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## §3.2 Space-time supersymmetry and Worldsheet supersymmetry

Type II string theory : $N=(1,1)$ supergravity on worldsheet.

$(b,c) \leftrightarrow (\beta,\gamma)_{left}$ $(\tilde{b},\tilde{c}) \leftrightarrow (\tilde{\beta},\tilde{\gamma})_{right}$

To obtain 4d Minkowski spacetime we should have

$(c,\tilde{c})=(9,9)$ $N=(1,1)$ SCFT (unitary, modular invariant) w/ well-defined $(-1)^F \in \{\pm i\}$, $(-1)^{\tilde{F}} \in \{\pm i\}$

with discrete spectrum of $L_0$ and $\tilde{L}_0$

$\Uparrow$ phenomenology $\qquad$ $\uparrow$ GSO proj.

$$\Rightarrow \mathcal{H}^{tot} = \oplus_i \begin{pmatrix} \text{irr. repr. of} \\ \text{left mover} \\ T(z), G(z), (-1)^F \end{pmatrix} \otimes \begin{pmatrix} \text{irr. repr. of} \\ \text{right mover} \\ \tilde{T}(\bar{z}), \tilde{G}(\bar{z}), (-1)^{\tilde{F}} \end{pmatrix}$$

(NS-type repr. and R-type repr.).

4d $N=2$ supersymmetry

$$\Longleftrightarrow \exists \begin{pmatrix} \mathcal{O}(z)\otimes \mathbb{1} & h_{\mathcal{O}}=3/8 & (-1)^F=+i \\ \mathcal{O}'(z)\otimes\mathbb{1} & h_{\mathcal{O}'}=3/8 & (-1)^F=-i \\ \mathcal{O}(z)\,\mathcal{O}'(0)\sim \frac{1}{z^{3/4}}+\cdots \end{pmatrix} \text{ and } \exists \begin{pmatrix} \mathbb{1}\otimes\tilde{\mathcal{O}}(\bar{z}) & \tilde{h}=3/8 & (-1)^{\tilde{F}}=i \\ \mathbb{1}\otimes\tilde{\mathcal{O}}'(\bar{z}) & \tilde{h}=3/8 & (-1)^{\tilde{F}}=-i \\ \tilde{\mathcal{O}}(\bar{z})\,\tilde{\mathcal{O}}'(0)\sim\frac{1}{(\bar{z})^{3/4}} \end{pmatrix}$$

in $\mathcal{H}_R\otimes\tilde{\mathcal{H}}_{NS}$ $\quad$ $(c,\tilde{c})=(9,9)$ SCFT $\quad$ in $\mathcal{H}_{NS}\otimes\tilde{\mathcal{H}}_R$

gravitino vertex operator

$$\left(e^{-\phi/2}\,e^{\frac{i}{2}\varepsilon\cdot H}\,\mathcal{O}(z)\right)\cdot\left(e^{-\tilde{\phi}}\,\tilde{\psi}_\mu\cdot\mathbb{1}\right) \qquad \left(e^{-\phi}\,\psi_\mu\cdot\mathbb{1}\right)\cdot\left(e^{-\tilde{\phi}/2}\,e^{\frac{i}{2}\varepsilon\cdot\tilde{H}}\,\tilde{\mathcal{O}}(\bar{z})\right)$$

$h=3/8$, $\mathbb{R}^{3,1}$, $\mathbb{R}^{3,1}$, $h=1/2$.

$$\left(e^{-\phi/2}\,e^{\frac{i}{2}\varepsilon'\cdot H}\,\mathcal{O}'(z)\right)\left(e^{-\tilde{\phi}}\,\tilde{\psi}_\mu\cdot\mathbb{1}\right) \qquad \left(e^{-\phi}\,\psi_\mu\,\mathbb{1}\right)\cdot\left(e^{-\tilde{\phi}/2}\,e^{\frac{i}{2}\varepsilon'\cdot\tilde{H}}\,\tilde{\mathcal{O}}'(\bar{z})\right)$$

$h=1$ $\quad$ $\tilde{h}=1$

SUSY generator

$$Q_\varepsilon^{I=1} = \oint dz\left(e^{-\phi/2}\,e^{\frac{i}{2}\varepsilon\cdot H}\,\mathcal{O}\right) \qquad Q_\varepsilon^{I=2} = \oint d\bar{z}\,e^{-\tilde{\phi}/2}\,e^{\frac{i}{2}\varepsilon\cdot\tilde{H}}\,\tilde{\mathcal{O}}(\bar{z})$$

$$\bar{Q}_{\varepsilon',I=1} = \oint dz\left(e^{-\phi/2}\,e^{\frac{i}{2}\varepsilon'\cdot H}\,\mathcal{O}'\right) \qquad \bar{Q}_{\varepsilon',I=2} = \oint d\bar{z}\,e^{-\tilde{\phi}/2}\,e^{\frac{i}{2}\varepsilon'\cdot\tilde{H}}\,\tilde{\mathcal{O}}'(\bar{z})$$

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4d $N=2$ space-time supersymmetry $\Rightarrow$ world sheet $N=(2,2)$ superconformal algebra ?

A. Sen Nucl. Phys. B278 ('86) 289
A. Sen Nucl. Phys. B284 ('87) 423

Banks Dixon Friedan Martinec Nucl. Phys. B299 ('88) 613.

consider

$$\langle \mathcal{O}(z_1)\, \mathcal{O}'(z_2)\, \mathcal{O}(z_3)\, \mathcal{O}'(z_4)\rangle = \left(\frac{z_{13}\; z_{24}}{z_{12}\, z_{14}\; z_{23}\, z_{34}}\right)^{3/4}.$$

in the $(c,\tilde{c})=(9,9)$ SCFT

Expand for small $z_{12}$ and $z_{34}$

$$\Rightarrow \langle \cdots \rangle = \frac{1}{(z_{12}\, z_{34})^{3/4}}\left(1+\frac{3}{4}\frac{z_{12}\, z_{34}}{z_{23}\, z_{24}}+\cdots\right)$$

weight $= 1 \in \frac{1}{2}\mathbb{Z}$

$$\Rightarrow \text{OPE} \quad \mathcal{O}(z)\, \mathcal{O}'(0) \sim \frac{1}{z^{3/4}}\left(1+\frac{1}{2} z J(0)+\cdots\right)$$

[w/ $J(z)J(0) \sim \frac{3}{z^2}$ (normalize $J(0)$ this way)]

$\to J(0)|0\rangle \notin \left(L_{-2}|0\rangle,\ G_{-3/2}|0\rangle \cdots\right) \longrightarrow$ expand the algebra.

$G$: eigenstate decomposit'n under $J_0 \Rightarrow G^{\pm}$

$\{T(z),\ G^{\pm}(z),\ J(z)\}$ forms an $N=2$ superconformal algebra.

(non-trivial)
- $J$: primary
- $J_0$ decomp. of $G$ only $G^{\pm}$ no $G^0$ etc

crossing symmetry

$$\left(\begin{array}{l}\langle \mathcal{O}(z)\, \mathcal{O}'(0)\, J(\zeta)\rangle \\ = \frac{3}{2}\frac{(z-0)^{1/4}}{(\zeta-0)(\zeta-z)}\end{array}\right) \longrightarrow \left\{\begin{array}{l} J(z)\, \mathcal{O}(0) \sim \frac{3/2}{z}\mathcal{O}(0) \\ J(z)\, \mathcal{O}'(0) \sim \frac{-3/2}{z}\mathcal{O}(0). \end{array}\right.$$

$$\Rightarrow \quad h=3/8 \cdot \left[\begin{array}{lll} \mathcal{O}=e^{\frac{i}{2}\sqrt{3}\varphi} & J=i\sqrt{3}\,\partial\varphi & \varphi(z)\varphi(0) \sim -\ln(z) \\ \mathcal{O}'=e^{-\frac{i}{2}\sqrt{3}\varphi} & & \Delta T(z) \sim -\frac{1}{2}(\partial\varphi)^2 \end{array}\right]$$

$$\Rightarrow \quad \mathcal{O}(z)\, \mathcal{O}(0) \sim z^{\frac{3}{4}}\left(e^{i\sqrt{3}\varphi}\right)$$

$h = 3/8 + 3/8 \Leftrightarrow -3/4 \quad 3/2 \in \frac{1}{2}\mathbb{Z}$

$$\left(\begin{array}{l} e^{\pm\frac{i}{2}\sqrt{3}\varphi}, \\ e^{\pm i\sqrt{3}\varphi} \end{array}\right)$$

called spectral flow operators.

They are in $\mathcal{H}_R \otimes \mathcal{H}_{NS}$, $\mathcal{H}_{NS} \otimes \mathcal{H}_{NS}$.

$\Rightarrow$ expand the algebra.

$$\left\{\begin{array}{l} T(z),\ G^{\pm}(z),\ J(z) \\ e^{\pm i\sqrt{3}\varphi(z)} \end{array}\right\} \text{PLUS}$$

singled-valued-ness of OPE in the $(c,\tilde{c})=(9,9)$ SCFT NS-NS sector

$$\left(e^{i\sqrt{3}\varphi(z)}\otimes\mathbb{1}\right)\left(e^{i\frac{Q}{\sqrt{3}}\varphi(0)}\cdot(\text{any neutral op})(0)\otimes\widetilde{\text{any op.}}(\bar{0})\right)\sim z^{Q}\cdot(\text{operator})$$

Summary

$\Rightarrow Q\in\mathbb{Z}$.

- The superchiral algebra contains
$$\{\underbrace{T(z),\ G^{\pm}(z),\ J(z)}_{N=2},\ e^{\pm i\sqrt{3}\varphi}\}\times\{\underbrace{\tilde{T}(\bar{z}),\ \tilde{G}^{\pm}(\bar{z}),\ \tilde{J}(\bar{z})}_{N=2},\ e^{\pm i\sqrt{3}\tilde{\varphi}}\}.$$
- $J$-charged $e^{i\frac{Q}{\sqrt{3}}\varphi}\times(\text{neutral})$ $\quad$ $\tilde{J}$-charged $e^{i\frac{\tilde{Q}}{\sqrt{3}}\tilde{\varphi}}(\text{neutral})$ $\quad Q\in\mathbb{Z}$ $\quad \tilde{Q}\in\mathbb{Z}$
- $\mathcal{H}_R\otimes\tilde{\mathcal{H}}_{NS}\ni\left(e^{\pm\frac{i}{2}\sqrt{3}\varphi}\otimes\mathbb{1}\right)$ $\quad$ $\tilde{\mathcal{H}}_{NS}\otimes\mathcal{H}_R\ni\mathbb{1}\otimes e^{\pm\frac{i}{2}\sqrt{3}\tilde{\varphi}}$

+ modular inv, unitary, discrete spectrum, $(c,\tilde{c})=(9,9)$.

converse : obvious $\left(\begin{array}{l}\text{gravitino vertex op. SUSY change}\\ \text{by}\ (e^{\pm\frac{i}{2}\sqrt{3}\varphi}\ \&\ e^{\pm\frac{i}{2}\sqrt{3}\tilde{\varphi}})\end{array}\right)$

$N=(2,2)$ SCA : not enough to guarantee 4d $\mathcal{N}=2$ SUSY
(eg. tensor of $N=2$ minimal models w/o orbifold projet'n.)

To explore further.

Lerche Vafa Warner
Nucl. Phys. B324 ('89) 427.

Rohm Witten
Annals Phys. 170 ('86) 454.

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## §3.4 Stable D-brane configuration

Type IIA/($CY_3 = M$)

D-brane change $v', v'' \in H^{even}(M;\mathbb{Q})$.

If $\mathrm{Arg}(Z(v')) = \mathrm{Arg}(Z(v''))$ : coexis.

If $\mathrm{Arg}(Z(v')) \neq \mathrm{Arg}(Z(v''))$ then. $|Z(v'+v'')| < |Z(v')| + |Z(v'')|$

$\Rightarrow$
- a bound state w/ change ($v'+v''=v$) formed if such a state exists.
- a non-BPS configurat'n. if such a state does not exist.

For which $v \in H^{even}(M:\mathbb{Q})$ does a BPS state exist?

D6-D4-D2-D0 bound state $\Leftrightarrow$ vector bdle on $M$ w/ connect'n satisfying E.o.M.

(Thm Donaldson-Uhlenbeck-Yau) $\Updownarrow$

holomorphic vector bdle on $M$ that is stable

$\exists$ known inequalities on characteristic classes.

D4-D2-D0 $\Leftrightarrow$ stable hol. vect. bdle on surface $\subset M$

⋮

(similar story for Type II/K3)

**Def** a holomorphic vector bdle $E$ on a Kähler mfd $(X, J)$ is stable iff any sub-bundle $F$ of $E$ satisfies

$$\frac{\int_X J^{n-1} \wedge c_1(F)}{\mathrm{rank}(F)} < \frac{\int_X J^{n-1} \wedge c_1(E)}{\mathrm{rank}(E)}$$

PLUS

Step 1

In the large vol limit

$$
\boxed{\begin{aligned}
Z &\propto (-i)^3 \frac{1}{3!}\left(\int_M J^3\right) \mathrm{rk} + \frac{(-i)^2}{2!}\int_M J^2 \wedge c_1 + \cdots \\
&= (-i)^3 \frac{1}{3!}\left(\int_M J^3\right) \times \mathrm{rk} \times \left[ 1 + \frac{i\left(\int_M J^2 \wedge c_1\right) 3}{\mathrm{rk} \int J^3} + \cdots \right]
\end{aligned}}
$$

So a brane $(M, E)$ state exists if any "sub-brane" $(M, F)$

satisfies $\quad \mathrm{Arg}\left(Z(v(M,F))\right) < \mathrm{Arg}\left(Z(v(M,E))\right)$

Step 2

characterize sub-bdles in this way

$$0 \longrightarrow F \longrightarrow E \longrightarrow G \longrightarrow \quad \text{all vector bdles on } X.$$

$$\Longleftrightarrow \quad h^1(X; F \otimes G^*) > 0$$

$$\Rightarrow \quad \chi(X; F \otimes G^*) < 0$$

$$\begin{array}{cl} \begin{bmatrix} * & 0 \\ * & * \end{bmatrix} & \begin{array}{l} \}\, G \\ \}\, F \end{array} \end{array}$$

connect'n, transit'n fns.

On $X = M = CY$.

$$0 > \chi(M; F \otimes G^*) = \int_M td(TM)\, ch(F)\, ch(G)^\vee = \int_M v(M,F) \wedge \left(v(M,G)\right)^\vee$$

Step 3

D-brane state w/ charge $v \in H^{even}(M; \mathbb{Q})$ exists iff.

any D-brane state w/ charge $v'$ such that $\int_X v' \wedge (v - v')^\vee < 0$

satisfies $\quad \frac{1}{\pi}\mathrm{Arg}\left(Z(v')\right) < \frac{1}{\pi}\mathrm{Arg}\left(Z(v)\right)$

(stability condition.)

Step 4

allow $\frac{1}{\pi}\mathrm{Arg}\left(Z(v; B+iJ)\right)$ to take value in $\mathbb{R}$ not in $\mathbb{R}/2\mathbb{Z}$

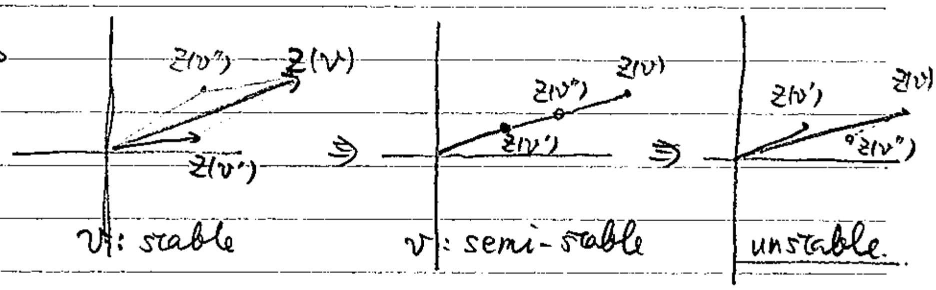
continuously as $(B+iJ)$ varies.

$\boxed{\text{Steps } 1 \sim 4}$ applicable to cases w/ $M = K3$, too.

## Wall of marginal stability

$v' \subset v$.

$(B+iJ)_0$




$$\text{state}(v) \longrightarrow \begin{pmatrix} \text{state}(v') \\ + \\ \text{state}(v'') \end{pmatrix}$$

BPS

[ each : BPS state
together: non-BPS config. ]

Supporting evidence,

$\mathrm{IIA}/(CY_3 = M)$ but with large $g_{sA}$

$\Rightarrow$ better to deal w/. the D6-D4-D2-D0 bound state (worldline in $\mathbb{R}^{3,1}$)

as a black hole in $\mathbb{R}^{3,1}$

$$\left( r \sim \sqrt{G_{4d}} \cdot m \ ; \quad \sqrt{G_{4d}} \propto g_{sA} \right)$$

For certain class of the $U(1)^{h^{11}(M)+1}$ electric & magnetic charges $v$,

the BH configuration is a multi-centered molecule.

The binding radius of the molecule approaches $\infty$

as $\mathrm{Arg}(Z(v'))$ approaches $\mathrm{Arg}(Z(v))$.

Denef Moore th/0702146. §3.2.