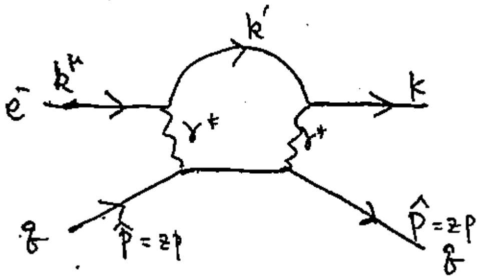
§ ~~11~~ 7.6 DGLAP equation.

< Dokshitzer – Gribov – Lipatov – Altarelli – Parisi >



$$(2\,\mathrm{Im}\,\mathcal{M}) = \int d\Pi\,(\mathcal{M}^\dagger \mathcal{M})$$

unitarity.

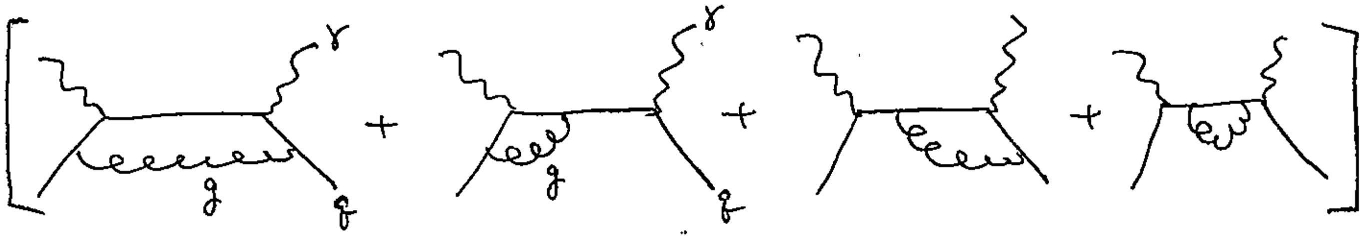
(total inelastic cross section

Im [forward amplitude.].

$\boxed{\text{Higher-order loop correction?}}$

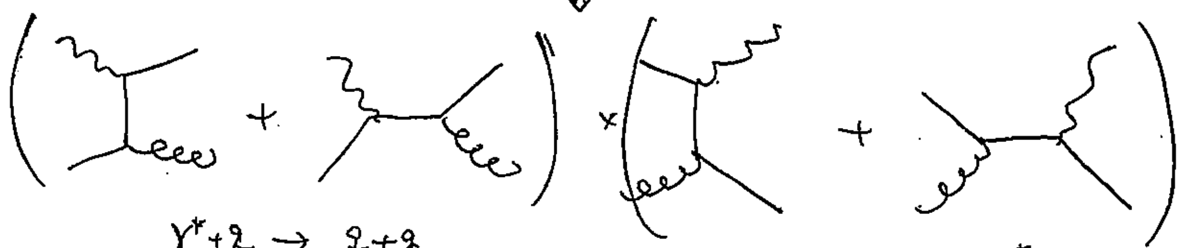
< ~~ignore~~ omit. lepton line. >

$[\gamma^* + q \to \gamma^* + q]$



$\Downarrow$

$\boxed{\text{real gluon emission.}}$



$\gamma^* + q \to q + g$ $\qquad$ $g + q \to q + \gamma^*$.

$$\Delta\left(2\,\mathrm{Im}\,T^{\mu\nu}\right) = \underbrace{\int_0^1 \frac{dz}{z}\, f_q(z)}_{\text{parton model.}} \times \left[(-) \times \left[i\mathcal{M}^{\mu\nu}(\gamma^* + q \to \gamma^* + q)\right]^{\text{(spin-average)}}\right.$$

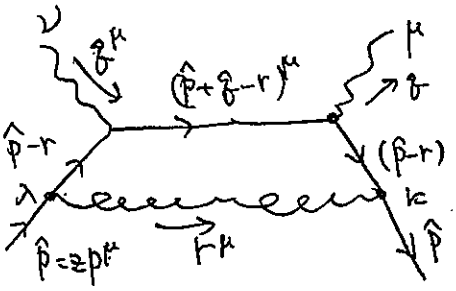
with on-shell propagators $\boxed{\dfrac{i}{p^2 - m^2 + i\epsilon} \Rightarrow 2\pi\delta(p^2 - m^2)}$

Cutkosky rule

Peskin – Schröder §7.3.

e.g. <u>1st graph.</u>

$$\Delta^{(1)}(2\mathrm{Im}\,T^{\mu\nu}) = \int_0^1 \frac{dz}{z} f_g(z) \times (-)\int \frac{1}{2}\sum_r \bar{u}_r(\hat{p})\left[(-igt^a\gamma^\kappa)\frac{\cancel{i(\hat{p}-r)}}{(\hat{p}-r)^2+i\varepsilon}(iQ_q\gamma^\mu)\frac{\cancel{i(\hat{p}+q-r)}}{(\hat{p}+q-r)^2+i\varepsilon}\right.$$



$$\left.(iQ_q\gamma^\nu)\frac{\cancel{i(\hat{p}-r)}}{(\hat{p}-r)^2+i\varepsilon}(-igt^b\gamma^\lambda)\right]u_r(\hat{p})$$

$$\times\left(\frac{-i\,\eta_{\kappa\lambda}\,\delta_{ab}}{r^2+i\varepsilon}\right)\frac{d^4r}{(2\pi)^4}$$

with $\begin{cases}\dfrac{i}{(\hat{p}+q-r)^2+i\varepsilon} \Rightarrow (2\pi)\,\delta\big((\hat{p}+q-r)^2\big)\\[2ex] \dfrac{i}{r^2+i\varepsilon} \Rightarrow (2\pi)\,\delta(r^2).\end{cases}$

spin average... $\mathrm{Tr}\left[\sum_r\big(u_r(\hat{p})\,\bar{u}_r(\hat{p})\big)\cdots\right] = \mathrm{Tr}\left[(\hat{\cancel{p}})\cdots\right].$

Feynman gauge.

If we were to use the tree level result.

$$\eta_{\mu\nu}(2\mathrm{Im}\,T^{\mu\nu}) = \frac{2\pi}{x}(F_2 - 6xF_1) \rightarrow -(4\pi)\,Q_q^2\left[f_q(x)+f_{\bar{q}}(x)\right]$$

$$\Rightarrow \boxed{-\frac{1}{4\pi}\eta_{\mu\nu}\Delta(2\mathrm{Im}\,T^{\mu\nu})} \quad \text{how is it like?}$$

$$\int\frac{d^4r}{(2\pi)^4}(2\pi)^2\delta(r^2)\,\delta\big((\hat{p}+q-r)^2\big) \Rightarrow$$

set an axis.

$\begin{cases}\hat{p} = z(p, p, \vec{0})\\ q^\mu = (Q^0, Q^3, \vec{0})\end{cases}$

$r^\mu = (r^0, r^3, \vec{r}_T)$.

- $|\vec{r}_T|$ set by $\delta(r^2)$
- focus on a region. $r^\mu \not\parallel p^\mu$ $(r^0 \sim r^3) \Leftrightarrow (r^+, r^-, \vec{r}_T) \sim (1, \lambda^2, \lambda)$.

$\Rightarrow (\hat{p}+q-r)^2 \simeq \chi z(2p\cdot q) + q^2$ if $(\Delta r)^\mu$ = small.

$r^\mu = \vec{r}_T + z(1-\chi)p^\mu + (\Delta r)^\mu$ $\Rightarrow [z\chi - x \simeq 0$

$$\boxed{-\frac{1}{4\pi}\left(2\,\mathrm{Im}\,T^{\mu\nu}\right)\eta_{\mu\nu}} \simeq Q_q^2 f_q(x) +$$

$$Q_q^2 \int \frac{dz}{z} f_q(z) \frac{[-g^2 C_2(R)]}{(-4\pi)} \int \frac{d(p\cdot r)}{(p\cdot q)} \frac{1}{8\pi} \times \left\{ 4 \frac{r\cdot q}{(\hat{p}\cdot r)} + 8 \frac{(\hat{p}\cdot q)\ (\hat{p}-r)\cdot q}{(\hat{p}\cdot r)(2\hat{p}\cdot q + q^2)} \times 2 \right\}$$

① ②+③

apart from $\frac{1}{(\hat{p}\cdot r)}$. use $r_\mu = (1-x)\hat{p}_\mu$.

$$\frac{(\hat{p}\cdot q)}{(\hat{p}\cdot r)} \times 4 \left\{ (1-x) + \frac{2x}{(1-x)} \right\} = \left\{ \frac{1+x^2}{1-x} \right.$$

$$\Rightarrow \text{"}(Q_q^2)\text{"}\left[f_q(x)\right] \cong Q_q^2 f_q(x)$$

$$+ Q_q^2 \frac{\alpha_s}{2\pi} C_2(R) \int \frac{d(p\cdot r)}{(p\cdot r)} \int_x^1 \frac{dz}{z} f_q(z) \left\{ \frac{1+(x/z)^2}{1-(x/z)} \right\}$$

$\Downarrow$

pure logarithmic divergence.
from a region w/ small $(p\cdot r)$.

Just like renormalization

$$\frac{4\pi}{g^2(p^2)} = \frac{4\pi}{g_0^2} + \frac{b}{2\pi}\ln\left(\frac{p^2}{\Lambda^2}\right) \Rightarrow \left[\frac{4\pi}{g_0^2} + \frac{b}{2\pi}\ln\left(\frac{\mu_R^2}{\Lambda^2}\right)\right] + \frac{b}{2\pi}\ln\left(\frac{p^2}{\mu_R^2}\right)$$

$$\left[\ldots\right] \equiv \left(\frac{4\pi}{g^2(\mu_R)}\right)$$

physical. observable parameter — renormalized coupling.

$$\boxed{\text{"}f_q(x;Q^2)\text{"} = f_q(x;\mu_F^2) + \frac{\alpha_s}{2\pi} C_2(R) \int_{\mu_F^2}^{Q^2} \frac{d(p\cdot r)}{(p\cdot r)} \int_x^1 \frac{dz}{z} f_q(z;\mu_F^2) \frac{1+(x/z)^2}{1-(x/z)}}$$

observable "renormalized"

μ_F: factorization scale.

$(-2\hat{p}\cdot r) \sim (\hat{p}-r)^2$: virtuality of parton q.

radiative corr. with $\boxed{\text{virtuality} \lesssim \mu_F^2}$: swept under the carpet. $f_q(z;\mu_F^2)$

near light cone.

- Wilson's interpretation of renormalization:

  momenta above μ_R: renormalized into $g(\mu_R)$

- (fluctuation / distribution) close to the light cone than μ_F^2:

  swept into / taken into account in $f_q(x;\mu_F^2)$

$$\boxed{\frac{\partial f_q(x;\mu_F^2)}{\partial \ln(\mu_F^2)} = \frac{\alpha_s}{2\pi} C_2(R) \int_x^1 \frac{dz}{z} \underbrace{\frac{1+(x/z)^2}{1-(x/z)}}_{\parallel} f_q(z;\mu_F^2)}$$

DGLAP eq

[weakly dep. on μ_F

$P(x/z)$ splitting function.

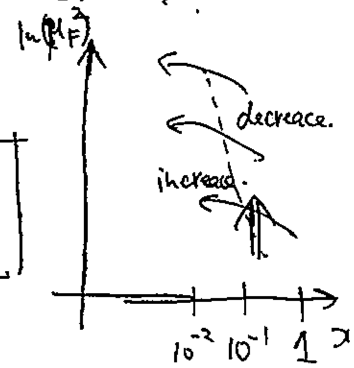
convolution form.

< * extra contribution at $\delta(1-x/z)$ >

Mellin transform.

$$\boxed{\frac{\partial \tilde{f}_q(j;\mu_F^2)}{\partial \ln(\mu_F^2)} = \frac{\alpha_s}{2\pi} C_2(R)\, \gamma(j)\, \tilde{f}_q(j;\mu_F^2)}$$

$$\boxed{\tilde{f}_q(j) + \tilde{f}_{\bar{q}}(j) = \frac{1}{2} A_j}$$



OPE

operator M.E.

$$\int T\{J^\mu(x) J^\nu(y)\} e^{+iq\cdot(x-y)} d^4y \Rightarrow \sum_j C_j(q^2;\mu_R^2) \left[\bar{\psi}\gamma^\mu (\not{D})^{j-1}\psi\right]_{\mu_R^2}$$

$$\Downarrow$$

$$\sum_j C_j(q^2;\mu_R^2) \langle h | [\quad]_{\mu_R^2} | h \rangle.$$

take care of UV DOF first

"IR" DOF later.

Parton model

$dz\, f_q(z;\mu_F^2)$ $\Rightarrow$

take care of collinear DOF first

$\Downarrow$

$dz\, f_q(z;\mu_F^2)\, \cancel{P(x/z)}\, \hat{\sigma}(z)$

hard scatter later.

the same thing.

Factorization into (hard part) × (non-perturbative part)

OPE makes it clear in the case of DIS.

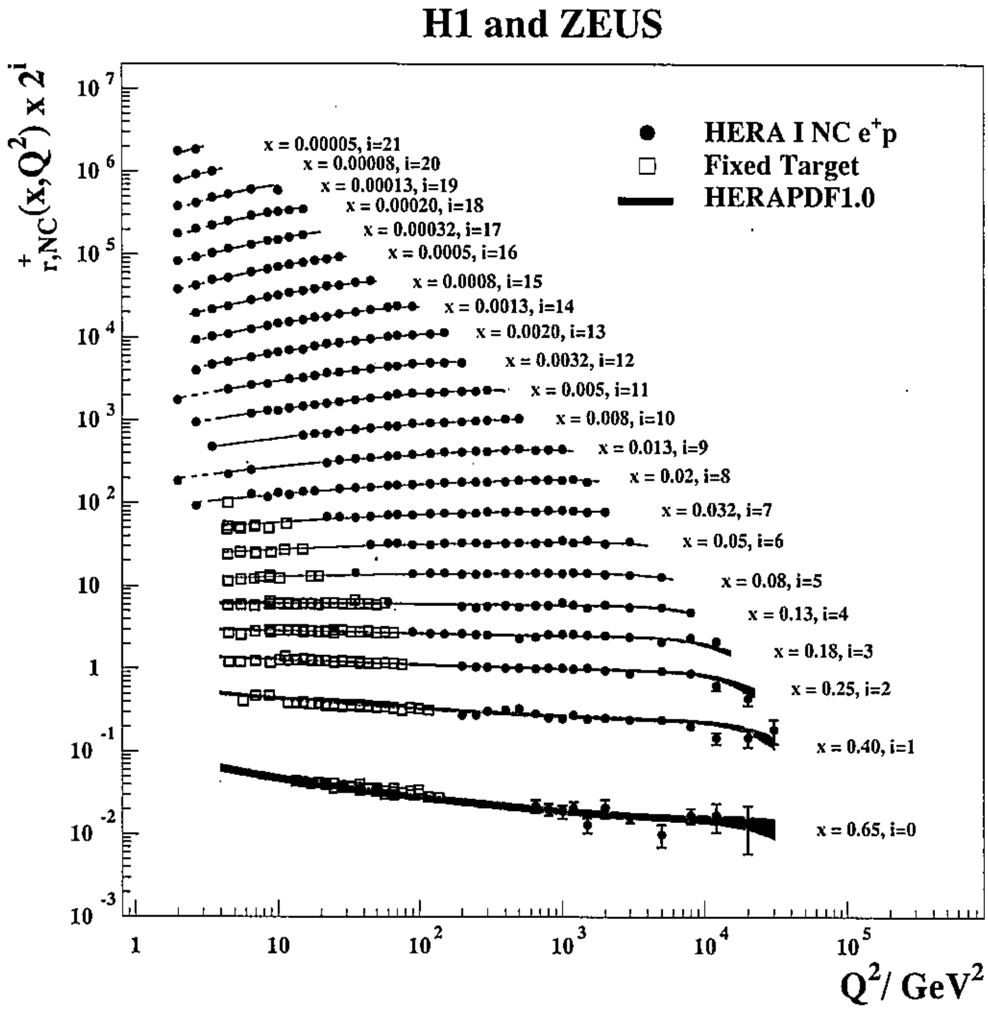


Figure 9. HERA combined NC $e^{+}p$ reduced cross section and fixed-target data as a function of Q^2. The error bars indicate the total experimental uncertainty. The HERAPDF1.0 fit is superimposed. The bands represent the total uncertainty of the fit. Dashed lines are shown for Q^2 values not included in the QCD analysis.

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