

§ 2 Pheno Perspectives. (incl. String Duality)

§ 2.1. The Doublet-Triplet Splitting Problem in SUSY GUT sets the KK scale.

$$\left(\begin{array}{c|c} SU(3)_c & \\ \hline & SU(2)_L \end{array} \right) \xrightarrow{U(1)_Y} \left(\begin{array}{c|c} \frac{2}{3} & \\ \hline \frac{-1}{3} & \frac{1}{2} \end{array} \right) \quad \Lambda_{GUT}^2 = \left(\begin{array}{c|c} (\Lambda^2)^{-2/3} & (3 \otimes 2)^{+1/6} \\ \hline & (\Lambda^2)^{+2} \end{array} \right) \quad \bar{5} = \left(\begin{array}{c} 3^{+1/3} \\ 2^{-2/3} \end{array} \right)$$

Georgi - Glashow SU(5)
{ U^c, Q, E^c }
{ D^c, L }

- How to break $SU(5) \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$?
- How to obtain just $(H_u(2) \text{ w/o } H_c(3))$ below $M_{GUT} \sim 10^{16} \text{ GeV}$?
 $(H_d(2) \text{ w/o } \bar{H}_c(\bar{3}))$

A direct mass term $H_c(3) - \bar{H}_c(\bar{3})$ is not natural ($M_2 \ll M_3$) ↑ coupling unification

Alternative ideas (see Watari's Ph.D thesis review part) yields dim=5 p. decay op.

In: Het/($M_s = CY_3$)

(a) $\pi_1(M_6) \neq \{1\}$ w/ non-trivial holonomy repr $\pi_1(M_6) \rightarrow \left(\begin{array}{c} U(1)_Y \times SU(5) \\ \times \\ SU(5)_{str} \end{array} \right)$

(b) $G_{str} = [SU(5) \times U(1)_Y] \quad H = SU(3) \times SU(2)$

WITTEN Nucl. Phys. B258 ('85) 75.
 WITTEN Nucl. Phys. B268 ('85) 79.

$(H_c(3) \text{ and } H_u(2) \text{ are } 0\text{-modes of different holes})$
 \Rightarrow different # of 0-modes.

Either way, the symmetry breaking scale $\approx 10^{16} \text{ GeV}$ (unification scale)

The same is true in Type II / CY_3 -orientifold. $M_{pe} \sim 10^2 M_{GUT}$

$$\left\{ \begin{array}{l} M_{GUT} \sim \frac{1}{R} \\ M_{pe}^2 \sim M_{str}^8 R^6 \frac{1}{g_s^2} \\ \frac{1}{\alpha} \sim \left(\frac{M_{str}^6 R^6}{g_s^2} \right)_{Het} \text{ or } \left(M_{str}^4 R^4 \frac{1}{g_s} \right)_{D7} \end{array} \right\} \Rightarrow \pi\text{'s not negligible}$$

$\frac{R}{2\pi\sqrt{\alpha'}} \text{ cannot be as large as } \frac{1}{M_{str}}$

§ 2.2 String Duality

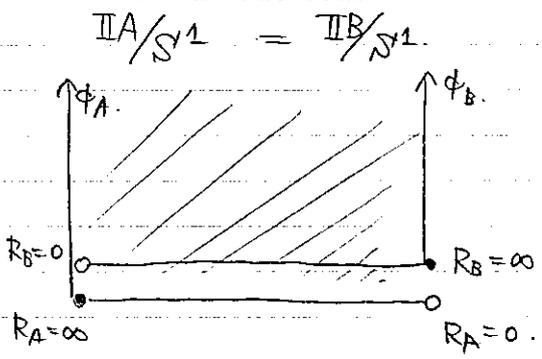
String compactifications with $N=1$ SUSY on $\mathbb{R}^{3,1}$:

$$\left(\begin{array}{ccc} \text{Het}/(M_6 = \text{CY}_3, \text{ gauge bundle}) & \text{IIA}/(M_6 = \text{CY}_3, D6, 06) & \text{IIB}/(M_6 = \text{CY}_3, D7, 07) \\ & M/(G_2\text{-hol mfd}) & F/(\text{ell. CY}_3) \end{array} \right)$$

Not all of them are distinct from one another.

(A)

Example: T-duality



$$M = (\text{discrete}) \backslash O(1, 1; \mathbb{R}) / O(1) \times O(1)$$

common moduli space

the same spectrum
the same physics.

$$(R_A, \phi_A, \begin{matrix} C^{(1)} \\ \mathbb{R}^2 \end{matrix}) \xrightleftharpoons{\text{map}} (R_B, \phi_B, C^{(0)})$$

should not double count.

Example: Het $E_8 \times E_8$ - Het $SO(32)$ duality (Polchinski II, p. 78)

$$\text{Het}/S^1 : (\mathfrak{g}_{99}, \mathfrak{g}_A, \phi)_{E_8 \times E_8} \xrightleftharpoons{\text{map}} (\mathfrak{g}_{99}, \mathfrak{g}_A, \phi)_{SO(32)}$$

$$\text{common moduli space} = (\text{discrete}) \backslash O(1, 17; \mathbb{R}) / O(1) \times O(17)$$

↑ $(\times \mathbb{R}_{>0})$
local field redefinition

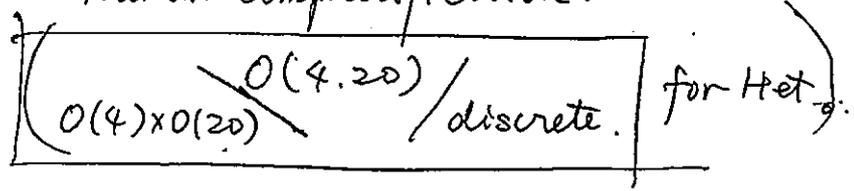
of Theorem: Let L be an even self-dual lattice of signature (r_+, r_-) .

Then $r_+ - r_- \equiv 0 \pmod{8}$. Moreover, if $r_+ > 0$ & $r_- > 0$, it is unique modulo isometry. So, $\text{II}_{1,1} \oplus (E_8 \oplus E_8) \cong \text{II}_{1,1} \oplus (D_{16}; \mathbb{Z}_2)$ lattice isometry.

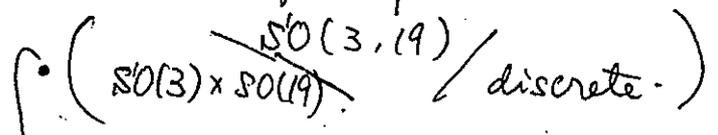
When $r_- = 0$, $L = E_8$ if $r_+ = 8$, $L = \begin{cases} E_8 \oplus E_8 & \text{if } r_+ = 16, \\ D_{16}; \mathbb{Z}_2 & \text{if } r_+ = 24. \end{cases}$ there are 24 L's if $r_+ = 24$.

B | Het / T⁴ - IIA / K3 duality (16-SUSY) K3: CY₂

Narain compactification.

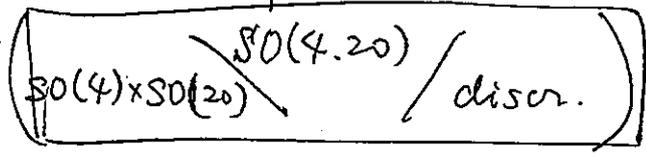


On the other hand. in IIA
moduli space of metric on K3



$$\left\langle \begin{aligned} h^0(K3) = h^4(K3) = 1 \\ h^2(K3) = 22. \end{aligned} \right\rangle$$

- B-field on the 22 2-cycles.
- 1 combination from ϕ_{100} & $vol(K3)$.



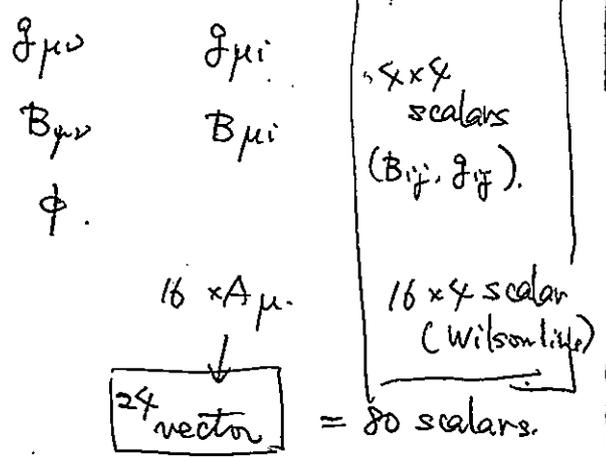
R^m nke $dy^k \wedge dy^l$
SU(2)-valued
not SO(4) valued.
half non-trivial.

CY₃: SU(3)-valued.
not SO(6)-valued
SO(6)-spinor = SU(4) - $\frac{1}{2}$
SU(3).
 $\langle \omega \rangle$ $\frac{3}{4}$ non-trivial
 $\frac{1}{4}$ unbroken.

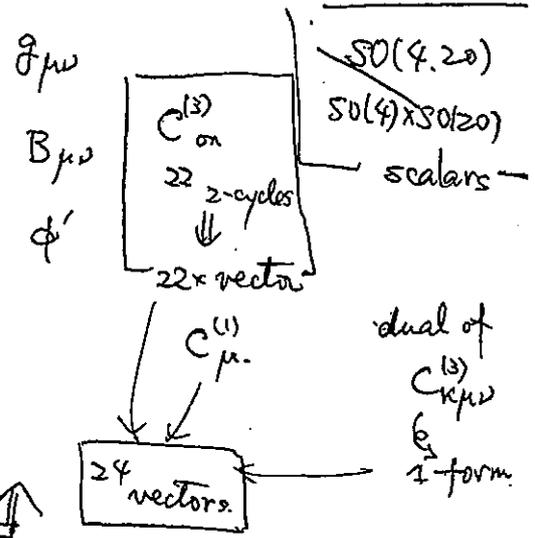
for IIA

dimensional reduction. (massless spectrum in 6D eff-theory)

Het side.



IIA side.



[the same massless spectrum in 6D.]

unbroken (enhanced) symmetry in Het.

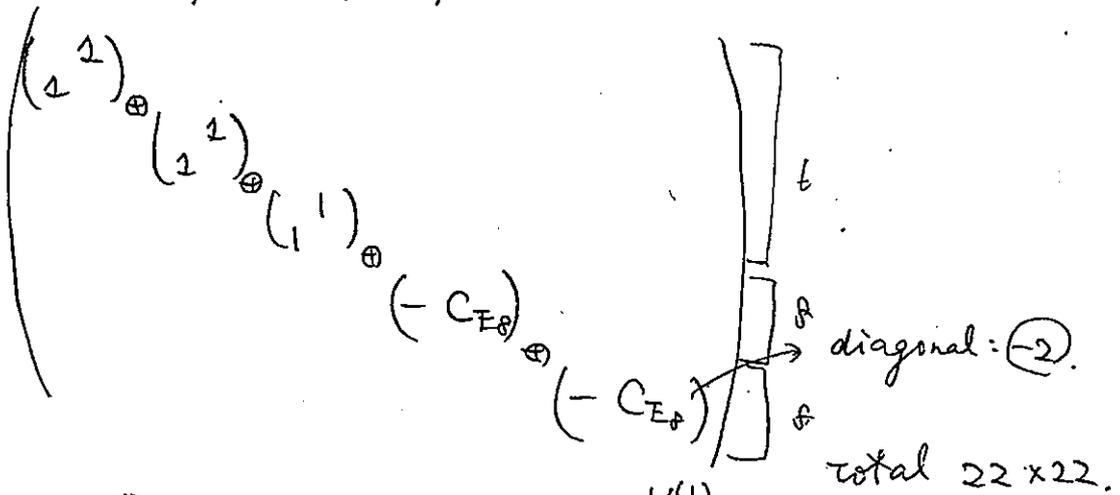
there must also be in IIA ?

(in high codim. subspace of $O(4,20)$ moduli space,
 $O(4) \times O(20)$)



(for a special choice of moduli of K3 (W, B))

Intersection form of 2-cycles. in K3.



$C^{(3)}$ on these 2-cycles \Rightarrow vector in D=6 eff. theory.

D2-branes wrapped on these 2-cycles \Rightarrow charged

\Rightarrow W-boson's of non-Abelian gauge theory under these $U(1)$'s.
 from SUSY 16-SUSY vector multiplet super YM.

non-Abelian enhanced

when those cycles collapsed to a point (0-size)

- A-D-E singularity (isolated singul. pts in cpx surface)
- ALE space of A-D-E type.
- multi centered Taub-NUT space of A-D-E type.

A_{N-1} singularity : $y^2 = x^2 + z^N$

D_N : $y^2 = -x^2z + z^{N-1}$

E_6 : $y^2 = x^3 + z^4$

E_7 : $y^2 = x^3 + xz^3$

E_8 : $y^2 = x^3 + z^5$

Evidence. not just the agreement of the massless state spectrum
of the moduli space.
but also of the BPS state spectrum.

Proof of the duality in terms of the world sheet field theory.

Harvey Strominger
1985

§2.2. (C)

~~§2.2.1~~ M-theory and Type IIA String Theory

Refs -
Polchinski Chap. 12
Giveon Kutasov Chap. 4.

M-theory : something whose L.T. effective theory is D=11 supergravity.

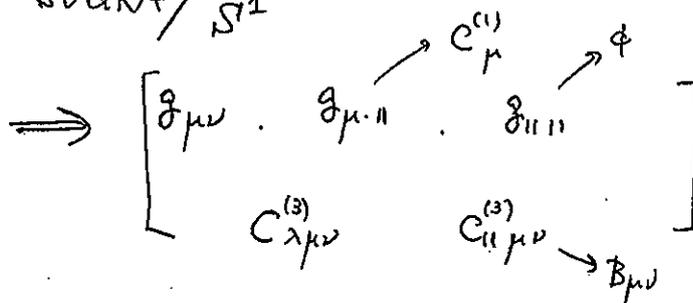
bosonic massless modes (d.o.f.)

soliton

$(g_{mn}, C_{lmn}^{(3)})$

M2-brane
M5-brane

• 11D SUGRA / S^1



massless d.o.f. of Type IIA.

- M2-branes wrapped on $S^1 \rightarrow F1$
- not = = = $\rightarrow D2$
- M5-branes wrapped on $S^1 \rightarrow D4$
- not = = = $\rightarrow NS5$

11D sugra.

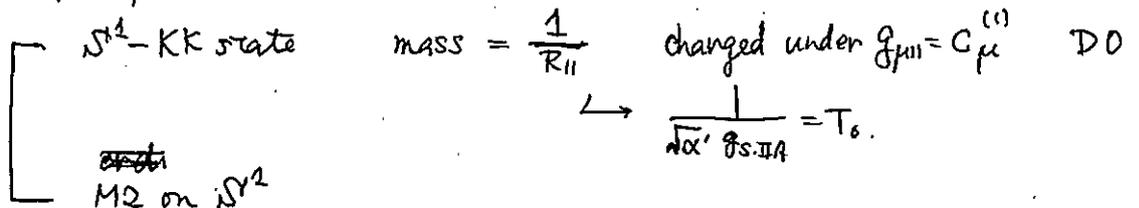
$S^1 = \frac{1}{2\kappa_{11}^2} \int d^4x \sqrt{-g} R + \dots$ $\frac{1}{2\kappa_{11}^2} \equiv \frac{(2\pi)}{l_{11}^9}$

$T_{M2} = \frac{2\pi}{l_{11}^3}$ $T_{M5} = \frac{2\pi}{l_{11}^6}$

IIA

Dp-brane
 $T_p = \frac{1}{(2\pi)^p (\alpha')^{\frac{p+1}{2}}}$

S^1 compactification. radius R_{11}



$\frac{2\pi}{(l_{11})^3} \times (2\pi R_{11}) \rightarrow \frac{1}{2\pi \alpha'}$

dictionary between $(l_{11}, R_{11}) \leftrightarrow (\alpha', g_{s, IIA})$

then it follows that....

M2 ~~or~~ w/o wrapping.

$$\left[\frac{(2\pi)}{(l_{11})^3} \rightarrow \frac{1}{2\pi\alpha'} \times \frac{1}{2\pi\sqrt{\alpha'} g_s \ell_A} = \frac{1}{(2\pi)^2 (\alpha')^{3/2} g_s \ell_A} \right]$$

T_{D2}

M5 wrapping

$$\left[\frac{2\pi}{(l_{11})^6} \times (2\pi R_{11}) \rightarrow \frac{1}{(2\pi)^5 (\alpha')^3 g_s^2 \ell_A} = T_{D5} \right]$$

M5 w/o wrapping

$$\frac{2\pi}{(l_{11})^6} \rightarrow \frac{(T_{D2})^2}{2\pi} = \frac{1}{(2\pi)^5 (\alpha')^3 g_s^2 \ell_A}$$

* multi centered Taub-NUT space. for M-theory — D6-brane in IIA

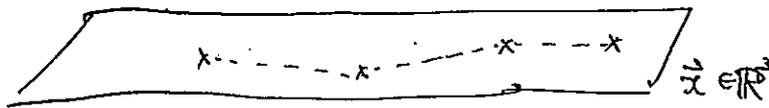
$(\vec{x} \in \mathbb{R}^3, \tau \in [0, 4\pi])$ coordinates.

(Sen. th/9707123.)

$$ds^2 = U(\vec{x})^{-1} (d\tau + A_i(\vec{x}) dx^i)^2 + U(\vec{x}) dx^i dx^i$$

$$\begin{cases} U(\vec{x}) = 1 + \sum_{I=1}^N \frac{m_I}{|\vec{x} - \vec{x}_I|} \\ \vec{\nabla} \times \vec{A} = -\vec{\nabla} U(\vec{x}) \end{cases}$$

smooth if \vec{x}_I 's ($I=1 \sim N$) are distinct from one another



S^2 -fibration over intervals between $\{\vec{x}_I\}$'s

N -center $\Rightarrow (N-1)$ -topological 2-cycles.

- large $|\vec{x}|$:
 $U(\vec{x}) \approx 1$
 const. radius.
 $M \rightarrow \text{IIA}$

- $\vec{x} \sim \vec{x}_I$:
 $U \rightarrow +\infty$
 $U^{-1} \rightarrow 0$
 S^2 radius $\Rightarrow 0$.

M-theory $(g_{mn}, C_{lmn}^{(3)})$ on A_{N-1} -type. (N -center) Taub-NUT space.

~~$\Rightarrow \nu(\vec{x}_I) \nu(\vec{x}_J)$~~

M2-brane on the 2-cycles.

\Rightarrow open string (F1 string)
 end points (center $\vec{x}_I \in \mathbb{R}^3$). D6-branes.

$S^1 U(N)$ gauge theory (A_{N-1})

* If we take $U(\vec{x}) = \sum_{I=1}^N \frac{m_I}{|\vec{x} - \vec{x}_I|}$ instead.

That's $\mathbb{C}^2 / \mathbb{Z}_N$ in the limit of $\vec{x}_I \rightarrow \vec{0}$.

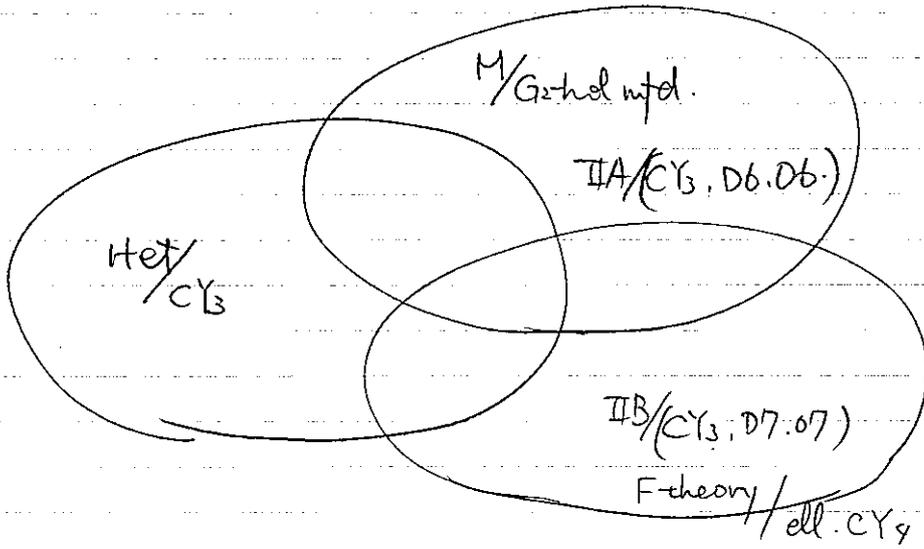
§ 2.2. (F)

The adiabatic "argument"

The $Het/T^3 - M\text{-theory}/K3$ duality suggests $IIA/(CY_3 w/ D6, \overline{06})$
 $Het/(T^3\text{-fibred } CY_3) - M\text{-theory}/(K3\text{-fibred } G_2\text{-hol mfd})$ duality.

§ 2.2. (G)

any restriction on types of degenerate fibrat'n??



("The" landscape of string vacua with $D=4$ $N=1$ SUSY)

(Tatar Watari th/0602238)

Low-energy physics

Microscopic formulation

Gauge group
Symmetry
Interact'n

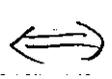


algebra

current algebra in Het.
intersect'n of topological cycles
in M-theory, F-theory.
open string reconnection in Type IIA, IB.

geometrization of gauge theory

particle multiplicity
(the number of generations)



topology

$\chi(M_6; Pa(V))$ in Het.
 $L_1 \cdot L_2$ in IIA.
 $\int_{S_1 S_2} c_1(\frac{F}{2\pi})$ in IB.
 $\int_S G^{(4)}$ in F-theory

coupling constant ↔ where in the moduli space

Our knowledge on the Standard Model does not ^{favour} one formulation of string theory over the others in a simplest possible way because of the string duality.

Perhaps "God is in the details"?

① $SU(5)$ GUT up-type Yukawa $\{u^c H_u \subset (10^{ab} 10^{cd} H(s)^e E_{abcde})$
is from the deformation of $E_6 \rightarrow U(2) \times SU(5)$ GUT
(open strings on Dbrane + Oplanes are not enough)

② In a common moduli space of duality, one formulation provides calculability in one corner while the other formulation provides calculability in another corner. ⇒ still makes sense with the SM which corner fits better.

§23. Axion in string theory

• Het / (CY₃ = M) ⇒ d¹⁰ f_{eff} tr_H(W^αW_α)

$$f_{\text{eff}} \sim \left(\underset{\uparrow}{S'} + \sum_{a=1}^{h-1} C_a T_a \right)$$

$$\left(\frac{1}{e^2} + ia \right) \quad \left[(\delta g_{\alpha\beta}) + i (\delta B_{\alpha\beta}) \right]$$

* da ~ (dB⁽²⁾)

• IIB / (CY₃ = W) D7 on S² f_{eff} ~ (T_S + ...)

$$\left[(\delta g_{\alpha\beta}) + i (\delta \tilde{C})_{\alpha\beta} \right]$$

* d(δC̃) = (dC⁽²⁾)

↑ scalar multiplets in R^{3,1} ⊗ (1,1) form on W₆

The imaginary parts of those chiral multiplets
 4D N=1
 can play the role of the PQ axion.

* They are not associated w/ SSB of a global U(1) sym.
 They are present already @ Mpl scale.

* They are from n-form fields in string theory.
 no need to impose a global U(1) symmetry by hand.
 their interact'n tightly constrained.

* Their axion decay constant cannot be chosen freely.

$$K_{\text{eff}}(T, \bar{T}, S, \bar{S}, \dots) \sim M_{\text{pl}}^2 \times \text{fun}(T\text{'s}, S\text{'s})$$

$$\left(\partial_T \partial_{\bar{T}} K_{\text{eff}} \right) \Big|_{T=\langle T \rangle} = M_{\text{pl}}^2 \cdot \left(\partial_T \partial_{\bar{T}} \text{fun} \right) \Big|_{T=\langle T \rangle}$$

T = <T> + δT. rescale (δT) for canonical kin. term.

$\frac{g^2 (\delta T)_{\text{can}}}{32\pi^2 F_a} \sim \frac{g^2 (\delta T)_{\text{can}}}{16\pi M_{\text{pl}} \sqrt{K_{T\bar{T}}}}$	$\Rightarrow \frac{1}{32\pi^2 F_a} \sim \frac{1}{M_{\text{pl}} \sqrt{(\partial_T \partial_{\bar{T}} \text{fun})}}$	(for more. Svrcek Witten '06) PLUS
?	?	

Such a large axion decay constant will result in too much axion dark matter (if we assume that $\langle a \rangle \sim \mathcal{O}(f_a)$ at the end of inflation).

⇒ option A $\langle a \rangle$ just happens to be much smaller than f_a .
(alternatively, we can invoke anthropic argument)

⇒ option B such an axion field is absent in the low-energy effective theory (somehow, large supersymmetric mass for the Kähler moduli)

Cosmology of saxion fields (\Rightarrow the real part of the Kähler moduli)
 \Uparrow (see Kawasaki Nakayama for pheno analysis.)
(or dilaton (het))

§ 2.4 flux and cpx str. moduli stabilizat'n

• $\mathbb{R}B/(W_0 = CY_3)$ think of $\langle H^{(3)} \rangle \in (2\pi)^2 \alpha' H^3(W; \mathbb{Z})$

$$\left. \begin{aligned} &= dB^{(2)} \text{ locally} \\ &\left\{ \begin{aligned} &dC^{(2)} \\ &\langle F^{(3)} \rangle \in (2\pi)^2 \alpha' H^3(W; \mathbb{Z}) \end{aligned} \right\} \end{aligned} \right\}$$

It is known that

$$\Delta W_{\text{eff. FD}} = \frac{M_{\text{pl}}^3}{\sqrt{4\pi}} \int_W (F^{(3)} - SH^{(3)}) \wedge \Omega_W$$

Gukov Vafa Witten th/9906070

$$S = (C^{(0)} + i \frac{1}{e\phi})$$

$$\left\langle \frac{b_2(W)}{\sum_{i=1}^{b_2(W)} (m_i - S^i n_i)} \prod_{j=1}^{h^{1,1}(W)} C^{(j)} \right\rangle$$

$\text{Span}_{\mathbb{Z}} \{ \gamma^i \mid i=1 \sim b_2 \}$
 $= H^3(W; \mathbb{Z})$

$$\left\{ \begin{aligned} F^{(3)} &\sim \gamma^i m_i \\ H^{(3)} &\sim \gamma^i n_i \\ \Omega_W(z) &\sim \gamma^i \Pi_i(z) \end{aligned} \right.$$

$$\Pi_i(z) : (\text{period integrals}) \quad C^{(j)} := \int_W (\gamma^i \wedge \gamma^j)$$

depend on the cpx str. of W in the family of $[W]_{\text{top}}$

$\Rightarrow \partial_z (\Delta W_{\text{eff. FD}}) = 0$ (F-term condition)

determines $\langle z^i \rangle, \dots, i=1 \sim h^{1,1}(W)$

Expanding $(\Delta W_{\text{eff. FD}})(z)$ with respect to the fluctuat'ns

$z^i = \langle z^i \rangle + \delta z^i$ around the vac. $\langle z^i \rangle$, we often see that the Hessian (the 2nd derivative matrix) is non-degenerate. (= (δz^i) 's have mass terms)

(eg. Giddings Kachru Polchinski and refs. earlier)

★ The typical mass scale $\sim \frac{(M_{KK})^3}{(M_{str})^2} \Rightarrow \frac{(M_{GUT})^3}{(M_{str})^2}$

somewhat below the GUT scale.

good for Right-handed ν mass.

Kachru Schultze Trivedi th/02.01.028

Tatar. Tsuchiya. Watari '09

★ They decay quickly because of the large mass.

Safe in cosmological thermal history.

(do not interfere with the BBN)

★ statistics of fluxes $\left[\begin{array}{l} H^{(3)} \in (2\pi)^3 \alpha' H^3(W; \mathbb{Z}) \\ F^{(3)} \in (2\pi)^3 \alpha' H^3(W; \mathbb{Z}) \end{array} \right] \Rightarrow$ statistics of cpx-str.

\Downarrow
statistics of eff. theory coupling constants (like Yukawa's)

★ There is a "corresponding" mechanism in Heterotic string.

$\langle H^{(3)} \rangle \longleftrightarrow$ non-Kählerity of the metric

$\langle F^{(3)} \rangle \longleftrightarrow \langle H^{(3)} \rangle$

IIb, Type I (SO(32)) \longleftrightarrow Het SO(32), $E_8 \times E_8$
S-dual

To what extent is there (are we convinced with the) string duality in the presence of fluxes?

★ In Type IIb, RR flux cannot be formulated in the NSR formalism. (in Polchinski).

("vertex operators w/ a branch cut develop a condensate") \Rightarrow use Green Schwarz formalism.

§ 2.5 Inflation and Volume (Kähler) Moduli Stabilization

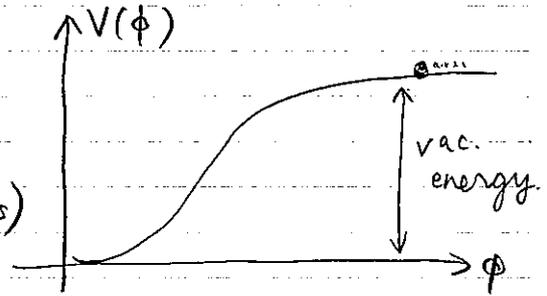
The slow-roll inflation is an idea

that there is an extra scalar field

(in addition to the Standard Model fields)

with very flat potential $V(\phi)$

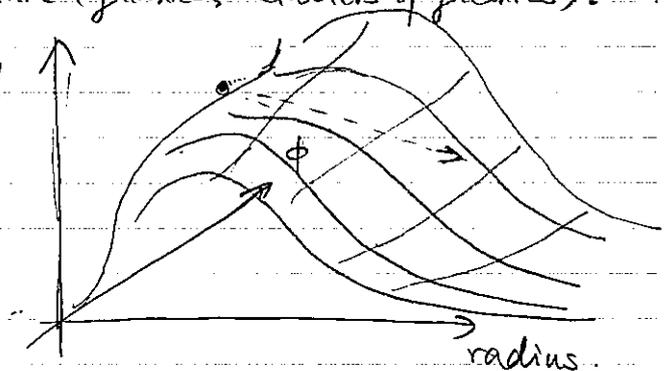
so the vac. energy inflates the universe \mathbb{R}^{3+1} to a large size during the slow roll period.



Its quantum fluctuation $(\delta\phi)$ and the energy fluctuation $(\frac{\partial V}{\partial\phi})(\delta\phi)$

results in the large scale structure (galaxies, clusters of galaxies).

Nice! But in string vacua obtained via compactifications, we always have volume (moduli) fields, and a potential $V(\phi, \text{radius})$ in the \mathbb{R}^{3+1} effective theory.



Moreover, at the $\text{radius} \rightarrow \infty$ limit, the potential $V(\phi, \text{radius}) \rightarrow 0$.

So... any period of $V(\phi) > 0$ (including "now") faces a danger of tunneling into inflation in $(9+1)$ dimensions (decompactification).

So, here is a tension (though not a contradiction or no-go)

★ $10^{-5} \sim \frac{\delta\rho}{\rho} \sim \frac{V(\phi)}{(M_{pl}^2)} \frac{1}{(\text{fine tuning})} \Rightarrow V(\phi)_{\text{infl}} \sim (10^{15} \sim 10^{16} \text{ GeV})^4$ w/o fine tuning

↑ measurement

★ If we adopt a scenario of SUSY GUT ($\Rightarrow M \sim 10^{16} \text{ GeV}$) and volume stabilization via $W \sim e^{-b_1 T} + e^{-b_2 T}$ (race track mechanism), the potential barrier $V(\phi, T)$ would not be as high as

Inflation and SUSY GUT cannot be discussed independently in String Compactification.