The D-Star and its Decays

– a phenomenological parton model of the bound states of heavy D-particles –

- Banks, Fischler, Klebanov & Susskind (BFKS) hep-th/9709091, 9711005
- G. Horowitz, EJM 9710217
- M. Li, EJM 9801070
- M. Li, EJM, V. Sahakian ?
- H. Liu, A. Tseytlin 9712063

⋮

1. YM Phase transitions & their SUGRA counterparts
2. Mean field theory & Black hole thermodynamics
3. Matrix string limit & the correspondence principle

- Matrix theory (BFSS) asserts that low-energy dynamics of many (say N) Dp-branes on $\tilde{T}^p$ reproduces 11D SUGRA on T^p in the infinite momentum frame (IMF). The appropriate kinematic regime is (BFSS, Sen, Seiberg, Maldacena, ...)

M-theory		SYM	
$g_s = \left(\frac{R}{\ell_{pl}}\right)^{3/2}$	$\to 0$	$\tilde{R}_i = \frac{\ell_{pl}^3}{R R_i}$	fixed
$R_1, \dots, R_p$	$\to 0$		
E	$\to 0$	$g_{YM}^2 = \frac{R^3 \tilde{R}_1 \cdots \tilde{R}_p}{\ell_{pl}^6}$	fixed
N	$\to \infty$		

- The 11th dimension is the hidden circle of $M \to IIA$ reduction, size $R = g_s^{2/3} \ell_{pl}$. The system carries longitudinal momentum $P = N/R$ in this direction.
- The generic object in the theory is a black hole. Want to describe it as a metastable bound state of the Dp-branes: a **D-star**. The above limit freezes stringy excitations, leaving YM on the branes.
- Expect that generic (thermal) state of SYM describes a BH in $D = 11-p$ dimensions in the IMF...

- Problem - typical BH has horizon size

$$r_0 \sim (G_D M)^{\frac{1}{D-3}} \gg R \qquad \left(G_D = \frac{R_1 \cdots R_p}{\ell_{pl}^{9}} , \quad D = 11-p \right)$$

and the entropy

$$S_{BH} \sim G_D^{-1} r_0^{D-2} \quad \text{can be} \gg S_{YM}(g, N, E, \tilde{R}_i)$$

thus typically the BH

a) does not fit in the longitudinal box of size R

b) has more available states than YM at a given energy

BFKS proposed a solution to these two problems :

a) for sufficient boost, "Lorentz contraction" will "squeeze" the BH into size R or less ; one needs

$$R > \underbrace{\left(\frac{M}{P}\right)}_{e^{-\alpha}} r_0 = \frac{S_{BH}}{P} = R \cdot \frac{S_{BH}}{N}$$

α = rapidity

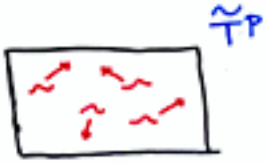
is $N > S_{BH}$ "fits" the BH into the box

b) thus, automatically have enough d.o.f.'s to describe the BH

YM picture :

$N \ll S_{BH}$

$(P \ll S/R)$



thermal gas of YM quanta = longitudinally wrapped membranes in matrix theory dictionary

$N \gg S_{BH}$

$(P \gg S/R)$

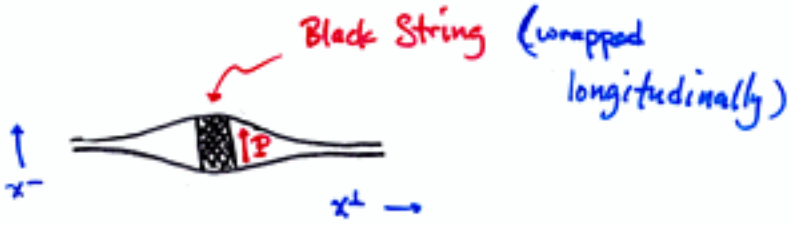
$T \gg \tilde{R}_i^{-1} \Longrightarrow$ system frozen into QM of zero modes = D-ptcles pointlike longitudinally

reason for transition : thermal wavelength grows larger than effective box size $\Rightarrow$ system freezes into moduli space dynamics

GR picture :

$N \ll S_{BH}$

$\uparrow$ PR



$N \gg S_{BH}$

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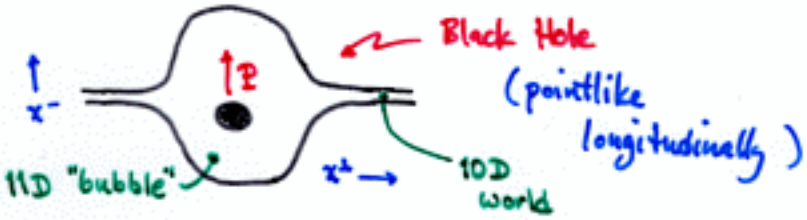
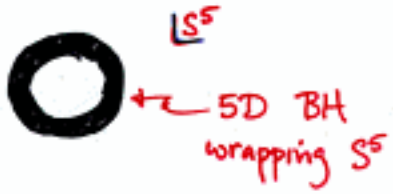
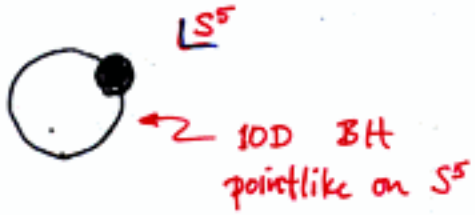


reason for transition : for given mass black object,
string has more entropy in fixed box size R for $P \ll S/R$
hole " " " " " " " " " $P \gg S/R$

Aside on AdS/CFT correspondence :

A similar transition occurs for AdS_5 Schwarzschild





$r_0 \gg R_{AdS} = R_{S^5}$

$r_0 \ll R_{AdS} = R_{S^5}$

$T_{YM} \sim T_H \gg R_{AdS}^{-1}$

YM interpretation : there exist long-lived, metastable states of strongly coupled 3+1d $\mathcal{N}=4$ SYM at large N, with

$$\left.\begin{aligned} E &\sim N^2 \frac{r_0^7}{R_{AdS}^8} \\ S &\sim N^2 \left(\frac{r_0}{R_{AdS}}\right)^8 \\ \text{lifetime} &\sim N^2 \frac{r_0^9}{R_{AdS}^8} \end{aligned}\right\} \quad \ell_s < r_0 < R_{AdS}$$

(get macroscopic strings for smaller energy)

these states spontaneously break the R-symmetry (localize on S^5) and are only present for strong coupling $(g_s N)^{1/4} = \frac{R_{AdS}}{\ell_s} \gg 1$

QM of SYM zero modes

Fix a superselection sector N. The ground state (at $E_{IMF}=0$) is the graviton of momentum $P=\frac{N}{R}$ = threshold bound state of N D-particles. One can roughly think of the e-values of the SYM matrices as position coords for the D-ptcles.

This state has a highly coherent wavefunction. Expect that if we pertwb it by adding energy, some coherence is lost.

- **Hypothesis #1:** As system disorders w/increasing energy, it maintains coherence over clusters of D-ptcles of size k

  ($k=N$ for threshold bound state
  $k=1$ at hole/string transition $N=S_{BH}$)

- **Hypothesis #2:** Mean field theory of clusters approximates the dynamics of BH's with $S_{BH} \lesssim PR = N$

- **Hypothesis #3:** Can use known (and expected) effective interactions of matrix theory at their limit of validity.

- Let $\frac{N}{k}$ = # clusters $\equiv S$ (we'll see that $S = S_{BH}$)
- Suppose cluster c.o.m. has wavefn. of spread $|\delta\vec{x}| \sim r_0$
- Heisenberg $\rightarrow$ $|\delta\vec{p}| \sim 1/r_0$, so

$$E_{kin} \sim (\#\text{ clusters}) \cdot \frac{\delta\vec{p}^{\,2}}{P} \sim S \cdot \frac{r_0^{-2}}{(N/SR)} \sim \left(\frac{S}{r_0}\right)^2 \Big/ \left(\frac{N}{R}\right)$$

- Simplest cluster interaction is via 1-loop matrix integral (ie static interaction of zero longitudinal momentum exchange)

$$V_{int}^{1\text{-loop}} \sim \left(\frac{N_1}{R}\vec{v}_1^{\,2}\right)\left(\frac{N_2}{R}\vec{v}_2^{\,2}\right) \frac{G_D}{R\, r_{12}^{D-4}}$$

Since clusters are comingled, need to include longitudinal momentum exchange (parton exchange between clusters)

- **Hypothesis #3a**: Interactions are approximately Lorentz covariant at sufficiently large N.

in rest frame
$$\int_0^{2\pi R} \frac{dx_{10}}{2\pi R} \sum_{n\in\mathbb{Z}} \frac{\exp[-ipx_{10}]}{[(x_{10}+2\pi R n)^2 + r^2]^{d/2}} = \frac{(2p/r)^{\frac{d-1}{2}}}{\sqrt{\pi}\,\Gamma(d/2)} K_{\frac{d-1}{2}}(pr)$$

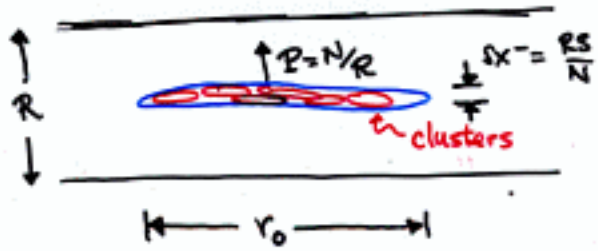
$$\sim \begin{cases} \frac{1}{R r^{d-1}} & pr \ll 1 \\ \exp[-pr] & pr \gg 1 \end{cases}$$

(want $d-1 = D-4$)

$\therefore$ In the boosted frame, longitudinal momentum exchange interactions are same as $V_{int}^{1\text{-loop}}$ up to scale of long. resolution $\delta x^- \sim \frac{RS}{N}$

YM picture :

(GR: )



$$E_{pot} \sim G_D \sum_{R\Delta p_- = 0}^{N/S} \sum_{a,b=1}^{S} \frac{(\vec{P}^2/P_{cl})_a \, (\vec{P}^2/P_{cl})_b}{R\, r_{ab}^{D-4}}$$

$$\sim E_{kin} \cdot \frac{G_D S}{r_0^{D-2}}$$

Virial theorem $E_{pot} \sim E_{kin} \Longrightarrow \boxed{S \sim G_D^{-1} r_0^{D-2}}$!!!

Moreover, $\frac{M^2}{P} = E_{IMF} \sim E_{kin} + E_{pot} \sim \left(\frac{S}{r_0}\right)^2 / P \Rightarrow \boxed{M = S/r_0}$!!!

Punchline: IMF description of a black hole is a "star" composed of D-particles. Simple arguments reproduce the expected mass-entropy and entropy-area scalings.

Comments :

- The interactions are a cartoon. In this regime, all loop interactions $\frac{(N_{cl} v^2)^{\ell-1}}{r^{\ell(D-4)}}$ are of the same order. The loop expansion breaks down, since nonabelian d.o.f.'s are light & excited (the state has a significant membrane component). Nevertheless, expect mean field theory to predict the scale at which this takes place.

  (cf. also refined analysis Liu-Tseytlin 9712063.)

- In order to claim $S = \#\text{clusters} = S_{BH}$, need to treat clusters as distinguishable objects. This is implied by the previous remark - clusters interact with a diffuse background of nonabelian matrix modes which break any permutation symmetry of clusters (each cluster interacts with a different set of off-diagonal matrix elements)

Charged Black Holes:

$$\left.\begin{aligned} M &\sim G_D^{-1}\, r_0^{D-3}\, \mathrm{ch}^2\gamma \\ Q &\sim G_D^{-1}\, r_0^{D-3}\, \mathrm{sh}\gamma\, \mathrm{ch}\gamma \\ S &\sim G_D^{-1}\, \underbrace{r_0^{D-2}}_{\text{Horizon area}}\, \mathrm{ch}\gamma \end{aligned}\right\} \circledast$$

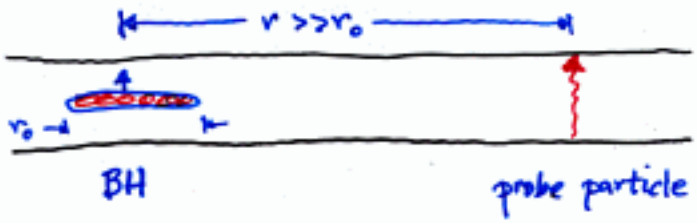
- One finds that Black String / Black Hole transition takes place at $P = S\,\mathrm{ch}\gamma / R$ (M. Li, EJM, H. Arodz)

- In YM, can introduce charge by giving D-particles net momentum along internal torus T^p. Virial estimate again reproduces scaling behavior $\circledast$ given hypotheses 1-3a.

- In all cases, $|\vec{\delta p}|_{\text{noncompact}} \sim \frac{1}{r_0\, \mathrm{ch}\gamma} \sim T_H$

  ie virial temperature of clusters is the Hawking temperature. Characteristic temp of emitted quanta must be this scale — **Hawking radiation is the "solar wind" of the D-star**

Probe dynamics



$$S_{probe} = P_-^{probe} \int d\tau \, \frac{1}{g_{--}} \left[(g_{+-} + g_{-i} v^i) - \left((g_{+-} + g_{-i} v^i)^2 - g_{--} (g_{++} + g_{ij} v^i v^j + g_{+i} v^i) \right) \right]^{1/2}$$

Expanding for large r, small v; with $g_{\mu\nu} \sim \eta_{\mu\nu} + a_{\mu\nu} \left(\frac{r_0}{r}\right)^{D-3}$

$$S_{probe} \sim P_-^{probe} \int d\tau \left[\tfrac{1}{2} v^2 + \tfrac{1}{2} \left(\tfrac{r_0}{r}\right)^{D-3} \left(a_{++} + a_{ij} v^i v^j + a_{+i} v^i - (a_{+-} + a_{-i} v^i) v^2 + \tfrac{1}{4} a_{--} v^4 \right) + \dots \right]$$

- Treat the source as a collection of clusters
- Allow both BH and probe to have charge ($\vec{v} \neq 0$ in T^P)
- Use leading matrix theory interaction to estimate terms in S_{probe}

$\Rightarrow$ qualitatively reproduce all terms in S_{probe} to order v^4

D-Stars, Matrix Strings, & the Correspondence Principle

Further support for the model comes from the matrix string limit. Shrinking an $S^1 \subset T^p$ (eg $R_p \ll R$) one finds

$$p+1 \text{ SYM on } \tilde{T}^p \longrightarrow 1+1 \text{ SYM on } \tilde{S}^1 \qquad \left(\tilde{R}_p = \frac{\ell_{pl}^3}{R R_p} \gg \tilde{R}_i\right) \quad i = 1 \ldots p-1$$

Dominant configurations are "Slinkies®", a "string bit" description of the IMF superstring

At the "correspondence point" $r_0 \sim \ell_{str} = \left(\frac{\ell_{pl}^3}{R_p}\right)^{1/2}$, a transition occurs from BH's ($r_0 > \ell_{str}$) to strings ($r_0 < \ell_{str}$) as the entropically favored objects (Horowitz - Polchinski).

At $r_0 \sim \ell_{str}$ the entropies match

$$S_{BH} \sim G_D^{-1} r_0^{D-2} \sim S_{str} \sim \sqrt{N_{osc}}$$

Now consider a BH which is simultaneously

a) at the correspondence point $r_0 \sim \ell_{str}$

b) boosted to the black hole / black string transition $N = S$ in a longitudinal box of size R

The IMF matrix string has temperature

$$T_{str} = \sqrt{N_{osc}} \Big/ \left(\frac{N}{R}\, \ell_{str}^2\right) \qquad (**)$$

The correspondence principle asserts that this is the (boosted) Hawking temperature (since a macroscopic string decays by emitting quanta at the Hagedorn temperature $T \sim \frac{1}{\ell_{str}} \sim \frac{1}{r_0} \sim T_H$)

$$T_H \sim e^{-\alpha} \cdot \frac{1}{r_0} \sim \frac{R}{r_0^2}$$

Comparing,

$$\frac{\sqrt{N_{osc}}}{N} \sim \frac{\ell_{str}^2}{r_0^2} \sim 1$$

- Thus, each of the N "string bits" or D-particles carries one "bit of information". This is clear from (**) – the thermal wavelength is $\lambda_T \sim \tilde{R}_p$ = one string bit; a macroscopic string is a random walk of S steps of size λ_T.
- The "Boltzmann" character of the D-particle gas is manifest in the fact that the D-particles are labelled by their position on the matrix string – this is an example of the "<u>nonabelian background</u>", ie the set of SU(N) twisted B.C.'s that intertwines the string bits as one circles $\tilde{S}^1$.

- Further boosting to $N \gg S_{BH}$ (but remaining at the correspondence point), the string temperature boosts down by a factor N/S_{BH}. The number of string bits contained in a thermal wavelength (coherence length) is $k = N/S_{BH}$; these are the "clusters". Their virial temperature is indeed the (boosted) Hawking temperature.

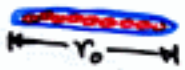
- Charged BH's at the correspondence point match perfectly with the cluster ansatz; one finds that at the juncture of the correspondence point and the hole/string transition, the matrix string has $N/S_{BH} \sim ch\gamma$ string bits in a thermal wavelength - exactly as predicted by the ansatz.

Summary:

- Black holes are "stars" composed of D-particles, membranes, etc. They are "supported" against collapse by the uncertainty principle and by the fact that gravity ceases to be the complete IMF description of interactions within the BH.

- Characteristic size of BH is governed by available energy/momentum resolution – r_0 in transverse direction, r_0 "Lorentz contracted" to $e^{-\alpha} r_0 = \frac{RS}{N}$ longitudinally.

- Transverse density of cluster centers never exceeds the "holographic bound" of one per transverse Planck volume.



- Locality/nonlocality – individual D-particles have spread $|\delta\vec{x}| \sim N^{1/9}$, whereas black hole wavefunction has $|\delta\vec{x}| \sim r_0$ independent of N. Any locality properties are a result of coherence of cluster wavefunction.

  cluster c.o.m.

  (same phenomenon occurs in
  - perturbative string theory
  - AdS/CFT)