QFT II/QFT   homework X (Dec. 07, 2020)

- Reports on these homework problems are supposed to be submitted through the U Tokyo ITC-LMS. We request that the file name includes the problem number, such as II-1***.pdf or ****-IV-2-IX-1.jpeg. The ITC-LMS will show who had submitted the file (student ID and name), so the file name will not have to contain your name or ID number. (this instruction may be updated later)

1. **Hyperfine structure, Lamb shift** [C or D]

   (evaluate for yourself whether your achievement is worth D or C)

   Explore as much as you like on hyperfine structure and/or Lamb shift, by reading textbooks or literatures. [Fine structure: see homework VII-1] Here are some suggestions on the literature, though they will probably not be the only options.

   (a) Hyperfine structure: a book by Bethe and Salpeter, "Quantum Mechanics of One- and Two-Electrom Atoms," Springer, 1957; a free copy of the relevant part of this book seems to be available on-line as a pdf file (hosted by uni-freiburg; I wish it is a leagal copy).

   (b) Lamb shift: a combination of Landau–Lifshitz vol.4 (QED) section 123 and Peskin– Schroeder section 7.5 (eq. (7.94)) will do the job; I am not sure, though, whether sufficient attention is paid to the sign ± 1 in Landau–Lifshitz vol.4 section 123. [Bjorken Drell section 8.7 should also contain relevant discussion, though I have not taken time to see it recently.]

2. **An approximation in the Bethe-Salpeter equation** [C]

   (a) In the Bethe–Salpeter equation applied to non-relativistic systems, we often re- place the propagator in the 2PI kernel K by its "approximation,"

   $$\frac{1}{(\omega'')^2 - |\vec{p}''|^2} \Rightarrow \frac{1}{-|\vec{p}''|^2}. \tag{1}$$

   Pick up a non-relativistic system of your favorite where a bound state is formed, and verify that the integral in the right-hand side of the Bethe–Salpeter equation

   $$G = (2\pi)^4 \delta^4(p - p') D_a D_b + D_a D_b \int \frac{d^4 p''}{(2\pi)^4} K G \tag{2}$$

   is dominated by a region where $(\omega'')^2 \ll |\vec{p}''|^2$.