

# A Quick Lecture on "Compactification for Physics in $\mathbb{R}^{3,1}$ "

## §1 Compactification in <sup>10D</sup> SUGRA Language

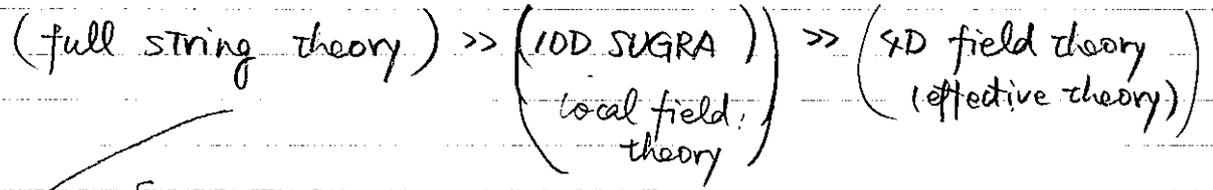
(Ref: GSW §13-16 for §1.1-1.3) ①

String vacua constructed via (free boson  $\mathbb{R}^{3,1}$ ) ⊗ (unitary compact CFT w/ modular invariance) are all qualified for the description of this universe.

"compact" <sup>def.</sup> = the spectrum of conformal weights is discrete.

OK. if  $\Delta h \gtrsim \alpha' \times (\text{TeV})^2$  ← LHC.

No guarantee that there is a cascade of hierarchy.



⊛ eg.  $S^1$ -compactification @ self-dual radius.  
 $\rightarrow$   $SU(2)$  gauge field in  $(8+1)$ -dim effective theory.  
 (24+1)-dim theory.  
 [ more examples ]

The subset of string vacua with the cascade of hierarchy ⊛ allows intuitive understanding of how the low-energy effective theory in  $\mathbb{R}^{3,1}$  is determined.

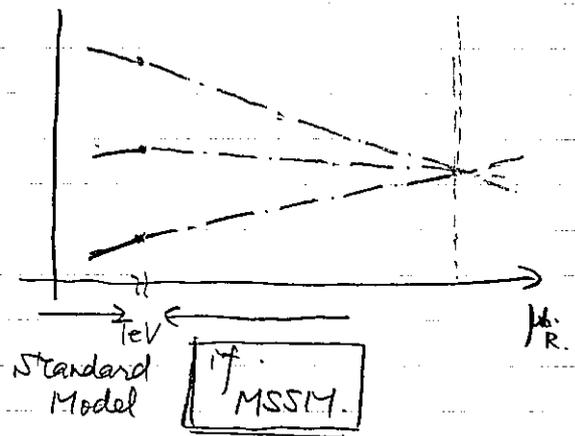
(+ motivated by SUSY GUT  $\Rightarrow$  §2.1 (doublet-triplet splitting problem.)

so... we focus on this subset of string vacua.

# §1.1 Bottom-up Motivations for Low-energy SUSY

◦ gauge coupling unification

is observed, if we assume that the spectrum of the SM is supersymmetrized soon above the TeV scale.



$$\frac{\partial}{\partial \ln \mu} \left( \frac{1}{\alpha_i(\mu)} \right) = \frac{b_i}{2\pi} \leftarrow \text{at 1-loop}$$

depends on the matter contents.

↑ more solid

◦ a stable particle for dark matter

The dark matter particle needs to be (almost) stable, and has to have been produced by the amount  $\rho_{DM}$  consistent with cosmological measurements today.

★ 100 GeV - 100 TeV stable particle thermal relic.  
(Griest Kamionkowski '90 PRL)

★ axion (a solution to the strong CP problem) is an alternative.

↓ less solid.

◦ Naturalness problem (a strategic question)

§ 1.2. Special Holonomy Mfds and Partial Supersymmetry Breaking

Remember in D=4 N=1 SUSY gauge theories...

$$\delta\lambda_\alpha = -\frac{1}{4} F_{\mu\nu} (\sigma^{\mu\nu})_{\alpha\beta} \xi^\beta + D \xi_\alpha$$

$$\delta\bar{\lambda}_{\dot{\alpha}} = -\frac{1}{4} F_{\mu\nu} (\bar{\sigma}^{\mu\nu})^{\dot{\alpha}\dot{\beta}} \bar{\xi}_{\dot{\beta}} + D \bar{\xi}^{\dot{\alpha}}$$

if  $\langle D \rangle = 0$  and  $\langle F_{\mu\nu} (\bar{\sigma}^{\mu\nu})^{\dot{\alpha}\dot{\beta}} \rangle = 0$  but  $\langle F_{\mu\nu} (\sigma^{\mu\nu})_{\alpha\beta} \rangle \neq 0$ .

(instanton configuration)

then half of the 4 SUSY charges remain unbroken:

- ↳ Euclidean 4-dim:  $SO(4) \cong SU(2) \times SU(2)$ 
  - $\xi_\alpha, \bar{\xi}_{\dot{\alpha}}$  pseudo real
  - $2 + 2$  (symplectic)
  - = 4 SUSY charges
- ↳  $SO(4)$  rotational symmetry is lost at the same time.

Analogy:

$\mathbb{R}^{3,1}$  replaced by  $M \times \mathbb{R}^{3,1}$   
 $\uparrow$  6-dim mfd.

non-trivial background (=vac. value) configuration of metric & gauge fields on M.

- may [ break  $SO(6)$  symmetry but preserve  $SO(3,1)$
- break a part of SUSY but preserve the rest of the SUSY ]

An example: Heterotic string

10D SUGRA action ( $\alpha'$ -expansion)

Polchinski §12.1 (12.1.39) } bosonic part only  
 string frame

$$\frac{1}{\sqrt{-\det(g)}} \mathcal{L} = \frac{1}{2\kappa_{10}^2} e^{-2\phi} \left[ R + 8(\partial\phi)^2 - \frac{1}{2} |\tilde{H}_3|^2 - (\dots) \text{Tr}(|F|^2) \right]$$

+(fermionic)

GSW §13 p.313 + p.323 } incl. fermionic terms  
 Einstein frame.

bosonic DOF  $(g_{MN}, \phi, B_{MN}) + (F_{MN}^a)$   
 massless

fermionic DOF  $(\Psi_M, \lambda) + (\chi^a)$   
 56 + 8 on-shell

SUSY transformation (10D N=1 SUGRA) (GSW p.416 (815))

$$\begin{cases} \delta \Psi_M = \frac{1}{\kappa_{10}} (\mathcal{D}_M \eta) + (\dots) e^{-\phi/2} (\Gamma's)_M{}^{NPQ} \eta \tilde{H}_{NPQ} + \dots \\ \delta \lambda = -\frac{1}{2\kappa_{10}^2} \Gamma^M (\partial_M \phi) \eta + (\dots) e^{-\phi/2} \Gamma^{MNP} \tilde{H}_{MNP} \eta + \dots \\ \delta \chi^a = -\frac{1}{4} (\dots) e^{-\phi/2} \Gamma^{MN} F_{MN}^a \eta + \dots \end{cases}$$

< fermionic terms, higher-order terms in  $\alpha'$  ignored >

(Ref: Polchinski appendix B.)

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$\eta$ : SUSY transformation parameter in 16-repr. of  $SO(9,1)$

Majorana Weyl spinor

In the irr. decomposition

$$SO(9,1) \longrightarrow SO(3,1) \times SO(6) \overset{\sim}{\sim} SU(4)$$

$$\left( \begin{array}{l} 16 \longrightarrow \left[ \begin{array}{l} (2_L, 1) \otimes \chi \\ \oplus (1, 2_R) \otimes \bar{\chi} \end{array} \right] \left. \begin{array}{l} \\ \end{array} \right\} \text{CPT} \\ \\ 16' \longrightarrow \left[ \begin{array}{l} (2_L, 1) \otimes \bar{\chi} \\ \oplus (1, 2_R) \otimes \chi \end{array} \right] \left. \begin{array}{l} \\ \end{array} \right\} \text{CPT} \end{array} \right)$$

$$SO(9,1) \longrightarrow SO(5,1) \times (SU(2) \times SU(2) \simeq SO(4))$$

$$\left( \begin{array}{l} 16 \longrightarrow \left[ \begin{array}{l} \chi \otimes (2, 1) \\ \oplus \chi' \otimes (1, 2) \end{array} \right] \left. \begin{array}{l} \supset \text{CPT} \\ \supset \text{CPT} \end{array} \right. \\ \\ 16' \longrightarrow \left[ \begin{array}{l} \chi \otimes (1, 2) \\ \oplus \chi' \otimes (2, 1) \end{array} \right] \left. \begin{array}{l} \supset \\ \supset \end{array} \right)$$

Via the separation of variables  $\eta(x, y) \Rightarrow \epsilon_\alpha(x) \otimes \eta_\alpha(y) + \bar{\epsilon}_{\dot{\alpha}}(x) \otimes \bar{\eta}_{\dot{\alpha}}(y)$   
 ((10D  $\rightarrow$   $\mathbb{R}^{3,1} \times$  (6D internal))

$$\begin{aligned} \psi_\mu(x, y) &\longrightarrow \psi_{\mu\alpha}(x) \eta_\alpha(y) + \bar{\psi}_{\mu\dot{\alpha}}(x) \bar{\eta}_{\dot{\alpha}}(y) \\ (\delta \psi_{\mu\alpha}(x)) \eta_\alpha(y) &= \frac{1}{\kappa_{10}} (D_\mu \epsilon_\alpha) \eta_\alpha(y) + \dots \quad \text{as in 4D SUGRA.} \\ \bar{\psi}_m(x, y) &\longrightarrow \epsilon_{\dot{\alpha}}(x) \cdot \bar{\psi}_m(y) + \bar{\epsilon}_{\dot{\alpha}}(x) \bar{\psi}_m(y) \\ \epsilon_{\dot{\alpha}}(x) \cdot (\delta \bar{\psi}_m(y)) &= \frac{1}{\kappa_{10}} \epsilon_{\dot{\alpha}}(x) \cdot (D_m \bar{\eta}_{\dot{\alpha}})(y) + \dots \end{aligned}$$

$\Rightarrow$  independent covariantly constant spinors  $(D_m \eta_{\dot{\alpha}}) = 0$  on  $M$   
 $i=1, 2, \dots, N$

give rise to  $\eta(x, y) \Rightarrow \epsilon_{\dot{\alpha}}(x) \eta_{\dot{\alpha}}(y) + c.c.$   
 $N$  extended SUSY in  $(3+1)$ -dim.  $\mathbb{R}^{3,1}$

$\delta_{Dir}$  internal mfd

Special holonomy manifold  $M_m$

Levi-Civita connection takes its value in  $\mathfrak{g} \subset \mathfrak{SO}(n)$ .  
 (Christoffel symbol)

$$0 = D_m \eta = \left( \partial_m \delta^\alpha_\beta + \omega_m^{ab} (S^{ab})^\alpha_\beta \right) \eta^\beta = 0.$$

$a, \beta$ : spinor index  
 $a, b, = 1, \dots, n$   
 $m = 1, \dots, n$   
 so, if  $\langle \omega_m^{ab}(\gamma) \rangle$  is less than  $\mathfrak{SO}(n)$  at any point in  $M_n$ ,  
 there is a chance to find  $\eta_0 \neq 0$  w/  $D_m \eta_0 = 0$

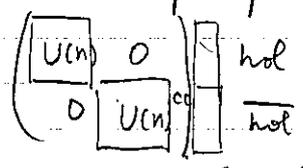
★ Kähler manifold ( $\Rightarrow$  cpx mfd by def)

$$U(n) \text{ holonomy} \subset SO(2n)$$

$$\Gamma_{\beta\gamma}^\alpha \neq 0, \quad \Gamma_{\bar{\beta}\bar{\gamma}}^{\bar{\alpha}} \neq 0$$

but all other  $\Gamma_{ne}^m$ 's vanish.

in the vector repr. of  $SO(2n)$



★ Calabi-Yau cpx n-fold (real 2n dimension)

$$SU(n) \subset U(n) \subset SO(2n)$$

$$\begin{matrix} SU(4) \\ \mathbb{R} \\ SO(6) \end{matrix} \rightarrow U(3) = SU(3) \times U(1)$$

n=3 the spinor repr. of  $SO(6) \cong SU(4)$   $4 \oplus \bar{4} \rightarrow (1 \oplus 3) \oplus (1 \oplus \bar{3})$

$$(|\uparrow\uparrow\uparrow\rangle, |\uparrow\downarrow\downarrow\rangle, |\downarrow\uparrow\downarrow\rangle, |\downarrow\downarrow\uparrow\rangle) \oplus (|\downarrow\downarrow\downarrow\rangle, |\downarrow\uparrow\uparrow\rangle, |\uparrow\downarrow\uparrow\rangle, |\uparrow\uparrow\downarrow\rangle)$$

$$({}^{\wedge^3}TM)^{\frac{1}{2}} \oplus ({}^{\wedge^3}TM)^{-\frac{1}{2}} \oplus (TM) \qquad ({}^{\wedge^3}TM)^{-\frac{1}{2}} \oplus ({}^{\wedge^3}TM)^{+\frac{1}{2}} \oplus (T^*M)$$

$S^+$  (rank=4 spinor bundle over  $M$ )  $S^-$

$SU(3)$  holonomy  $\Leftrightarrow D_m \eta_0 = (2m + \omega_m) \eta_0$  (4x4)  
 $\hookrightarrow$  value in (3x3) part  
 $\Leftrightarrow ({}^{\wedge^3}TM)$  is a trivial bundle.

n=2 (complex surface)

The spinor repr. of  $SO(4) \simeq SU(2) \times SU(2)$  is  $(2,1) \oplus (1,2)$

$(|\uparrow\uparrow\rangle, |\downarrow\downarrow\rangle) \oplus (|\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle)$

$\parallel$   
 $S_+^{\frac{1}{2}} = (\wedge^2 TM)^{\frac{1}{2}} \oplus (T^*M)^{\frac{1}{2}}$        $\parallel$   
 $S_-^{\frac{1}{2}} = (\wedge^2 TM)^{\frac{1}{2}} \otimes TM = (\wedge^2 TM)^{\frac{1}{2}} \otimes (T^*M)$

If  $\langle \omega_m \rangle$  the spin connect'n is with in

$SU(2) \subset U(2) \subset SO(4)$   
 $\hookrightarrow$  one of the two  $SU(2) \times SU(2) \simeq SO(4)$

then  $D_m \eta_0 = 0$  condition becomes  $d\eta_0 = 0$   
in the other  $SU(2)$  doublet.

n=1 :  $T^2$  (complex torus)  $\Rightarrow$  transl.-inv. metric  $\Rightarrow \langle \omega_m \rangle = 0$

All the SUSY charges remain unbroken.

- [ Heter/T<sup>2</sup>  $\Rightarrow$  16 SUSY charges in (7+1)-dim eff. theory
- [ Type II/T<sup>2</sup>  $\Rightarrow$  32 SUSY charges in (7+1)-dim eff. theory

n=2 : K3 surface (cpx surface M w/ trivial  $(\wedge^2 TM)$  &  $h^{1,0} = 0$ )

- [ Heter/K3  $\Rightarrow$  (0,1) SUSY
  - [ Type IIA/K3  $\Rightarrow$  (1,1) SUSY in (5+1)-dim eff theory
  - [ Type IIB/K3  $\Rightarrow$  (0,2) SUSY
- $\uparrow$   $\uparrow$   
# SUSY charges in  $\mathfrak{g}$  of  $SO(5,1)$       # SUSY charges in  $\mathfrak{g}$  of  $SO(5,1)$

If  $h^{1,0} > 0 \Rightarrow D_m \eta = 0$  w/  $\eta \in (T^*M \simeq S_-)$ . More SUSY charges in (5+1)-dim. theory.

(What is the possible value of  $h^{1,0}(M)$  if  $(\wedge^2 TM)$  is trivial? K3=M  $\Rightarrow$   $h^{1,0}=0$  T<sup>2</sup>=M  $\Rightarrow$   $h^{1,0}=2$ . what else?)

n=3 Calabi-Yau 3-fold  $\left( \begin{array}{l} \text{cpx 3-dim. Kähler } M \text{ w/} \\ \text{trivial } (N^3 TM) \text{ \& } h^{1,0} = 0 \end{array} \right)$

- Het/ $CY_3 \Rightarrow N=1$  SUSY
- IIA/ $CY_3 \Rightarrow N=2$  SUSY in (3+1)-dim effective theory
- IIB/ $CY_3 \Rightarrow N=2$  SUSY

If  $h^{1,0} > 0$  then the eff. theory has  $\overset{h^{1,0} (x2)}{\text{more SUSY charges}}$ .

- $M = K3 \times T^2 \Rightarrow h^{1,0} = 1 \quad N = 1+1 \quad (2+2)$
- $M = T^6 \Rightarrow h^{1,0} = 3 \quad N = 1+3 \quad (2+6)$
- what else?

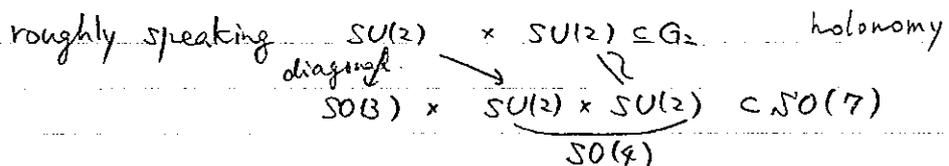
★  $G_2$ -holonomy:  $M$  is a real mfd of  $\dim = 7$ .

$SO(7)$  has just one spinor repr  $\underline{\delta}$ .

(see E. Witten hep-th/0108165 pp. 7-8)

$\text{Res}_{G_2}^{SO(7)}(\underline{\delta}) = \underline{7} \oplus \underline{1}$

(32 D SUGRA)/ $M \Rightarrow N=1$  SUSY charges in (3+1)-dim eff. theory  
 $(32 \times \frac{1}{8} = 4)$



spinor  $\underline{\delta} = \left( \underline{2} \otimes \underline{2} \otimes \underline{1} \right) \oplus \left( \underline{2} \otimes \underline{1} \otimes \underline{2} \right)$

↑ non-trivial holonomy

$1 \oplus \underline{3}$  under the diagonal  $SU(2) \subseteq SO(3) \times SU(2)$ .

↑ non-trivial holonomy  
 trivial holonomy cov. const. spinor lives here.

### § 1.3. Low-energy Spectrum of Closed String

IIA 10D SUGRA:

$$(g_{MN}, \phi, B_{MN}; C^{(4)}, C^{(3)})_{\text{boson}} + (\text{fermion})$$

We just need to work out bosonic DOF's in a supersymmetric compactification. (SUSY determines fermionic spectrum.)

Consider IIA/ $M_6 = CY_3$

$$\left\{ \begin{array}{l} M_6 = \text{cpx str } \& \text{ Kähler metric given } \Leftrightarrow \langle g_{mn}(y) \rangle \neq 0 \text{ given.} \\ \langle \phi \rangle = \text{const. over } M_6. \quad \langle B_{mn}(y) \rangle \neq 0 \text{ harmonic form.} \\ \langle C^{(4)} \rangle = \langle C^{(3)} \rangle = 0. \end{array} \right.$$

a sufficient condition for  $N=2$  SUSY in (3+1)-dim. effective theory.

$$(\text{field})(x,y) = \langle \text{field} \rangle(y) + (\text{field fluctuation})(x,y)$$

$$\begin{array}{c} \nearrow \\ \sum_i (\text{low-energy field})_i(x) (\text{wave fun})_i(y) \\ \text{separat'n of variables.} \end{array}$$

In many cases (with an appropriate gauge fixing)

the equations on the wave functions after the separation of variables is the Laplace equation.

$$\left\{ \begin{array}{l} (\delta\phi)(x,y) = \phi(x) \cdot 1 + (\text{KK-modes}) \\ (\delta B_{\mu\nu})(x,y) = b_{\mu\nu}(x) \cdot 1 + (\text{KK-modes}) \\ (\delta B_{\mu n})(x,y) = (\text{KK-modes}) \quad \text{no harmonic 1-form on } M \quad (h^{1,0}(M)=0) \\ (\delta B_{mn})(x,y) = \sum_{a=1}^{h^{1,1}(M)} \left( (\omega^a)_{mn}(y) \beta^a(x) \right) + (\text{KK-modes}) \\ \left( H^{1,1}(M; \mathbb{R}) = \text{Span}_{\mathbb{R}} \{ \omega^a \mid a=1,2,\dots, h^{1,1}(M) \} \right) \end{array} \right.$$

- $(\delta C_{\mu}^{(1)}) = c_{\mu}(x) \cdot 1 + (\text{KK-modes})$
- $(\delta C_m^{(1)}) = (\text{KK-modes}) \leftarrow (\text{no harmonic 1-form on } M)$
- $(\delta C_{\mu\nu\lambda}^{(3)}) = \text{non dynamical} + (\text{KK-modes})$
- $(\delta C_{\mu\nu m}^{(3)}) = (\text{KK-modes})$
- $(\delta C_{\mu mn}^{(3)}) = \sum_{a=1}^{h^{1,1}} (A_{\mu}^a(x) (\omega_{mn}^a)(y)) + (\text{KK-modes})$
- $(\delta C_{mnl}^{(3)}) = \sum_{i=1}^{b_3(M)} (C_{\bar{z}}(x) (\gamma_{i, mnl}^{\bar{z}})(y)) + (\text{KK-modes})$
- $(\delta g_{\mu\nu}) = (\mathbb{R}^{3,1} \text{ graviton}) + (\text{KK-modes})$
- $(\delta g_{\mu n}) = (\text{KK-modes})$
- $(\delta g_{\alpha\bar{\beta}}) = \sum_{a=1}^{h^{1,1}} (\alpha^a(x) \omega_{\alpha\bar{\beta}}^a(y)) + (\text{KK-modes})$
- $(\delta g_{\alpha\bar{\beta}}) = \sum_{i=1}^{h^{2,1}(M)} (z_i(x) \langle g_{\alpha\bar{\beta}} \rangle(y) (J_i)^{\delta}_{\bar{\beta}}(y)) + (\text{KK-modes})$

$\{J_i\}$  : basis of  $H^2(M; TM)$   
 $\Downarrow$  Calabi-Yau.  
 $\{J_i\}$  basis of  $H^{2,1}(M; \mathbb{C})$  ( $3 \approx 1^2 \bar{3}$  in  $SU(3)$ )

via  $\left[ (d\bar{z}^{\bar{\beta}} J^{\delta}_{\bar{\beta}}) \frac{\partial}{\partial z^{\delta}} \right] \mapsto \Omega_{\alpha\gamma\delta} J^{\delta}_{\bar{\beta}} = \gamma_{\alpha\gamma\bar{\beta}}$   
 $\left[ (\gamma_{\alpha\gamma\bar{\beta}} dz^{\alpha} dz^{\gamma} d\bar{z}^{\bar{\beta}}) \right]$

the tangent space of the moduli space of complex structure of  $M$ .

( K. Kodaira } "Cplx Mfd" [Iwanami Pub. Co.]  
 Lecture note @ U. Tokyo.  
 「複素多様体と複素構造の変形」

Refs. E. Witten. Phys. Lett. B155 ('85) 151.  
Grimm Louis th/0403067.

SUMMARY

IIA / (M<sub>6</sub> = C^3)

massless fields in R<sup>3,1</sup>

(graviton) . C <sub>μ</sub> (from C <sup>(1)</sup> )	TR <sup>3,1</sup> N=2 SUSY
h <sup>1,1</sup> × (A <sub>μ</sub> <sup>a</sup> (from C <sup>(3)</sup> ), α <sup>a</sup> (metric), β <sup>a</sup> (B <sup>(2)</sup> ))	sugra mult.
h <sup>2,1</sup> × (z <sup>i</sup> (from δg <sub>z<math>\bar{z}</math>), C<sub>i</sub>, C<sub>i'</sub> (from C<sup>(3)</sup>))</sub>	vector mult.
(φ, b <sub>μν</sub> , C <sub>100</sub> , C <sub>1'0</sub> )	hyper mult.

linear/tensor  
(mult)  
≈ hyper mult.

h<sup>1,1</sup>(M<sub>6</sub>) vector multiplets U(1)  
h<sup>2,1</sup>(M<sub>6</sub>) + 1 hyper multiplets

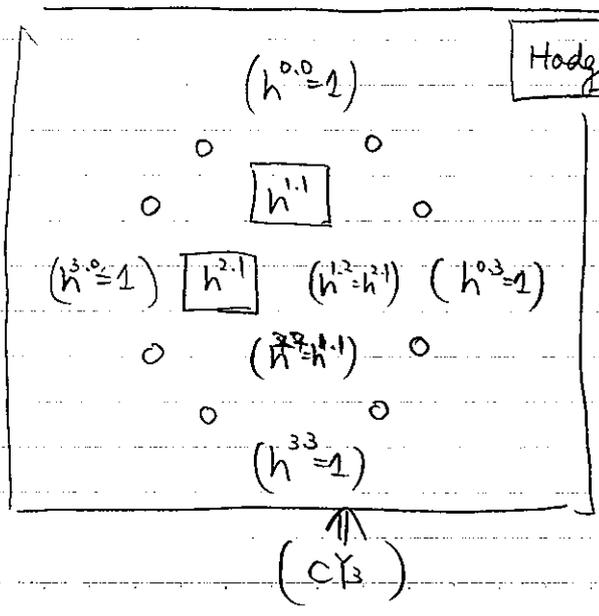
mirror symmetry

II B / (W<sub>6</sub> = C^3) then

(g<sub>MN</sub>, φ, B<sub>MN</sub>; c<sup>(0)</sup>, c<sup>(2)</sup>, c<sup>(4)</sup>) ⇒

TR<sup>3,1</sup> N=2 SUSY

1 SUGRA multiplet  
h<sup>2,1</sup>(W<sub>6</sub>) vector multiplets U(1)  
h<sup>1,1</sup>(W<sub>6</sub>) + 1 hyper multiplets



h<sup>p,q</sup> := dim<sub>C</sub> (H<sup>p,q</sup>(M; C))

H<sup>p,q</sup>(M; C) ≈ H<sup>q</sup>(M; N<sup>p</sup>T<sup>\*</sup>M)  
if M: Kähler mfd.

• h<sup>p,q</sup> = h<sup>q,p</sup> (complex conjugation)

if C<sub>n</sub>: { h<sup>n,0</sup> = 1  
h<sup>p,q</sup> = h<sup>n-p,n-q</sup>  
h<sup>n,q</sup> = h<sup>0,q</sup>

← (N<sup>n</sup>T<sup>\*</sup>M is trivial)  
← (Serre duality)

Het 10D SUGRA

$(g_{MN}, \phi, B_{MN}, A_M^a)$

↳ gauge field in  $SO(32)$  or  $E_8 \times E_8$ .

A class of  $N=1$  SUSY preserving background configurat'n

$$\begin{cases} M_6 = CY_3. & (\text{Cpx str \& Kähler metric given.}) \\ \langle \phi \rangle = \text{const.} & \langle B^{(2)} \rangle \text{ harmonic on } M_6. \\ F_{\alpha\beta}^a = F_{\bar{\alpha}\bar{\beta}}^a = 0. & F_{\alpha\beta}^a g^{\alpha\bar{\beta}} = 0 \text{ on } M_6. \end{cases} \quad (*)$$

sufficient condition for  $N=1$ . (not necessary)

(\*) needs to be subject to a constraint.

$c_2(TM) = c_2(V)$

↳ the 2nd Chern class of the vector bundle that  $\langle F_{\alpha\beta}^a \rangle \neq 0$  determines.

$N=1$  SUSY  
The spectrum in  $\mathbb{R}^{3,1}$  (in the repr.  $\frac{1}{30} \text{Tr adj.}$ )

- SUGRA multiplet  $(g_{\mu\nu})$
  - $h^{1,1}$  chiral multiplet  $((\alpha^a(x) + i\beta^a(x))$  from  $(\delta g_{\alpha\beta}, \delta B_{\alpha\bar{\beta}}) \propto \omega_{\alpha\bar{\beta}}^a$ )  
Kähler (moduli)
  - $h^{2,1}$  " " "  $(z^i$  from  $(\delta g_{\alpha\bar{\beta}}) \propto \langle g_{\alpha\bar{\beta}} \rangle J_{\bar{\beta}}^{\alpha}$ )  
Cpx str (moduli)
  - 1 chiral multiplet  $(\phi_{40}, b_{\mu\nu})$  ...  $(\phi_{40} = \delta\phi_{10D} + \text{lin comb of } \alpha^a)$   
universal
- and....

if  $G_{\text{str}} \times H \subset G$  and  $\text{Res}_{G_{\text{str}} \times H}^G(\text{adj}) = (\text{adj}, 1) + (1, \text{adj}) + \bigoplus_{a \in A} (P_a, R_a)$

then.

- 1. vector multiplet  $A_{\mu}^a$  in  $(1, \text{adj})$
  - $H^2(M_6; P_a(V))$  chiral multiplets in repr.  $R_a$  of  $H$
  - $H^2(M_6; \text{adj}(V))$  chiral multiplets neutral under  $H$
- vect. bundle (moduli)

Comment

- Ricci flat & Kähler  $\implies \det(TM)$  is trivial  
( $c_2(TM)$  is)
- $\det(TM)$  is trivial & Kähler  $\implies$  Ricci flat.

Kähler  $\implies R_{\alpha\bar{\beta}\gamma\bar{\delta}} = K_{\alpha\bar{\gamma}}R_{\beta\bar{\delta}} - K_{\alpha\bar{\delta}}R_{\beta\bar{\gamma}} - K_{\beta\bar{\gamma}}R_{\alpha\bar{\delta}} + K_{\beta\bar{\delta}}R_{\alpha\bar{\gamma}}$

so...

(Wess Bagger appendix C)

$$R_{\gamma\bar{\delta}} = R^{\alpha\bar{\beta}}R_{\alpha\bar{\beta}\gamma\bar{\delta}} = K^{\beta\bar{\alpha}}R_{\alpha\bar{\beta}\gamma\bar{\delta}} = K^{\beta\bar{\alpha}}R_{\alpha\bar{\beta}\delta\bar{\gamma}} = R^{\beta\bar{\alpha}}R_{\delta\bar{\gamma}}$$

Ricci flat  $\iff$  traceless.

(related discuss will be found in §3.1)

Comment

$M$ : compact Kähler manifold.  
 If  $h^{2,0}(M) = 0$ .  
 $\exists$  holomorphic embedding  $M \rightarrow \mathbb{P}^N$ .

(rough idea)

In the long exact sequence associated with  $0 \rightarrow \mathbb{Z} \rightarrow \mathcal{O}_M \rightarrow \mathcal{O}_M^{\otimes 2} \rightarrow 0$ .

$$H^2(M; \mathcal{O}_M) \rightarrow H^1(M; \mathcal{O}_M^{\otimes 2}) \rightarrow H^2(M; \mathbb{Z}) \rightarrow H^2(M; \mathcal{O}_M)$$

$$H^1(M; \mathcal{O}_M^{\otimes 2}) \twoheadrightarrow H^2(M; \mathbb{Z}) \text{ surjective.}$$

$$\begin{array}{c} \mathbb{R} \\ \hline H^{0,2}(M; \mathbb{C}) \\ \hline \mathbb{C} \end{array}$$

So, pick a class  $[w] \in H^2(M; \mathbb{Z})$  in the Kähler cone.

$$\implies \exists L_w \text{ on } M. \text{ so } c_2(L_w) = [w].$$

Use  $\mathbb{P}(M; L_w)$  to construct  $M \rightarrow \mathbb{P}^{N = \dim[\mathbb{P}(M; L_w)] - 1}$ .

key words:  $\left\{ \begin{array}{l} \text{Kodaira embedding theorem (above)} \\ \text{Chow's theorem} \end{array} \right.$

$\hookrightarrow$  [every complex analytic submanifold of a projective space is algebraic.]

Refs. for p. 14, p. 15:  
 Polchinski Chaudhuri Johnson th/9602052.  
 Giveon Kutasov th/9802067. (Chapt I+II)

§ 1.4. D-brane.

D-branes placed in a flat space-time.

Type IA string theory.

D0-brane. D2-brane. D4-brane. D6-brane. D8-brane  
 are stable.

$\eta \in \underline{16}$  of  $SO(9,1)$   
 $\eta' \in \underline{16}'$  of  $SO(9,1)$  } SUSY transform parameters  
 of 10D (2,2)-SUSY SUGRA.  
 total = 32 SUSY charges.

Dp-brane stretched in

$x^0, x^1 \sim x^p$  directions

preserve SUSY charges with  $\eta = \underbrace{\Gamma^{012 \dots p}}_{\text{odd } P\text{'s}} \eta'$  (\*)  
 odd P's. opposite chirality

Type IB string theory

Dp-branes with odd p's.

$\eta^1, \eta^2 \in \underline{16}$  of  $SO(9,1)$  SUSY transform parameters.

A Dp-brane stretched in the  $x^0, x^1 \sim x^p$  directions.

preserves SUSY charges with  $\eta^1 = \underbrace{\Gamma^{012 \dots p}}_{\text{even } P\text{'s}} \eta^2$  (\*)  
 even P's. the same chirality

(\*) : can be derived from the algebra  $\{Q^\dagger, Q\} = P^M P_M + (\text{central changes})$

just like in Wess-Bagger Chap. I.

(must be written somewhere in Polchinski (Chap. 13?))

★ If a  $D_p$ -brane and a  $D_{p'}$ -brane coexist.

the remaining SUSY changes are

$$\begin{aligned} \eta &= \Gamma^{01 \dots p} \eta' \\ \Leftrightarrow \eta &= \Gamma^{01 \dots p'} \eta' \end{aligned}$$

$$\Rightarrow \Gamma^{(p'+1)(p'+2) \dots p} \eta' = \eta' \text{ follows.}$$

Because  $(\Gamma^1 \Gamma^2)^2 = \Gamma^1 \Gamma^2 \Gamma^1 \Gamma^2 = -(\Gamma^1)^2 (\Gamma^2)^2 = -1$ .

$(\Gamma^1 \Gamma^2)$  has only two eq. values:  $\pm i$ .  $\Gamma^{12} \eta' \neq \eta'$ .

$$\Gamma^{1234} \Gamma^{1234} = (-1)^{3+2+1} (\Gamma^1)^2 (\Gamma^2)^2 (\Gamma^3)^2 (\Gamma^4)^2 = +1$$

$(\Gamma^{1234})$  has two eq. values  $\pm 1$ .

"So" the condition  $\Gamma^{1234} \eta' = \eta'$

reduces the SUSY changes by half.

$$\boxed{D3-D7 : N=2 \text{ SUSY gauge theory in } \mathbb{R}^{3,1}}$$

★ If a  $D_p$ -brane and another  $D_p$ -brane intersect with an angle...

the condition for remaining SUSY can be worked out

by taking the common subset of the preserved SUSY

changes of both D-branes.

The result is ... a part of the statement in p. (16) (17)



Type IIA / (M<sub>6</sub> = CY<sub>3</sub>)

\*a

When n<sub>1</sub> × D6-branes are wrapped on a slag 3-cycle L<sub>1</sub>.  
n<sub>2</sub> × D6-branes are wrapped on a slag 3-cycle L<sub>2</sub>.  
⋮

Def: a 3-cycle L ⊂ M is said to be Lagrangian if the symplectic form J of M becomes trivial when pulled back to L at each point in L. It is further called special Lagrangian if Re(Ω)|<sub>L</sub> ∝ Im(Ω)|<sub>L</sub> over L.

the effective theory in R<sup>3,1</sup> has N=1 SUSY with the gauge group U(n<sub>1</sub>) × U(n<sub>2</sub>) × ...

The matter field in the (n<sub>1</sub>, n̄<sub>2</sub>) repr.  
#[chiral multiplet (n<sub>1</sub>, n̄<sub>2</sub>)] - #[chiral multiplet (n̄<sub>1</sub>, n<sub>2</sub>)] = L<sub>1</sub> · L<sub>2</sub> (the intersection number)

Berkooz Douglas Leigh th/96.06.139

The matter field in the (adj(n<sub>1</sub>), 1) repr.  
#[chiral (adj(n<sub>1</sub>), 1)] = b<sub>2</sub>(L<sub>1</sub>)