QFT II/QFT

"Category D" homework problems (ver 1 (Sep/28, 2020))

- Reports on these homework problems are supposed to be submitted through the U Tokyo ITC-LMS. We request that the file name includes the problem number, such as II-1***.pdf or ****-IV-2-IX-1.jpeg. The ITC-LMS will show who had submitted the file (student ID and name), so the file name will not have to contain your name or ID number. (this instruction may be updated later)

[D-2] see homework E-4.

[D-3] R_ξ **Gauge**

Read appropriate resources to learn how to deal with massive vector fields in a non-abelian gauge theory with spontaneous symmetry breaking (key word: R_ξ-gauge), and write a summary as a report. For example, [Peskin Schroeder] textbook §20.1, §20.2 and §21.1 (based on §9.4 and 16.2) cover this subject.

[D-4] see homework V-1.

[D-5] see homework X-1.

[D-1] **Spinor Helicity formalism**

Conventions

In this note, we use the following convention.[1]

$$\gamma^\mu = \begin{pmatrix} & \sigma^\mu \\ \bar{\sigma}^\mu & \end{pmatrix}, \qquad \sigma^\mu = (\mathbf{1}_{2\times 2}, -\vec{\tau}), \qquad \bar{\sigma}^\mu = (\mathbf{1}_{2\times 2}, \vec{\tau}), \tag{1}$$

so that $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} = 2\mathrm{diag}(+,-,-,-)$. A four component spinor Ψ is split into two component spinors as

$$\Psi = \begin{pmatrix} \psi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}, \qquad \overline{\Psi} := \Psi^\dagger \gamma^0 = (\bar{\psi}_{\dot{\alpha}}, \chi^\alpha), \qquad \alpha = 1,2, \quad \dot{\alpha} = 1,2. \tag{2}$$

Individual entries of the 2×2 matrices σ^μ and $\bar{\sigma}^\mu$ are denoted by $(\sigma^\mu)_{\alpha\dot{\alpha}}$ and $(\bar{\sigma}^\mu)^{\dot{\alpha}\alpha}$.

The two component spinors, ψ_α, $\bar{\chi}^{\dot{\alpha}}$ etc. are in separate irreducible spinor representations of $\mathrm{Spin}(1,3)$. There are two irreducible spinor representations of $\mathrm{Spin}(1,3)$, $\mathbf{2}_R$ and $\mathbf{2}_L$. When the Lorentz transformation of the spinor ψ_α is given by $\psi_\alpha \to \psi'_\alpha = (\rho(g))_\alpha^{\ \beta}\psi_\beta$, the

[1] This choice of σ^μ and $\bar{\sigma}^\mu$ is to follow the convention of a textbook "*Supersymmetry and Supergravity*," by J. Wess and J. Bagger.

Lorentz transformation of the spinor χ^α is $\chi^\alpha \to (\chi')^\alpha = \chi^\beta \rho(g)_\beta^{\ \alpha}$. In the sense that we can use the same 2×2 matrix (representation), we say that the spinors ψ_α and χ^α are in the representation, $\mathbf{2}_R$. Similarly, $\bar{\psi}^{\dot{\alpha}}$ and $\bar{\chi}_{\dot{\alpha}}$ are in the other spinor representation $\mathbf{2}_L$.

Using Spin(1,3) invariant anti-symmetric tensors[2] $\epsilon_{\alpha\beta}$ and $\epsilon_{\dot{\alpha}\dot{\beta}}$,

$$\epsilon_{12} = \epsilon_{\dot{1}\dot{2}} = -1, \quad \epsilon_{21} = \epsilon_{\dot{2}\dot{1}} = +1, \qquad \epsilon_{\alpha\beta}\epsilon^{\beta\gamma} = \delta_\alpha^{\ \gamma}, \quad \epsilon_{\dot{\alpha}\dot{\beta}}\epsilon^{\dot{\beta}\dot{\gamma}} = \delta_{\dot{\alpha}}^{\ \dot{\gamma}}, \tag{3}$$

spinor indices are raised and lowerd by

$$\psi^{\alpha\beta} = \epsilon^{\alpha\beta}\psi_\beta, \qquad \psi_\alpha = \epsilon_{\alpha\beta}\psi^\beta, \qquad \bar{\psi}_{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}}\bar{\psi}^{\dot{\beta}}, \qquad \bar{\psi}^{\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}}\bar{\psi}_{\dot{\beta}}. \tag{4}$$

The fact that both $\overline{\Psi}\Psi$ and $\overline{\Psi}\gamma_5\Psi$ are Lorentz invariant is equivalent to a statement that both

$$\chi\psi := \chi^\alpha\psi_\alpha = \epsilon_{\alpha\beta}\chi^\alpha\psi^\beta, \qquad \bar{\psi}\bar{\chi} := \bar{\psi}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}}\bar{\psi}_{\dot{\alpha}}\bar{\chi}_{\dot{\beta}} \tag{5}$$

are Lorentz invariant.

Later on, we will use the following relations,

$$\bar{\sigma}^{\mu\dot{\alpha}\alpha} = \epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\alpha\beta}\sigma^\mu_{\beta\dot{\beta}}, \qquad \sigma^\mu_{\alpha\dot{\alpha}} = \epsilon_{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}}\bar{\sigma}^{\mu\dot{\beta}\beta}, \tag{6}$$

$$\sigma^\mu_{\alpha\dot{\alpha}}\bar{\sigma}_\mu^{\dot{\beta}\beta} = 2\delta_\alpha^{\ \beta}\delta_{\dot{\alpha}}^{\ \dot{\beta}} \qquad \text{and} \qquad \sigma^\mu_{\alpha\dot{\alpha}}\bar{\sigma}^{\nu\,\dot{\alpha}\alpha} = 2\eta^{\mu\nu}. \tag{7}$$

On shell states (recap)

Solution to the Dirac equation

$$(i\gamma^\mu\partial_\mu - m)\, e^{-ip\cdot x}u(p) = 0, \qquad (i\gamma^\mu\partial_\mu - m)\, e^{-ip\cdot x}v(p) = 0, \tag{8}$$

are in the form of

$$u(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\xi \\ \sqrt{p\cdot\bar{\sigma}}\xi \end{pmatrix}, \qquad v(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\eta \\ -\sqrt{p\cdot\bar{\sigma}}\eta \end{pmatrix}, \qquad p^2 = m^2 \tag{9}$$

for some $SO(3) \subset SO(1,3)$ spinors ξ and η. There are two positive energy solutions (due to the choice of ξ) and two negative energy solutions (due to the choice of η).

When we consider the case the mass parameter vanishes, $m = 0$, things simplify a lot. The mode function $u(p)$ vanishes in the lower two components in the positive energy states with

[2] The tensor $\epsilon_{\alpha\beta}$ can be regarded as the Clebsh–Gordon coefficient of $\mathbf{2}_R \otimes \mathbf{2}_R \cong \mathbf{1} \oplus \mathbf{3}$ for the $\mathbf{1}$ component.

helicity $h = +1/2$, while the upper two components of $u(p)$ vanish in the positive energy states with helicity $h = -1/2$. An anti-particle state with momentum $p^\mu = (E, \vec{p})$ with helicity $h = \pm 1/2$ corresponds to $v(p)$ with the two component spinors $\sqrt{p \cdot \sigma}\eta$ and $\sqrt{p \cdot \bar{\sigma}}\eta$ in the $\mp 1/2$ eigenspace of $\vec{p} \cdot \vec{\tau}$ [as explained in the lecture]. For this reason, the two component spinor field ψ is the annihilation operator of right-handed ($h = +1/2$) particles and creation operator of left-handed ($h = -1/2$) anti-particles, while $\bar{\chi}$ is the annihilation operator of left-handed ($h = -1/2$) particles and creation operator of right-handed ($h = +1/2$) anti-particles.

Step 1

Now, let us introduce a treatment of spinors for **massless** and **on-chell** states that is much more simple than kepping track of all the $\sqrt{p \cdot \sigma}$'s and $\sqrt{p \cdot \bar{\sigma}}$'s above. Note first that the on-shell massless condition $p^2 = 0$ for a four momentum p^μ is equivalent to a condition that the following 2×2 matrix

$$p_{\alpha\dot{\alpha}} := p_\mu \sigma^\mu_{\alpha\dot{\alpha}} \tag{10}$$

has a vanishing determinant. This matrix is rank-1, unless $p^\mu = 0$. So, there must be[3] some two-component spinors λ_α and $\tilde{\lambda}_{\dot{\alpha}}$ for a given on-shell massless p^μ so that

$$p_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}}. \tag{11}$$

Lorentz transformation on p^μ in the vector representation of $SO(1,3)$ is equivalent to the Lorentz transformation on λ_α and $\tilde{\lambda}_{\dot{\alpha}}$ that are in the $\mathbf{2}_R$ and $\mathbf{2}_L$ representations combined.

Solutions to the massless Dirac equation are given by

$$e^{-ip\cdot x}\begin{pmatrix} \lambda_\alpha \\ 0 \end{pmatrix}, \qquad e^{-ip\cdot x}\begin{pmatrix} 0 \\ \tilde{\lambda}^{\dot{\alpha}} \end{pmatrix}, \qquad e^{ip\cdot x}\begin{pmatrix} 0 \\ \tilde{\lambda}^{\dot{\alpha}} \end{pmatrix}, \qquad e^{ip\cdot x}\begin{pmatrix} \lambda_\alpha \\ 0 \end{pmatrix} \tag{12}$$

for $h = \pm 1/2$ particle states and $h = \pm 1/2$ anti-particle states, respectively. To see this, it is enough, in the case of $h = +1/2$ particle state as an example, to see that[4]

$$(p \cdot \bar{\sigma})^{\dot{\alpha}\alpha}\lambda_\alpha = \tilde{\lambda}^{\dot{\alpha}}\lambda^\alpha\lambda_\alpha = 0. \tag{13}$$

[3] The choice of λ and $\tilde{\lambda}$ is not unique for a given four momentum p^μ; we can replace λ and $\tilde{\lambda}$ by $\lambda \times a$ and $\tilde{\lambda}/a$, respectively.

[4] Spinors λ_α and $\tilde{\lambda}_{\dot{\alpha}}$ are $\mathbb{C}$-valued, just like $u(p)$ and $v(p)$ are. The fermion fields ψ_α and $\bar{\chi}^{\dot{\alpha}}$ are Grassmann, because they are given by those solutions multiplied by fermion annihilation/creation operators.

Proof for the three other solutions are the same. The following short-hand notation is used frequently in the following:

$$|p\rangle := \begin{pmatrix} \lambda_\alpha \\ 0 \end{pmatrix}, \qquad |p] := \begin{pmatrix} 0 \\ \tilde{\lambda}^{\dot{\alpha}} \end{pmatrix}, \qquad \langle p| := (\lambda^\alpha, 0), \qquad [p| := (0, \tilde{\lambda}_{\dot{\alpha}}). \tag{14}$$

Exercise

1. If you find any statements that are non-trivial for you, verify them, whenever you can. [If you understand the meaning of the statements, but do not understand why, it is an option to accept the statement and move on.] If you find a statement whose meaning is not clear, ask others what it means. .... This "Exercise" applies also to the rest of this note.

2. Verify, in the simplest example of $p^\mu = (E, 0, 0, E)$ that the solutions derived in the two different ways—one is those using $u(p)$ and $v(p)$, and the other using λ_α and $\tilde{\lambda}_{\dot{\alpha}}$—are the same. [Here, we need to fix an ambiguity in choosing λ_α and $\tilde{\lambda}_{\dot{\alpha}}$ for a given momentum p^μ. We do so as in the next exercise problem.]

3. Compute explicitly the following:

$$\lambda_\alpha = \tilde{\lambda}_{\dot{\alpha}} = \exp\left[i\frac{\varphi}{2}\tau_3\right] \exp\left[i\frac{\theta}{2}\tau_2\right] \begin{pmatrix} \sqrt{2E} \\ 0 \end{pmatrix}, \tag{15}$$

which is for a massless on-shell four momentum $p^\mu = E(1, \sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$.

Step 2

The Feynman rule for QED is given by

$$\frac{i\not{p}}{p^2 + i\epsilon}, \qquad \frac{-i\eta_{\mu\nu}}{p^2 + i\epsilon} \tag{16}$$

for a fermion and photon propagator, and

$$-i\gamma^\mu e Q_\Psi \tag{17}$$

for a vertex,[5] as usual. For fermion external states, see Table 1

Step 3: First example, s-channel scattering.

Consider $e^+ + e^-$ scattering procsses in the s-channel, as in Figure 1 (a). The particle 1

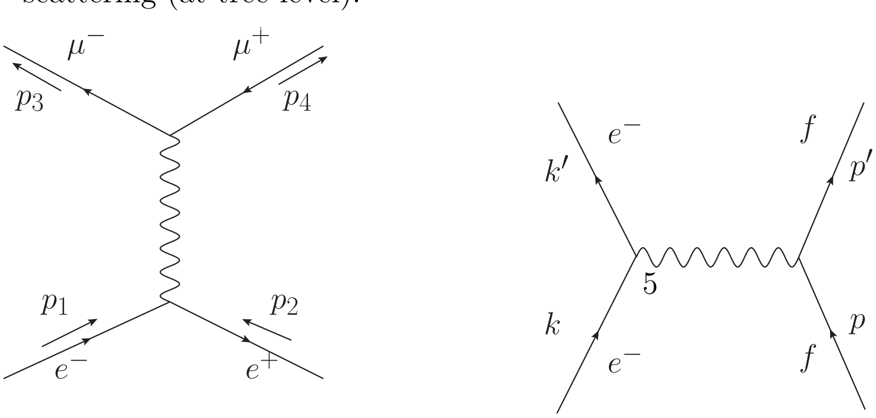
[5]Convention: the covariant derivative for Ψ is $D_\mu = \partial_\mu + ieQ_\Psi A_\mu$, where Q_Ψ is the electric charge of Ψ.

	$m \neq 0$	$h = +1/2$	$h = -1/2$
particle incoming	$u(p)$	$\vert\lambda\rangle$	$\vert\lambda]$
anti-particle incoming	$\bar{v}(p)$	$\langle\lambda\vert$	$[\lambda\vert$
particle outgoing	$\bar{u}(p)$	$[\lambda\vert$	$\langle\lambda\vert$
anti-particle outgoing	$v(p)$	$\vert\lambda]$	$\vert\lambda\rangle$

Table 1: Feynman rule assigns to fermion external states four component spinors shown in the second column, in QED with $m \neq 0$ fermions. When the fermions are massless (when the massless approximation can be made), however, we can use the ones in the 3rd and 4th columns instead.

(a) (b)

Figure 1: (a) is the only one Feynman diagram contributing to the $e^- + e^+ \to f^- + f^+$ scattering (at tree level), whereas (b) is the one contributing to the $e^- + f \to e^- + f$ scattering (at tree level).



is an electron with incoming momentum p_1^μ and the particle 2 a positron with momentum p_2^μ. As a first application of the spinor-helicity formalism, we consider this s-channel process that goes to $\mu^+ + \mu^-$. The particle 3 is μ^- with outgoing momentum p_3^μ and the particle 4 μ^+ with momentum p_4^μ. The Feynman rule given above immediately leads us to the following expression for the scattering amplitude of $e^-_{h=+1/2} + e^+_{h=-1/2} \longrightarrow \mu^-_{h=+1/2} + \mu^+_{h=-1/2}$ at energy scale $\sqrt{s}$ much higher than the muon mass m_μ:

$$\begin{aligned} & i\mathcal{M}\left(e^-_{+1/2}(p_1) + e^+_{-1/2}(p_2) \longrightarrow \mu^-_{+1/2}(p_3) + \mu^+_{-1/2}(p_4)\right) \\ & = \frac{-i}{s+i\epsilon}\left(-ieQ_e[2|\gamma^\mu|1\rangle\right)\left(-ieQ_\mu[3|\gamma_\mu|4\rangle\right). \end{aligned} \tag{18}$$

Here, $|i\rangle$, $|i]$ etc. stand for $|p_i\rangle$, $|p_i]$ etc. It is straightforward to write down the expression for three other processes

$$\begin{aligned} & e^-_{+1/2} + e^+_{-1/2} \rightarrow \mu^-_{-1/2} + \mu^+_{+1/2}, \\ & e^-_{-1/2} + e^+_{+1/2} \rightarrow \mu^-_{+1/2} + \mu^+_{-1/2}, \\ & e^-_{-1/2} + e^+_{+1/2} \rightarrow \mu^-_{-1/2} + \mu^+_{+1/2}. \end{aligned}$$

The scattering amplitude vanishes, when the initial states are in the helicity configuration of $e^+_{+1/2} + e^+_{+1/2}$ or $e^-_{-1/2} + e^+_{-1/2}$. Similarly, the final states never come out in the hilicity configuration of $(+1/2, +1/2)$ or $(-1/2, -1/2)$. Let us now work on the amplitude (19).

Exercise

1. Verify $[p_i|\gamma^\mu|p_j\rangle = \langle p_i|\gamma^\mu|p_j]$. Use (6).

2. We will use this notation heavily in the following:

$$\langle ij\rangle := \langle p_i p_j\rangle := \lambda(p_i)^\alpha \lambda(p_j)_\alpha, \qquad [ij] := [p_i p_j] := \tilde{\lambda}(p_i)_{\dot\alpha}\tilde{\lambda}(p_j)^{\dot\alpha}. \tag{19}$$

   Note that $\langle p_i p_i\rangle$ and $[p_i p_i]$ vanish, because the spinors $\lambda_\alpha(p_i)$ and $\tilde{\lambda}(p_i)_{\dot\alpha}$ are not Grassmann. Now, verify the following relation (using (7)):

$$\langle ij\rangle[ji] = 2p_i \cdot p_j. \tag{20}$$

3. Verify, using (7), that

$$[i|\gamma^\mu|j\rangle\ [k|\gamma_\mu|l\rangle = [i|\gamma^\mu|j\rangle\langle l|\gamma_\mu|k] = -2\langle jl\rangle[ik]. \tag{21}$$

4. Derive, for the amplitude (19), that

$$i\mathcal{M} = \frac{i(e^2 Q_e Q_\mu)}{s} 2[23]\langle 41\rangle, \qquad |\mathcal{M}|^2 = 4(e^2 Q_e Q_\mu)^2 \frac{u^2}{s^2} = 4(e^2 Q_e Q_\mu)^2 \cos^4\left(\frac{\theta}{2}\right). \quad (22)$$

5. Verify that $i\mathcal{M}$ for the process $e^-_{+1/2} + e^+_{-1/2} \to \mu^-_{-1/2} + \mu^+_{+1/2}$ is given by (22) with $[23]\langle 41\rangle$ replaced by $[24]\langle 31\rangle$.

If there is a complex scalar field Φ wich QED charge Q_Φ, the scattering amplitude of $e^-_{+1/2}(p_1) + e^+_{-1/2}(p_2) \to \Phi^-(p_3) + \Phi^+(p_4)$ at high energy ($E \gg m_\Phi$) is given by

$$i\mathcal{M} = \frac{-i}{s + i\epsilon} \left(-ieQ_e[2|\gamma^\mu|1\rangle\right) \left(-ieQ_\Phi(p_3 - p_4)_\mu\right). \quad (23)$$

So,

$$i\mathcal{M} = \frac{i(e^2 Q_e Q_\Phi)}{s}[2|(\not{p}_3 - \not{p}_4)|1\rangle = \frac{i(e^2 Q_e Q_\Phi)}{s}[2|\left(2\not{p}_3 - \not{p}_1 - \not{p}_2\right)|1\rangle = \frac{i(e^2 Q_e Q_\Phi)}{s} 2[23]\langle 31\rangle, \quad (24)$$

and we arrive at the result

$$|\mathcal{M}|^2 = (e^2 Q_e Q_\Phi)^2 \frac{4ut}{s^2} = (e^2 Q_e Q_\Phi)^2 \sin^2(\theta), \quad (25)$$

from which we can recover the result of homework IV-2 by taking average over the initial state spin configuration.

All the computations above only reproduce the results that we have already been familiar with (either in the lecture, or in homework IV-2). You will have noticed, however, that once you start to use the formula (20, 21), it is much easier and faster in this spinor–helicity formalism to compute the scattering amplitudes without summing/averaging over the spin configuration. This spinor–helicity formalism is only for massless states, but still this is very useful.

Step 3.5 forward–backward asymmetry ($\simeq$ homework IV-3)

There is a vector boson—called Z-boson—in the Standard Model in addition to photon, and it couples to a Dirac fermion f (such as $f = e, \mu, \tau, u, d, \cdots$) as in

$$\mathcal{L}_{\rm int} \ni -\psi^\dagger \bar{\sigma}^\mu Z_\mu g^R \psi - \chi \sigma^\mu Z_\mu g^L \bar{\chi}. \quad (26)$$

In the case of photon, the coupling constants satisfies the relation $g^R = g^L = eQ_f$, but the coupling constants of the Z boson does not satisfy this condition.

Exercise

1. Verify that the scattering amplitude for $e^-_{+1/2} + e^+_{-1/2} \to f_{+1/2} + \bar{f}_{-1/2}$ is given by

$$|\mathcal{M}|^2 = u^2 \left| \frac{Q_e Q_f e^2}{s} + \frac{g_e^R g_f^R}{s - m_Z^2} \right|^2 , \tag{27}$$

and the amplitude for $e^-_{+1/2} + e^+_{-1/2} \to f_{-1/2} + \bar{f}_{+1/2}$ by

$$|\mathcal{M}|^2 = t^2 \left| \frac{Q_e Q_f e^2}{s} + \frac{g_e^R g_f^L}{s - m_Z^2} \right|^2 , \tag{28}$$

when the center of mass energy $\sqrt{s}$ of these scattering processes is much higher than the mass of $e^\pm$ and $f, \bar{f}$. The distribution of the final states as a function of the scattering angle θ therefore depends, in a very non-trivial way, on the coupling constants g_f^R, g_f^L of the final state fermions, and on the energy scale $\sqrt{s}$ relatively to the mass of the Z boson.

Step 4: t-channel scattering

Consider, now, another 2-body to 2-body scattering between e^- and another fermion f with an electric charge Q_f at energy scale much higher than the masses of $e^\pm$ and $f, \bar{f}$. [we ignore Z-boson exchange in this Step 4.] Feynman diagram in Figure 1 (b) allows us to write down the amplitude right away.

$$i\mathcal{M}\left(e^-_{+1/2}(k) + f_{+1/2}(p) \to e^-_{+1/2}(k') + f_{+1/2}(p')\right) = \frac{-i}{t + i\epsilon} \left(-ieQ_e[k'|\gamma^\mu|k\rangle\right) \left(-ieQ_f[p'|\gamma_\mu|p\rangle\right), \tag{29}$$

$$i\mathcal{M}\left(e^-_{-1/2}(k) + f_{+1/2}(p) \to e^-_{-1/2}(k') + f_{+1/2}(p')\right) = \frac{-i}{t + i\epsilon} \left(-ieQ_e\langle k'|\gamma^\mu|k]\right) \left(-ieQ_f[p'|\gamma_\mu|p\rangle\right). \tag{30}$$

The amplitudes with $f_{-1/2}$ in the initial (and final) states are also obtained in a similar way.

Exercise

1. Verify, in the two scattering processes above, that

$$i\mathcal{M} = \frac{i(e^2 Q_e Q_f)}{t} \left\{ \begin{array}{l} 2[k'p']\langle pk\rangle, \\ 2\langle k'p\rangle[p'k], \end{array} \right. \qquad |\mathcal{M}|^2 = 4(e^2 Q_e Q_f)^2 \left\{ \begin{array}{l} s^2/t^2, \\ u^2/t^2, \end{array} \right. \tag{31}$$

respectively.

2. Verify, in the $e^-_{h_e=\pm 1/2}(k) + \Phi(p) \longrightarrow e^-_{h_e=\pm 1/2}(k') + \Phi(p')$ scattering, that

$$i\mathcal{M} = i\frac{e^2 Q_e Q_\Phi}{t} \left\{ \begin{array}{l} 2[k'p]\langle pk\rangle, \\ 2\langle k'p\rangle [pk], \end{array} \right. \qquad |\mathcal{M}|^2 = 4(e^2 Q_e Q_\Phi)^2 \frac{s(-u)}{t^2}. \tag{32}$$

Remark The matrix elements have different dependence on the kinematical varaibles depending on whether the target particle has spin 1/2 or spin 0. This aspect was crucial in determining the spin of "quarks" within a hadron. We just need to measure the energy and momentum distribution of an outgoing electron in a electron–hadron collision experiment; in this kind of experiments called deep inelastic scattering, it is conventional to introduce a kinematical parameter

$$y := \frac{s+u}{s} = 1 + \frac{u}{s}. \tag{33}$$

In terms of this y,

$$\frac{s^2+u^2}{t^2} = \frac{t^2}{s^2}\left[1 + (1-y)^2\right], \qquad \frac{s(-u)}{t^2} = \frac{t^2}{s^2}\left[1-y\right]. \tag{34}$$

The y-dependence was determined in deep inelastic scattering experiments, and the spin of quarks was determined to be 1/2.

Remark See references in the homework [E-1] for advantages of the spinor-helicity formalism in uncovering theoretical aspects of scattering amplitudes in general.