§ 2.6 BPS states of 4d space-time SUSY

4d $N=2$ SUSY algebra $\quad I, J = 1, 2$

$$\{Q^I_\alpha, \bar{Q}_{\dot{\beta} J}\} = 2\sigma^\mu_{\alpha\dot{\alpha}} P_\mu \delta^I_J \qquad \{Q^I_\alpha, Q^J_\beta\} = \epsilon_{\alpha\beta}\epsilon^{IJ} Z$$

central change.

$$\langle P^0 \rangle \geq |Z| \quad \text{(BPS bound)}$$

IIA/(CY$_3$=M) $\Rightarrow$ 4d $N=2$ SUSY. $U(1)^{h^{11}(M)+1}$: Dp-brane on p-cycle in M.

$\Rightarrow$ U(1)-changed particle excitath. BPS

$\Updownarrow$

IIB/(CY$_3$=M) + D(p+3)-brane wrapped on p-cycle in M, filling $\mathbb{R}^{3,1}$ $\Rightarrow$ 4d $N=1$ SUSY gauge theory

[$U(1)^{h^{11}(M)+1}$: orientifold projection $\Rightarrow$ some of them odd]

☆ Naive derivatn of the formula for "Z"

IIA/CY$_3$=M $\to$ 4d. $N=2$

start from 10d. IIA SUGRA SUSY algebra.

$$\begin{pmatrix} \{Q_-, Q_-^\dagger\} & \{Q_-, Q_+^\dagger\} \\ \{Q_+, Q_-^\dagger\} & \{Q_+, Q_+^\dagger\} \end{pmatrix} = \begin{bmatrix} (\Gamma^M\Gamma^0)_{UL} P_M & (*)_{UR} \\ (*)_{DL} & (\Gamma^M\Gamma^0)_{DR} P_M \end{bmatrix}$$

$$(*)_{UR} = (\Gamma^{10}\Gamma^0)_{UR} Z^{(0)} + \frac{1}{2}(\Gamma^{MN}\Gamma^0)_{UR} Z^{(2)}_{MN} + \frac{1}{4!}(\Gamma^{KLMN}\Gamma^{10}\Gamma^0)_{UR} Z^{(4)}_{KLMN}$$

$$(*)_{DL} = (\Gamma^{10}\Gamma^0)_{DL} Z^{(0)} + \frac{1}{2}(\Gamma^{MN}\Gamma^0)_{DL} Z^{(2)}_{MN} + \frac{1}{4!}(\Gamma^{KLMN}\Gamma^{10}\Gamma^0)_{DL} Z^{(4)}_{KLMN}.$$

add. $+ \frac{1}{6!}(\Gamma^{ABKLMN})_{UR/DL} Z^{(6)}_{ABKLMN}$

Giveon Kutasov p.35 eq (32)

Polchinski (B.2.9)

(contribuths from F1, NS5: omitted.)

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$$
\begin{array}{c|cc|cc}
 & \multicolumn{2}{c|}{Q_-^\dagger} & \multicolumn{2}{c}{Q_+^\dagger} \\
 & Q_-^\dagger(2_R,1\subset\bar{4}) & Q_-^\dagger(2_L,1\subset 4) & Q_+^\dagger(2_R,1\subset 4) & Q_+^\dagger(2_L,1\subset\bar{4}) \\
\hline
Q_- \to \; Q_-(2_L,1\subset 4) & & & & \\
Q_- (2_R,1\subset\bar{4}) & & & & \\
\hline
Q_+ \to \; Q_+(2_L,1\subset\bar{4}) & & & & \\
Q_+(2_R,1\subset 4) & & & &
\end{array}
$$

$\Gamma^\mu\Gamma^0 P_\mu$

$(\Gamma^{10}\Gamma^0)Z^0 + (\Gamma^{mn}\Gamma^0)Z^2 + (\Gamma^0\Gamma^{klmn}\Gamma^0)Z^4 + (\Gamma^{abklmn}\Gamma^0)Z^6$

$\boxed{\text{read out}}$

$$
\begin{array}{cc|cc}
\{Q^1_\alpha, \bar{Q}_{\dot\beta,1}\} & 0 & 0 & \{Q^1_\alpha, Q^2_\beta\} \\
0 & & \{\bar{Q}_{\dot\alpha,1}, \bar{Q}_{\dot\beta,2}\} & 0 \\
\hline
 & & \{Q^2_\alpha, \bar{Q}_{\dot\beta,2}\} & 0 \\
 & & 0 &
\end{array}
$$

read out (ignore Γ^{10} and Γ^0)

$$Z_{4d} \sim \langle\downarrow\downarrow\downarrow|\,(*)\,|\downarrow\downarrow\downarrow\rangle$$

$$
\begin{aligned}
(*) = {} & T_{D0}N_{D0} + T_{D2}N_{D2,i}\int_{C^i}(d\xi^1\wedge d\xi^2)\,\frac{\partial y^{m_1}}{\partial\xi^1}\frac{\partial y^{m_2}}{\partial\xi^2}\,e_{m_1}{}^{a_1}e_{m_2}{}^{a_2}\left(\gamma^{a_1}\gamma^{a_2}\right) \\
& + T_{D4}N_{D4}{}^{i}\int_{\Sigma_i} d\xi^1\wedge\cdots\wedge d\xi^4\,\frac{\partial y^{m_1}}{\partial\xi^1}\cdots\frac{\partial y^{m_4}}{\partial\xi^4}\,e_{m_1}{}^{a_1}\cdots e_{m_4}{}^{a_4}\left(\gamma^{a_1a_2a_3a_4}\right) \\
& + T_{D6}N_{D6}\int_M d\xi^1\wedge\cdots\wedge d\xi^6\,\frac{\partial y^{m_1}}{\partial\xi^1}\cdots\frac{\partial y^{m_6}}{\partial\xi^6}\,e_{m_1}{}^{a_1}\sim e_{m_6}{}^{a_6}\left(\gamma^{a_1a_2a_3a_4a_5a_6}\right)
\end{aligned}
$$

By using
$$
\left\{
\begin{aligned}
\gamma^{4,5} &= \tau^{1,2}\otimes\mathbb{1}\otimes\mathbb{1} \\
\gamma^{6,7} &= \tau^3\otimes\tau^{1,2}\otimes\mathbb{1} \\
\gamma^{8,9} &= \tau^3\otimes\tau^3\otimes\tau^{1,2}
\end{aligned}
\right.
$$

$$\boxed{Z_{4d} \sim \left(N_{D0} - i\,N_{D2,i}\frac{\mathrm{vol}(C^i)}{l_s^2} - N_{D4}{}^{j}\frac{\mathrm{vol}(\Sigma_j)}{l_s^4} + i\,\frac{\mathrm{vol}(M)}{l_s^6}\cdot N_{D6}\right)\times\frac{2\pi}{l_s g_s}}$$

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D-brane charge $\Leftrightarrow$ $(N_{D0}, N_{D2,i}, N_{D4}^{i}, N_{D6}) = v$.

a pair of D-branes : stable coexistence if $\mathrm{Arg}(Z(v_1)) = \mathrm{Arg}(Z(v_2))$

instability $|Z(v_1)| + |Z(v_2)| \geq |Z(v_1+v_2)|$

☆ Dp and D(p+2) or Dp and D(p+6) not a stable config.

☆ D4 and D0, D6 and D2 not a stable config.

D4 and $\overline{D0}$ D6 and $\overline{D2}$ coexist and stable.

in a flat spacetime

$\varepsilon' = \Gamma^0 \varepsilon$ & $\varepsilon' = \Gamma^{01234}\varepsilon \Rightarrow \boxed{\varepsilon = \Gamma^{1234}\varepsilon}$

$\varepsilon' = -\Gamma^0 \varepsilon$ & $\varepsilon' = \Gamma^{01234}\varepsilon \Rightarrow \boxed{\varepsilon = -\Gamma^{1234}\varepsilon}$

on a background w/ δ parameters

special holonomy : the covariantly const spinor should be part of "the δ parameters".

(either non-BPS or forming a bound state.)

☆ include B-field ver.

for a D-brane system wrapped on holomorphic cycles in a Calabi-Yau n-fold X ...

$$Z \simeq \frac{2\pi}{l_s g_s}\int_X e^{-t} \wedge v = (-1)^n \frac{2\pi}{l_s g_s}\int_X e^{t} \wedge v^{\vee}$$

$t := (B+iJ)/l_s^2$ 2-form on X

$v^{\vee}$: for $v \in H^{even}(X;\mathbb{Q})$ multiply $(-1)^k$ to the $2k$-form component.

$\boxed{n=3 \quad (X = CY_3)}$

$$\left[\begin{aligned} & v_{D6}(V_X) = ch(V_X)\,\Gamma_c(TX) \\ & v_{D4}(S, V_S) = (i_S)_*\left[ch(V_S)\, e^{\frac{1}{2}c_1(TS)} \sqrt{\left(\frac{\hat{A}(TS)}{\hat{A}(N_{S|X})}\right)_{hol}}\right] = i_{S*}\left[ch(V_S)\sqrt{\frac{td(TS)}{td(N_{S|X})}}\right] \end{aligned}\right.$$

S : surface in X $\quad V_S$: bdle on S

$$v_{D2}(C, V_C) = (i_C)_*\left[ch(V_C)\, e^{\frac{1}{2}c_1(TC)}\right]$$

C : curve in X, V_C : bdle on C

$$v_{D0} = [pt]$$

$$td(E) = 1 + \frac{1}{2}c_1(E) + \frac{c_2(E) + c_1(E)^2}{12} + \frac{c_1(E)c_2(E)}{24} + \cdots$$

$$\hat{A}(E_{hol}) = 1 - \frac{1}{12}ch_2(E) + \left(\frac{1}{2\cdot 12^2}(ch_2(E))^2 + \frac{1}{2^3\cdot 15}ch_4(E)\right) + \cdots$$

$$\Gamma_c(E) = \exp\left[\frac{-\gamma}{2\pi i}ch_1(E) + \frac{\zeta(2)}{(2\pi i)^2}ch_2(E) - \frac{2!\,\zeta(3)}{(2\pi i)^3}ch_3(E) + \frac{3!\,\zeta(4)}{(2\pi i)^4}ch_4(E) \cdots\right]$$

$$E = TX,\ X = CY_3 \Rightarrow \Gamma_c(TX) = \left[1 + \frac{1}{24}c_2(TX) - \frac{\zeta(3)}{(2\pi i)^3}c_3(TX)\right].$$

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$Z(v_{D0}) \sim 1.$

$Z(v_{D2}(C, V_c)) \sim \left[\left(-t - \frac{1}{2}K_C\right)\right]_C^{\times(rk)} + \left[c_1(V_c)\right]_C$

$\left[c_1(V_S)\cdot\left(-t-\frac{1}{2}K_{C_{VS}}\right)+\left(c_1(V_S)^2-c_2(V_S)\right)\right]_{S'}$

$Z(v_{D4}(S', V_S)) \sim \left[\frac{t^2}{2} - \frac{1}{2}c_1(TS)\wedge t + \frac{c_2(TS)+3c_1(TS)^2}{24}\right]_{S'}^{\times(rk)} + \left[c_1(V_S)\left(-t-\frac{1}{2}K_S\right) + ch_2(V_S)\right]_{S'}$

$Z(v_{D6}(V_X)) \sim \left[-\frac{t^3}{3!} - t\wedge\frac{c_2(TX)}{24} - \frac{\zeta(3)}{(2\pi i)^3}c_3(TX)\right]_X^{\times(rk)} + \left[c_1(V_X)\left(\frac{t^2}{2}+\frac{c_2(TX)}{24}\right)\right]_X$

$+\left[ch_2(V_X)\wedge(-t)\right]_X + \left[ch_3(V_X)\right]_X$

$= \left[-\frac{t^3}{3!} - t\wedge\frac{c_2(TX)}{24} - \frac{\zeta(3)}{(2\pi i)^3}c_3(TX)\right]_X + Z(v_{D4}(c_1(V_X), \mathbb{1}))$

$+ \left(c_2(V_X) - c_1(V_X)^2\right)\cdot t + \frac{1}{2}\left(c_3(V_X) - c_2(V_X)c_1(V_X)\right)_?$

on $X = CY_3$

$(rk \times D4 \text{ on } S') + (N_0 \times D0) \longrightarrow (rk \cdot D4\text{-branes on } S' \text{ w/ } V_S)$

if $rk\cdot N_0 > 0$. $\quad Z(v_{D4}(S', V_S))$

$ch(V_S) = rk + 0 + \underbrace{N\cdot[pt]}_{= -c_2(V_S)}$

$\boxed{n=2 \quad (X = K3)}$

$$\begin{cases} v_{D4}(V_X) = ch(V_X)\left(\Gamma_c(TX) = \sqrt{\hat{A}(TX)} = \sqrt{td(TX)} = 1 + \frac{c_2(TX)}{24}\right) \\ v_{D2}(C, V_c) = (i_C)_*\left[ch(V_c)\, e^{\frac{1}{2}c_1(TC)}\right] \\ v_{D0} = [pt] \end{cases}$$

(D4 and D0 : the same as above.)

D0 in D4 $\longleftrightarrow$ $ch_2(V) > 0$

$\overline{\text{D0}}$ in D4 $\longleftrightarrow$ $ch_2(V) < 0$ (masses just add up.)

(instanton # ~ $-ch_2$)

("Z" for 6d (1,1) SUSY algebra central charge)

$\left(ch_3(V) = \frac{1}{6}c_1(V)^3 - \frac{1}{2}c_2(V)\cdot c_1(V) + \frac{1}{2}c_3(V)\right)$ $\left(ch_4(V) = c_1^4 + \frac{1}{6}c_3c_1 - \frac{1}{6}c_4 + \frac{1}{12}c_2^2 - \frac{1}{6}c_2c_1^2\right)$

PLUS

☆ An expression that is believed to be correct.

$$|Z_{4d}| = M_{pl}\sqrt{8\pi}\cdot\left|(v_I X^I + m^I F_I)\, e^{+\hat{K}/2}\right|$$

$$\hat{K} = -\ln\left(i\left(X^I \bar{F}_I - \bar{X}^I F_I\right)\right)$$

$$\left(t = \frac{(B+iJ)}{\ell_s^2} = D_i t^i \qquad H^2(X;\mathbb{Z}) = \mathrm{Span}\{D_i\} \right)$$

$$X^I = (1, t^i)^T \qquad F_I = \left((2 - t^i\partial_i)\mathcal{F},\ \partial_i \mathcal{F}\right)^T$$

$$\mathcal{F} = \int_X \left(\frac{t^3}{3!} - \frac{c_2(TX)\wedge t}{24} - \frac{\zeta(3)}{(2\pi i)^3}\frac{c_3(TX)}{2}\right) - \frac{1}{2} a_{ij} t^i t^j + \frac{1}{(2\pi i)^3}\Sigma_\beta n_\beta \mathrm{Li}_3\left(e^{2\pi i\langle\beta, t\rangle}\right)$$

choose e.g. $a_{ij} = \begin{cases} i<j & \frac{1}{2}\int_{D_i} c_1(TD_i)\cdot D_j \\ j<i & \frac{1}{2}\int_{D_j} c_1(TD_j)\cdot D_i \end{cases}$ (or mod. $+\mathbb{Z}$, ij symmetric)

$$\left(M_{pl}\sqrt{8\pi}\,|e^{\hat{K}/2}| \underset{\text{large vol}}{\simeq} \frac{M_{pl}\sqrt{8\pi}}{\sqrt{i\int_X (t-\bar{t})^3/3!}} = \frac{M_{pl}\sqrt{8\pi}}{\sqrt{8\,\mathrm{vol}(X)/\ell_s^6}} = \frac{2\pi}{\ell_s g_s} \right)$$

$$(N_{D0};\ -N_{D2,i};\ N_{D6},\ N_{D4}^{i}) \begin{bmatrix} 1 \\ t^i + g(C^i) - 1 \\ -t^3/3! - t\wedge\frac{c_2(TX)}{24} - \frac{\zeta(3)}{(2\pi i)^3} c_3(TX) \\ \int_{S_i} \frac{t^2}{2} - \frac{1}{2}c_1(TS)\wedge t + \frac{c_2(TS)+3c_1(TS)^2}{24} \end{bmatrix}$$

$$= (N_{D0},\ -N_{D2,i};\ N_{D6},\ N_{D4}^{i}) \begin{bmatrix} 1 & 0 & 0 & 0 \\ g(C_i)-1 & \mathbb{1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \chi & & 0 & \mathbb{1} \end{bmatrix} \begin{bmatrix} 1 \\ t^j \\ (2-t^i\partial_i)\mathcal{F} \\ \partial_j \mathcal{F} \end{bmatrix}$$

$+ch_2(TS_i)$ $\quad \left(a_{ij} - \frac{1}{2}\int_{D_i} c_1(TD_i)\cdot D_j\right)$

$$\underbrace{\qquad\qquad\qquad\qquad}_{(v_I,\ m^I)}$$

IIB/($CY_3 = W$) D3-branes on a sLag L $\Rightarrow$ $Z_{4d} = M_{pl}\sqrt{8\pi}\left|e^{\hat{K}/2}(v_I X^I + m^I F_I)\right|$

$\hat{K} = -\ln\left(i\int_W \Omega_W \wedge \bar{\Omega}_W\right)$ $\quad (v_I X^I + m^I F_I) = \int_L \Omega_W^{(3,0)}$

$\boxed{(v, m) \Leftrightarrow H_3(W;\mathbb{Z})}$ $\quad$ ($L \to L'$ by an $SU(3)$ rot'n) $\Leftrightarrow$ $\mathrm{Arg}(Z(L)) = \mathrm{Arg}(Z(L'))$ [coexist PLUS stable.]

§ 2.7

§ 2.2. E

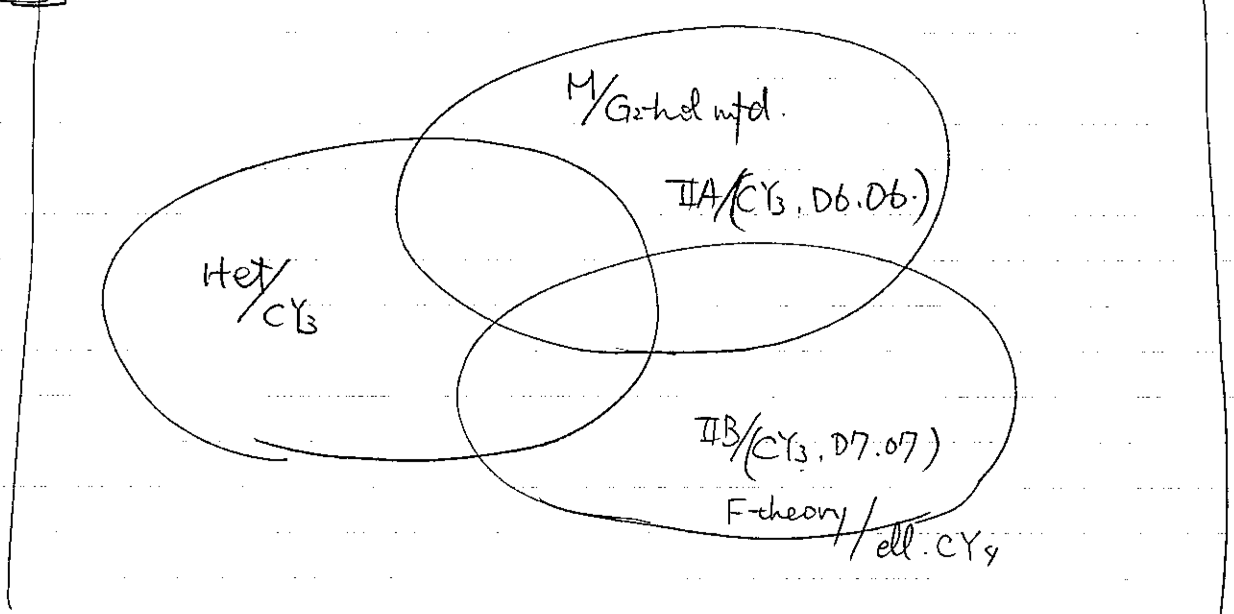
The adiabatic "argument"

The Het/T^3 — M-theory/K3 duality suggests $\mathrm{IIA}/(CY_3$ w/ D6, O6$)$

Het/(T^3-fibred CY_3) — M-theory/(K3-fibred G_2-hol mfd) duality.

§ 2.2. G

any ristriction on types of degenerate fibrat'n ??



("The" landscape of string vacua with D=4 N=1 SUSY)

(Tatar Watari th/0602238)

Low-energy physics

Microscopic formulation

Gauge group, Symmetry, Interact'n $\Longleftrightarrow$ algebra
- current algebra in Het.
- intersect'n of topological cycles in M-theory, F-theory
- open string reconnection in Type IIA, IIB.

geometrization of gauge theory.

particle multiplicity (the number of generations) $\Longleftrightarrow$ topology
- $\chi(M_6 ; \rho_a(V))$ in Het.
- $L_1 \cdot L_2$ in IIA.
- $\int_{S_1 \cdot S_2} c_1\left(\frac{"F"}{2\pi}\right)$ in IIB.
- $\int_S G^{(4)}$ in F-theory

coupling constant $\Longleftrightarrow$ where in the moduli space

Our knowledge on the Standard Model does not ~~one~~ favour one formulation of string theory over the others in a simplest possible way because of the string duality.

Perhaps "God is in the details"?

◎ SU(5) GUT up-type Yukawa $\int u^c H_u c$ $(10^{ab}\, 10^{cd}\, H(5)^e\, \epsilon_{abcde})$ is from the deformation of $E_6 \to U(2) \times SU(5)_{GUT}$ (open strings on Dbrane + Oplanes are not enough)

◎ In a common moduli space of duality, one formulation provides calculability in one corner while the other formulation provides calculability in another corner. $\Rightarrow$ $\boxed{\text{still makes sense with the SM which corner fits better.}}$

§3. To Go beyond the LO SUGRA Approximation.

§3.1 Kähler (SUSY) vs non-Kähler (non-SUSY) and α'-corrections.

Refs.

Gross, Witten Nucl. Phys. B277 ('86) 1.
Grisaru, van de Ven, Zanon Phys. Lett. B173 ('86) 423
Freeman, Pope Phys. Lett. B174 ('86) 48.
Nemeschansky, Sen Phys. Lett. B178 ('86) 365.
L. Witten, E. Witten Nucl. Phys. B281 ('87) 109

Yau's theorem (Calabi conjecture)

Let (M, ω, J) be a Kähler manifold.

Pick up a cohomology class $[\omega_*] \in H^{1,1}(M;\mathbb{R})$.
(with positivity constraint)

Choose any $\mathbb{R}$-valued closed (1.1)-form ρ on M
whose cohomology class $[\rho]$ is equal to $2\pi c_1(TM)$.

Then there is a unique choice from

$$[\omega_*] = \left\{ \begin{array}{l|l} \mathbb{R}\text{-valued (1.1)-form} & \omega_0 \text{ is a } \mathbb{R}\text{-valued closed (1.1)-form on } M \\ \omega_0 + i\partial\bar\partial\varphi & \text{w/ } [\omega_0]=[\omega_*] \\ & \forall\varphi.\ \mathbb{R}\text{-valued fcn. globally defined over } M \end{array} \right\}$$

(and the Kähler metric corresponding to $(\omega_0 + i\partial\bar\partial\varphi)$)

so that $\mathrm{Ric}(\omega_0 + i\partial\bar\partial\varphi) = \rho$.

Cor: use it in the case $c_1(TM) = 0 \in H^{1,1}(M;\mathbb{R})$
$\rho = 0$ in particular. $\Rightarrow$ $\exists$ Ricci flat metric in any $[\omega_*]$
by exploiting +(exact 2-form)

analogous to extracting a harmonic diff. form
from a cohomology class (fix the (+ exact form) freedom)
on a Kähler mfd.

Now think of ($g=0$?) higher α' corrections. (Type II string theory)

- $\underline{\beta = 0}$ in non-lin σ-model (only the UV of 1+1-dim field theory matters?) (g : irrelevant?)

$$S \sim \int d^2\sigma \frac{1}{4\pi\alpha'} G_{MN}(X)(dX)(dX) + \cdots$$

$$\Rightarrow \left(R^2/\alpha'\right) \sim \frac{1}{g^2} \qquad \text{perturbation in } \left(\alpha'/R^2\right)$$

- string S-matrix calculation $\Longrightarrow$ effective action on the spacetime.

$\Downarrow$

equation of motion

conjectured to agree?

At the leading order (1-loop in NLSM perturbation)

$$\begin{cases} \beta_\phi & 0 = \mathcal{O}(\delta c) - \frac{\alpha'}{2}\nabla^2\phi \\ \beta_{G_{MN}/\alpha'} & 0 = R_{MN} \end{cases}$$

Ricci-flat metric & $\phi = $ (const) on M. is a solut'n.

At higher order (until 4-loop in NLSM)

$$\begin{cases} 0 = \nabla^2\phi + (\alpha')^3 (R^4)_{\text{scalar}} \\ 0 = R_{MN} + (\alpha')^3 (R^4)_{MN} \end{cases}$$

For a generic Ricci flat metric, $(R^4)_{\text{scalar}} \neq 0$. $\int_M (R^4)_{\text{scalar}} \neq 0$.

although $(R^4)_{\text{scalar}} = 0$. $\underline{\text{if}}$ the Ricci-flat metric is $\underline{\text{also Kähler}}$.

When $\int_M (R^4)_{\text{scalar}} \neq 0$, $\int_M \nabla^2\phi = \nabla^2_{\mathbb{R}^{3,1}}\left(\int_M \phi\right) + \underbrace{\int_M (\nabla_M^2 \phi)}_{=0}$.

$$\text{so} \quad (\nabla^2_{\mathbb{R}^{3,1}}\phi) + \boxed{\frac{\int_M (R^4)_{\text{scalar}}}{\text{vol}(M)}(\alpha')^3}\,\phi = 0$$

$\sim (\alpha')^3/R^8$

tremendous $\underline{\text{space-time variat'n of } \phi \text{ is required on } \mathbb{R}^{3,1}}$.

If Kähler, $\phi = $ const. is fine. Now $\text{Ric}(\omega_0 + i\partial\bar\partial\varphi) + \Delta\beta = 0$ is the condition for the metric.

modified version of Yau's thm. (?)

[Gross Witten], [Freeman Pope]

The $\boxed{0 = R_{MN} + (\alpha')^3 (R^4)_{MN} + \cdots}$ condition at any finite order.

if M is Kähler and $c_1(TM) = 0$ $\left(\begin{array}{c} \text{so we can take} \\ \text{the LO metric } \tilde{G} \text{ (and } \tilde{K}) \text{ to be} \\ \text{Ricci flat.} \quad (\tilde{\omega}_0) \end{array} \right)$

◎ The $N=(2,2)$ NLSM on $(1+1)$-dim. has a notion of

$\beta_K = \beta_K^{(1)} + \Delta\beta_K$. $\quad \Delta\beta_K = \sum_{n=2}^{\infty} \Delta\beta_K^{(n)}$. $\quad \boxed{\Delta\beta_K^{(n)} \propto (\alpha')^{n-1}.}$

[β_K : given for individual local patches of M.]

☆ $\beta_K^{(1)} = 0$ when we choose the LO metric to be Ricci flat.

☆ for $\Delta\beta_K^{(n)}$ (the n-loop contribution in the NLSM)
with $n \geq 2$, [non-trivial] $\Delta\beta_K^{(n)}$: well-defined scalar fcn on M.
given explicitly by $G_{i\bar{j}}$, $R_{i\bar{j}k\bar{l}}$. (See [Nemeschansky Sen]).
(no need for patches covering M.)

☆ $\Delta\beta_K^{(n)} \sim (\alpha')^{n-1} (R^{n-1})_{\text{scalar}}$.

◎ Given the property above there is a systematic way to determine

$$\left\{ \begin{array}{l} K = \tilde{K} + \sum_{n=2}^{\infty} \delta K^{(n)} \\ G_{i\bar{j}} = \tilde{G}_{i\bar{j}} + \sum_{n=2}^{\infty} \delta G_{i\bar{j}}^{(n)} \\ \omega_{i\bar{j}} = (\tilde{\omega}_0)_{i\bar{j}} + \sum_{n=2}^{\infty} \partial\bar{\partial}_{\bar{j}} (\delta K^{(n)}) \end{array} \right. \longleftarrow \left\{ \begin{array}{l} \delta K^{(n)} = (\alpha')^n (\partial^{2(n-1)} \text{ on } \tilde{G}) \\ \delta G_{i\bar{j}}^{(n)} = (\alpha')^n (\partial^{2n} \text{ on } \tilde{G}) \end{array} \right.$$

at any finite order n. (☆)

At $\mathcal{O}((\alpha')^{n-1})$. $\quad 0 \overset{\text{(require)}}{=} \underbrace{\left((\Delta\beta_K)^{(n)}\big|_{K=\tilde{K}}\right)}_{\text{globally defined scalar on } M} + \Delta_{\tilde{G}}(\delta K^{(n-1)}) + \left(\begin{array}{c} \text{calculable terms} \\ \text{from } \delta G^{(l<n-1)}, \tilde{G}, \\ \Delta\beta_K^{(l<n)} \end{array} \right)$

$\longrightarrow$ @$(\alpha')^{n-1}$

$\Rightarrow$ $\boxed{\delta K^{(n-1)} \sim (\alpha')^{n-1} \text{ determined locally} ; (\delta K_\alpha^{(n-1)} - \delta K_\beta^{(n-1)}) \sim \text{hol} + \overline{\text{hol}} \text{ allowed}}$

$\Rightarrow$ no need for $\mathbb{R}^{3,1}$-dependent bg. for $\beta = 0$!

$\Rightarrow$ $G_{i\bar{j}}$ is not Ricci flat. $(\Delta\beta_K^{(n)})$ is cancelled by changing $\begin{array}{l} G \text{ from } \tilde{G} \\ K \text{ from } \tilde{K} \end{array}$ by $\begin{array}{l} \delta G^{(n-1)} \\ \delta K^{(n-1)} \end{array}$
and changing $\beta_K^{(1)}$ by $\Delta_{\tilde{G}}(\delta K^{(n-1)})$.

The non-$\Delta_{\tilde{G}}(\delta K^{(n-1)})$ terms in (☆) $\Rightarrow$ $^\exists(\text{const} = a_n) + \Delta_{\tilde{G}} \left(\begin{array}{c} ^\exists \text{global def} \\ \text{scalar on } M =: \varphi_n \end{array} \right)$
(see [Nemeschansky Sen])

$\Rightarrow$ $\boxed{\delta K^{(n-1)} = -\varphi_n - \frac{a_n}{\dim_{\mathbb{C}}(M)} \tilde{K}}$ $(+\text{hol} + \overline{\text{hol}})$

$\Rightarrow$ $\boxed{[\omega] = [\tilde{\omega}_0] - \sum_{n=2}^{\infty} \frac{a_n}{\dim_{\mathbb{C}}(M)} [\omega_0]}$ differ by scaling in $H^2(M; \mathbb{R})$. (?)

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The $\boxed{0 = \nabla^2\phi + (\alpha')^3 (R^4)_{\text{scalar}} + \cdots}$ condition
$\boxed{\frac{1}{\alpha'}(\text{correctn to } C)}$

$\Longleftrightarrow$ belief.

compute S-matrix to higher order in $(\alpha'\partial^2)$.
then write down the effective action
and then derive equation of motns.
finally make sure that a chose bg satisfy the E.o.M's.
(modify the bg. if necessary)

cf. Sen Phys. Lett. B170 ('86) 370.
Does the 0-mode counting in [Gross Witten] and [Freeman Pope] work for higher $(\alpha'\partial^2)$?

Unbroken supersymmetry in the effective theory on $\mathbb{R}^{3,1}$?

$\begin{bmatrix} \delta\Psi_M \\ \delta\lambda \\ \vdots \end{bmatrix}$ will receive $(\alpha'\partial^2)$-corrections.
because the 10D action does.

(supersymmetry-preserving bg. often satisfies equatn of motions.)

Type II orientifolds ? Het ?
(just orientifold proj. on equatn of motions?)

(cf. Becker et.al th/0204254)

PLUS