}{1+A_{(1)}|_{(2-n/2)^{-1}\to\ln(\Lambda^2/\mu^2)}}. \tag{26}$$

The vacuum polarization of the vector field arises from vector / ghost loop (Figure 5), Dirac fermion loop, and scalar loop (like in Figure 1); their divergent part are given by

$$\Pi_{(1);(d)} \sim \frac{g^2}{(4\pi)^2}\Gamma\left(2-\frac{n}{2}\right)\left(\frac{13}{6}-\frac{\xi}{2}\right)T_G, \tag{27}$$

$$\Pi_{(1);(e)} \sim \frac{g^2}{(4\pi)^2}\Gamma\left(2-\frac{n}{2}\right)\left(-\frac{4}{3}\right)\sum_f T_{Rf}, \tag{28}$$

where f runs over different Dirac fermions in the representation R_f under the gauge group. If a complex scalar field in representation R is in a theory, its contribution is 1/4 of that of Dirac fermion, just like in QED (see the problem VII-2). This means that

$$Z_3 \sim 1 + \Pi_{(1)}|_{(2-n/2)^{-1}\to\ln(\Lambda^2/q^2)}, \qquad \frac{Z_3}{Z_3^{(\mu)}} \sim \frac{1+\Pi_{(1)}|_{(2-n/2)^{-1}\to\ln(\Lambda^2/q^2)}}{1+\Pi_{(1)}|_{(2-n/2)^{-1}\to\ln(\Lambda^2/\mu^2)}}. \tag{29}$$

(a) Use those information to compute the beta-function. It should be

$$\beta_g \equiv \frac{\partial g}{\ln(E^2)} = -\frac{1}{2}g\frac{g^2}{(4\pi)^2}b_0 \qquad \Longleftrightarrow \qquad \frac{\partial}{\partial\ln(E)}\left(\frac{1}{\alpha}\right) = \frac{b_0}{2\pi}, \tag{30}$$

with

$$b_0 = \frac{11}{3} T_G - \sum_{i \in Wf} \frac{2}{3} T_{R_i} - \sum_{j \in s} \frac{1}{3} T_{R_j}. \tag{31}$$

Here, i runs over all the Weyl fermion species in a non-Abelian gauge theory (like, left-handed and right-handed u-quarks, those of d-quarks etc. in QCD ($G = \mathrm{SU}(3)$)), and j over all complex scalar fields charged under the gauge group G (like the Higgs boson for $G = \mathrm{SU}(2) \times \mathrm{U}(1)$). Note that b_0 can be positive (and hence β_g can be negative) in non-Abelian gauge theories, because $T_G > 0$.

(b) (if you have enough time) It may be fun to derive (22, 23, 25, 27, 28). [Explicit calculation is found in the Peskin–Schroeder textbook section 16.5, where the Feynman gauge $\xi = 1$ is used.]

(c) (not a problem) Note that the beta function does not depend on the fermion we chose in the Feynman diagram Figure 4. Although particle spiecies dependence remains through $C_2(R)$ in (22, 25), they eventually cancel, and do not remain in b_0. Note also that the gauge parameter dependence still remains in Z_2, Z_3 etc., but they also cancel eventually in b_0.

6. **2-loop in the ϕ^4 theory** [C]

The Peskin–Schroeder textbook Chapter 10.5 explains how 2-loop computations can be carried out in dimensional regularization, using the four-point function in the $\lambda\phi^4$ theory. It will take much less time to go throught it, if you just try to follow the logic, without trying to verify all the details in the calculation. Now, here is a homework (report) problem: what should the $\mathcal{O}(1/\epsilon^2)$ term be in the $\mathcal{O}(\lambda^3)$ counter term $\delta_\lambda^{(2)}$? [It is all right to trust and use all the results written in Chapter 10.5 of Peskin–Schroeder without verifying for yourself.]