

### §3. Theoretical Miscellany

#### §3.1 Kähler (susy) vs non-Kähler (non-SUSY) and $\alpha'$ -corrections.

Rfs. Gross, Witten Nucl. Phys. B277 ('86) 1

Grisaru, van de Ven, Zahon Phys. Lett. B173 ('88) 923

Freeman, Pope Phys. Lett. B174 ('88) 88

Nemeschansky, Ne'eman Phys. Lett. B178 ('88) 365

L.Witten, E.Witten Nucl. Phys. B281 ('87) 109

#### Yau's theorem (Calabi conjecture)

Let  $(M, \omega, J)$  be a Kähler manifold.

Pick up a cohomology class  $[\omega_*] \in H^{1,1}(M; \mathbb{R})$ .

(with positivity constraint)

Choose any  $\mathbb{R}$ -valued closed  $(1,1)$ -form  $\rho$  on  $M$ .

whose cohomology class  $[\rho]$  is equal to  $2\pi c_1(TM)$ .

Then there is a unique choice from

$$[\omega_*] = \begin{cases} \text{$\mathbb{R}$-valued $(1,1)$-form} \\ \omega_0 + i\partial\bar{\partial}\varphi \end{cases} \quad \begin{array}{l} \omega_0 \text{ is a $\mathbb{R}$-valued closed $(1,1)$-form on } M, \\ \text{and } [\omega_0] = [\omega_*] \end{array} \quad \begin{array}{l} \varphi \text{ is an } \mathbb{R}\text{-valued function, globally defined} \\ \text{over } M \end{array}$$

(and the Kähler metric corresponding to  $(\omega_0 + i\partial\bar{\partial}\varphi)$ ).

so that  $\text{Ric}(\omega_0 + i\partial\bar{\partial}\varphi) = \rho$ .

Cor: use it in the case  $c_1(TM) = 0 \in H^{1,1}(M; \mathbb{R})$

$\rho = 0$  in particular.

<sup>3</sup>Ricci flat metric

In any  $[\omega_*]$

by exploiting + (exact 2-form)

analogous to extracting a harmonic diff. form

from a cohomology class (fix the (+ exact form) freedom)  
on a Kähler mfd.

Now, think of ( $g=0?$ ) higher  $\alpha'$  corrections (Type II string theory)

- $\beta = 0$  in non-lin.  $\sigma$ -model. (only the UV of 1+1-dim field theory matters?)  
( $g$ : irrelevant?)

$$\begin{aligned} S &\sim \int d^3\sigma \frac{1}{8\pi\alpha'} G_{MN}(X)(dX/dX) + \dots \\ &\Rightarrow (R/\alpha') \sim \frac{1}{g^2} \quad \text{perturbation in } (\alpha'/R^2) \end{aligned}$$

↓  
conjectured to agree?

↓  
equation of motion

At the leading order (1-loop in NLSM perturbation)

$$\begin{cases} 0 = D^2\phi \\ 0 = R_{MN} \end{cases} \quad \begin{array}{l} \text{Ricci-flat metric } \delta\phi = \text{const.} \text{ on } M. \\ \text{is a soln.} \end{array}$$

At higher order (until  $\infty$ -loop in NLSM)

$$\begin{cases} 0 = D^2\phi + (\alpha')^3 (R^4)_{\text{scalar}} \\ 0 = R_{MN} + (\alpha')^3 (R^4)_{MN} \end{cases}$$

For a generic Ricci-flat metric,  $(R^4)_{\text{scalar}} \neq 0$ .  $\int_M (R^4)_{\text{scalar}} \neq 0$ .

although  $(R^4)_{\text{scalar}} = 0$ . if the Ricci-flat metric is also Kähler.

When  $\int_M (R^4)_{\text{scalar}} \neq 0$ ,  $\int_M D^2\phi = D^2 \left( \int_M \phi \right) + \int_M (D_M \phi) = 0$

so  $(D_{R^4}^2 \phi) + \boxed{\frac{\int_M (R^4)_{\text{scalar}}}{\text{vol}(M)} (\alpha')^3 \phi} = 0$

$\sim (\alpha')^3 / R^8$

tremendous space-time variation of  $\phi$  is required on  $\mathbb{R}^8$ !

If Kähler  $\phi = \text{const.}$  is fine. Now  $\text{Ric}(w_0 + \partial\bar{\partial}\phi) + 4\phi = 0$  is the condition for the metric.

modified version of Yau's thm. (?)  
[Gross Witten], [Freeman Pope]

The  $0 = R_{MN} + (\alpha')^3 (R^4)_{MN} + \dots$  condition at any finite order.

If  $M$  is Kähler and  $c_1(TM) = 0$  (so we can take the LO metric  $\tilde{G}$  (and  $\tilde{K}$ ) to be Ricci flat:  $\tilde{\omega}$ )

- ④ The  $N=(2,2)$  NLSM on  $(1+1)$ -dim has a notion of

$$\beta_K = \beta_K^{(1)} + \Delta\beta_K. \quad \Delta\beta_K = \sum_{n=2}^{\infty} \Delta\beta_K^{(n)}$$

the  $\beta$ -fans of the Kähler potential defined on a covering of  $M$ .

\*  $\beta_K^{(1)} = 0$  when we choose the LO metric to be Ricci flat

\* for  $\Delta\beta_K^{(n)}$  (the  $n$ -loop contribution in the NLSM)

with  $n \geq 2$ , we don't need a covering.  $\Delta\beta_K^{(n)}$  is a

well-defined scalar fan on  $M$ , given explicitly by

$G_{ij}$  and Ricci's. (See [Nemshansky Sen])

$$*\Delta\beta_K^{(n)} \sim (\alpha')^n (R^n) \text{ scalar}$$

- ⑤ Given the property above, there is a systematic way to determine

$$\begin{cases} K = \tilde{K} + \sum_{n=2}^{\infty} \delta K^{(n)} \\ G_{ij} = \tilde{G}_{ij} + \sum_{n=2}^{\infty} \delta G_{ij}^{(n)} \\ \omega_{ij} = (\tilde{\omega}_0)_{ij} + \sum_{n=2}^{\infty} \partial_i \bar{\partial}_j \varphi^{(n)} \end{cases} \quad \text{at any finite order } n$$

so that  $(\Delta\beta_K^{(n)})|_{K=\tilde{K}}$  is cancelled by  $(\Delta\beta_K^{(m)})$  with  $m < n$  and  $\beta_K^{(1)}$

with  $K = \tilde{K} + \sum_{n=2}^{n-m} \delta K^{(n)}$  inserted.

\* no need to invoke  $\mathbb{R}^4$ -dependent background for  $\beta = 0$

\*  $\varphi^{(n)}$  is in the form of  $(\alpha')^{n-1}(\text{const}) \cdot \tilde{K} + (\text{globally defined scalar on } M)$

[Nemshansky Sen]

\* so,  $G_{ij}$  is not Ricci flat metric any more.

the difference  $[\omega_{ij}] - [\tilde{\omega}_{ij}]$  is not zero (the contribution.)  $\xleftarrow{(\alpha')^n(\text{const}) \tilde{K}} \text{true}$

The  $0 = (\nabla^2 \phi) + (\alpha')^3 (R^4) \text{ scalar} + \dots$  condition: ??

Does the 0-mode counting argument in [Gross Witten] and [Freeman Pope] work for higher order in  $(\alpha'^2)$ ? 11.18

## Non-geometric Phase

Witten - th/9301042

CY<sub>3</sub> on world sheet by linear O-model.

e.g. quintic  $\phi_i$  ( $i=1 \sim 5$ ) +1 charge under  $U(1)$   
 $\Phi$  -5 charge.

$$\left\{ \begin{array}{l} D\text{-term} \quad \sum_i |\phi_i|^2 - 5|P| - r = 0 \\ \text{superpotential} \quad W = \Phi \cdot F^{(5)}(\phi_i) \end{array} \right. \quad \begin{array}{l} \uparrow \text{FI parameter.} \\ \downarrow \text{homog. deg. 5.} \end{array}$$

$$\Rightarrow \begin{cases} r \gg 0 \text{ (large vol)} \Rightarrow (F(\phi_v) = 0) \subset \mathbb{P}^{\infty} \\ r \ll 0, \quad W = \langle \phi \rangle \cdot F^{(0)}(\phi_v) \quad w/ \quad \cancel{\langle \phi_v \rangle = 0.} \\ \text{(non-geometric)} \end{cases}$$

Q. Are all non-geometric CFT understood as some limit  
of geometric CFT?  $\rightarrow$  Polchinski 1987.

→ Polchinski 198

If the geometry  $X := \{ F^{(5)}(\phi) = 0 \text{ in } \mathbb{P}^4 \}$   
 has a locus of singular pts;  
 then (the max. mfd.)  $\sim (X_{\text{sing.}})$

## Orbifolds / fractional branes

\* toroidal orbifold:

resolution of singularity  $\Rightarrow$  smooth CY.

(particular choice of Kähler moduli)

(particular choice of  $\equiv$  CY topology)

twisted sector field very  $\cong$  resolution

- Het orbifold

particular limit of vector bundle moduli.

- IB D3-brane at an orbifold singularity (fractional brane)

e.g. D3-brane at  $\mathbb{C}^3/\mathbb{Z}_3$  orbifold singl.

3-types. (orbifold conditions).

interpretation

$\mathbb{C}^3/\mathbb{Z}_3$  blow-up  $\Rightarrow \begin{matrix} \mathbb{P}^2 \\ \cup \\ \mathbb{P}^1 \end{matrix}$  appear

$$\text{3-types} \Rightarrow \begin{cases} D7 + \overline{D5} + \frac{1}{2} D3 \\ 2 \overline{D7} + D5 + \frac{1}{2} D3 \\ D7 \end{cases}$$

stable config.

Douglas Greene Morrison th/9709.151

Douglas et.al. th/00 02 037 00 03 263

Douglas th/00 11 017

§ 3.4 If we had a little more time

Other materials to be included would be...

(A) Relations among "Modular Invariance", "Branchi id's", and  
"Anomaly Inflow"

Refs. here and there in the literature from late 80's ~ early 90's

Green Harvey Moore th/9605033

Aldazabal Badagnani Ibanez Uranga th/9908071

(B) Is the modular invariance enough? at  $g=1$

Moore Seiberg Comm. Math. Phys. 123 ('89) 177.

Phys. Lett. B212 ('88) 851.

Nucl. Phys. B313 ('89) 16.

(C) Representation theory of the  $N=2$  superconformal algebra &  
spectral flow.

skipped because of the quick course lecture.

held at KEK in August '18.

(D) Off-shell amplitudes in string theory & Mass renormalization

A.Sen et.al. th/1311.1257, th/1401.7014, th/1408.0571