IN A THEORY WITH GRAVITY, ONE DOESN'T HAVE GAUGE-INVARIANT LOCAL OPERATORS $\phi(x)$.

THE ONLY KNOWN OBSERVABLES ARE "GLOBAL."

FOR INSTANCE, IN AN ASYMPTOTICALLY FLAT WORLD, ONE HAS THE S-MATRIX = RESPONSE TO SOURCES THAT CREATE AND

ANNIHILATE PARTICLES IN THE FAR PAST AND FUTURE, RESPECTIVELY.

BEYOND THE S-MATRIX ONE MAYBE HAS GLOBAL OBSERVABLES LIKE

$$\int d^4x \sqrt{g}\, R^{10}$$

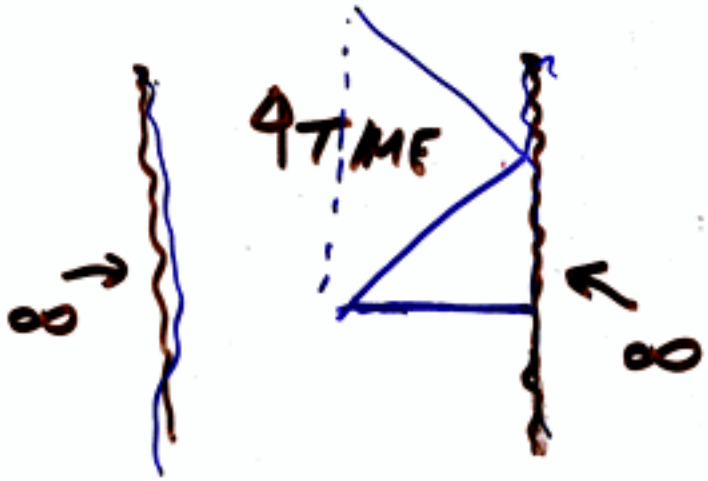
BUT THIS ISN'T CLEAR, AND MAY DEPEND ON DETAILS OF A THEORY

(NO EVIDENCE FOR IT IN STRING THEORY)

IF THE COSMOLOGICAL CONSTANT IS NEGATIVE, THERE IS NO NOTION OF AN S-MATRIX, BECAUSE THE BASIC PREREQUISITE IS NOT OBEYED: INTERACTIONS DON'T EFFECTIVELY TURN OFF IN THE FAR PAST OR FUTURE.

AdS HAS A PECULIAR BOUNDARY AT SPATIAL INFINITY

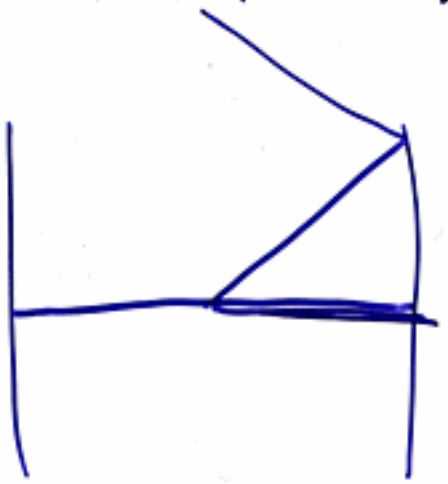
MASSLESS PARTICLES ARE FOREVER BOUNCING BACK AND FORTH; MASSIVE ONES ARE TRAPPED INSIDE.



NOT LIKE A CONVENTIONAL INFINITE VOLUME SYSTEM.

CONFORMAL INFINITY....

$$ds^2 = -(dt)^2\left(1 - \frac{1}{\ }\right) + \frac{dr^2}{1+\frac{1}{r^2}} + r^2 d\Omega^2$$



$$\left(\frac{1}{r^2}\right)^2$$

the bdry has not quite a metric but a <u>conformal structure</u>

AdS_{n+1} $\quad$ $SO(2, n)$ isometries

Bdry

M_n $\quad$ $SO(2,n)$ conformal symmetries

$\wr ?$

Minkowski

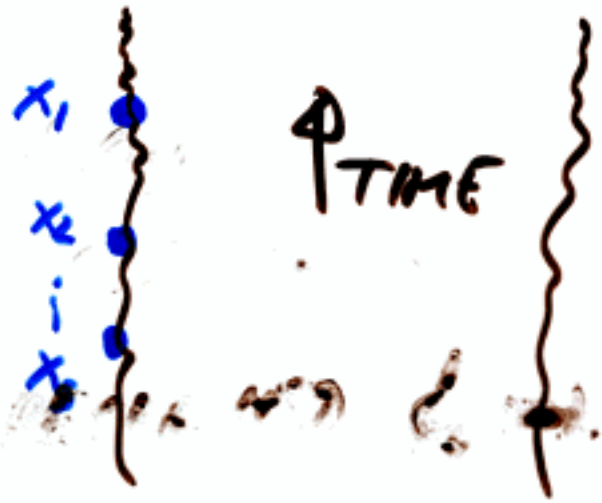
INSTEAD OF EMISSION AND ABSORPTION IN FAR PAST OF FUTURE, IT IS MORE NATURAL TO CONSIDER EMISSION AND ABSORPTION AT FINITE BOUNDARY POINTS.

BY ALLOWING EMISSION AT x AND ABSORTION AT y ONE GETS A SHORT DISTANCE SINGULARITY IF $x \to y$

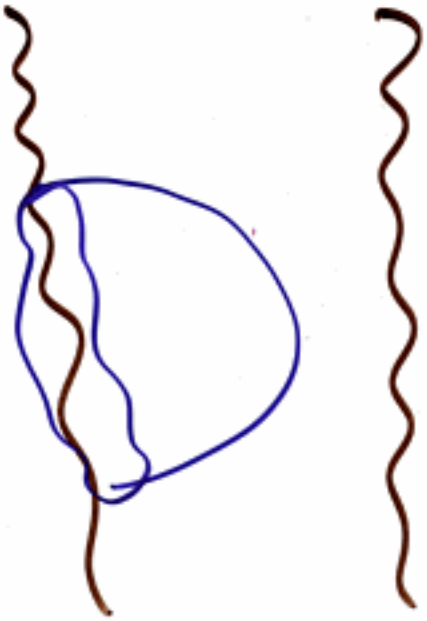
FOR $\Lambda < 0$ ONE GETS NOT AN S-MATRIX BUT, IN A WAY, SOMETHING MORE INTERESTING:

CORRELATION FUNCTIONS OF A CONFORMAL FIELD THEORY.

$$\langle \mathcal{O}_1(x_1)\, \mathcal{O}_2(x_2) \cdots \mathcal{O}_s(x_s) \rangle$$



TECHNICALLY ABSORPTION AND EMISSION ON BOUNDARY ARE STUDIED BY RESPONSE OF SYSTEM TO BOUNDARY BEHAVIOR



$$\theta(y) \sim \frac{\delta}{\delta \phi_0(y)}$$

$$\phi(x) \to \phi_0(x)$$

ON BOUNDARY

WITH SPECIFIED $\phi_0(x)$

SO FROM A GRAVITIONAL THEORY ON AdS_{n+1}

(OR $AdS_{n+1} \times X$, X=COMPACT)

WE GET A CONFORMAL FIELD THEORY ON

M_n = THE BOUNDARY OF AdS_{n+1}

APPARENTLY, THE GRAVITY THEORY IS EQUIVALENT TO THE C.F.T.

EVIDENCE FOR
"COMPLETENESS"
COMES NOTABLY
FROM MAP

BLACK HOLE THERMODYNAMICS

$\Longleftrightarrow$

THERMODYNAMICS
OF C.F.T.

SO WE HAVE THESE GLOBAL OBSERVABLES:

(Λ = COSMOLOGICAL CONSTANT)

$\Lambda = 0 \Rightarrow$ S-MATRIX

$\Lambda < 0 \Rightarrow$ CONFORMAL FIELD THEORY

THE SECOND IS MORE INTERESTING IN THAT IT GIVES A MICROSCOPIC DEFINITION OF A GRAVITATIONAL THEORY.

IT IS ILLUMINATING TO SAY "STRING THEORY ON $AdS_5 \times S^5$ IS EQUIVALENT TO $N=4$ SUPER YANG MILLS" ON THE BOUNDARY OF AdS_5" SINCE THAT IS A MICROSCOPIC DEFINITION

SAYING "STRING THEORY IN ASYMPTOTICALLY FLAT SPACE IS EQUIVALENT TO ITS S-MATRIX" ISN'T ILLUMINATING ....

AT THIS STAGE, WE MIGHT WANT TO REFLECT ON THE MEANING OF THE CLAIM ('t HOOFT, SUSSKIND) THAT THE WORLD IS "HOLOGRAPHIC"... AND DESCRIBED BY A THEORY ASSOCIATED WITH THE BOUNDARY OF SPACETIME

....

THIS MUST MEAN NOT JUST THAT THE OBSERVABLES ARE AT INFINITY

- STANDARD AS THERE ARE NO LOCAL OBSERVABLES

- BUT THAT IN SOME SENSE THE DEGREES OF FREEDOM ARE AT INFINITY... OR MORE EXACTLY THAT THE THEORY CAN BE DEFINED

IN TERMS OF DEGREES OF FREEDOM AT INFINITY.

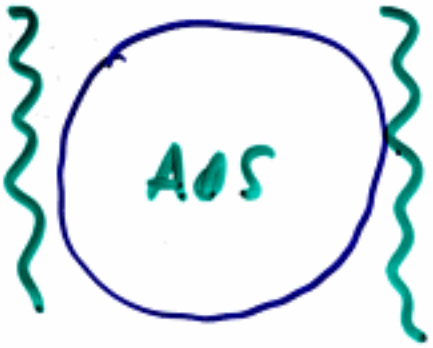
THE RELATION TO BOUNDARY CONFORMAL FIELD THEORY DOES THAT BEAUTIFULLY WHEN $\Lambda < 0$.

IT IS A HARD-TO-FORMULATE NOTION SINCE IN ANY CASE ONE CAN'T LOCALIZE THE DEGREES OF FREEDOM

THE CFT/AdS CORRESPONDENCE IS AN AMAZING WAY TO MAKE PRECISE — IF $\Lambda < 0$ — THE IDEA THAT THE DEGREES OF FREEDOM ARE AT INFINITY.

IT ACTUALLY INCORPORATES THE "INFORMATION" BOUND THAT IS PART OF THE HOLOGRAPHIC PACKAGE ....

BECAUSE OF AN AMAZING U.V. / I.R. CONNECTION:



$$\mathcal{O}(x) = \frac{\delta}{\delta \phi_0(x)}$$

$$\phi \to \varepsilon^{-\gamma} \phi_0(x)$$

INFRARED EFFECTS IN AdS ARE MAPPED TO ULTRAVIOLET EFFECTS ON THE BOUNDARY ....

MANY ASPECTS:
e.g. dimensions of operators

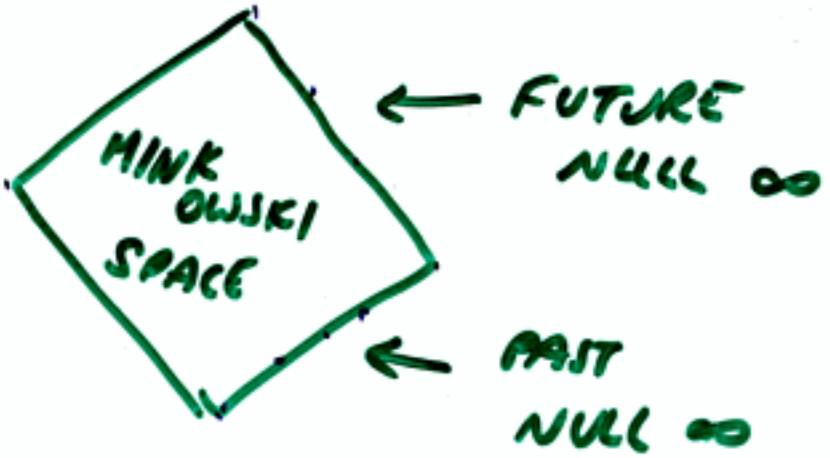
SO ONE GETS A HOLOGRAPHIC DESCRIPTION, FOR $\Lambda < 0$, IN A VERY SATISFACTORY WAY.

THE REAL OBSTACLE TO AN ANALOGOUS SUCCESS WHEN $\Lambda = 0$ SEEMS TO BE THAT THE NATURAL BOUNDARY OF MINKOWSKI SPACE IS NOT AT SPATIAL INFINITY BUT AT PAST AND FUTURE NULL INFINITY

THIS IS SEEN IN THE PENROSE DIAGRAM

$\Lambda < 0$

$\Lambda = 0$



EMISSION AND ABSORPTION AT PAST AND FUTURE INFINITY JUST LEAD TO THE S-MATRIX (OF MASSLESS PARTICLES)

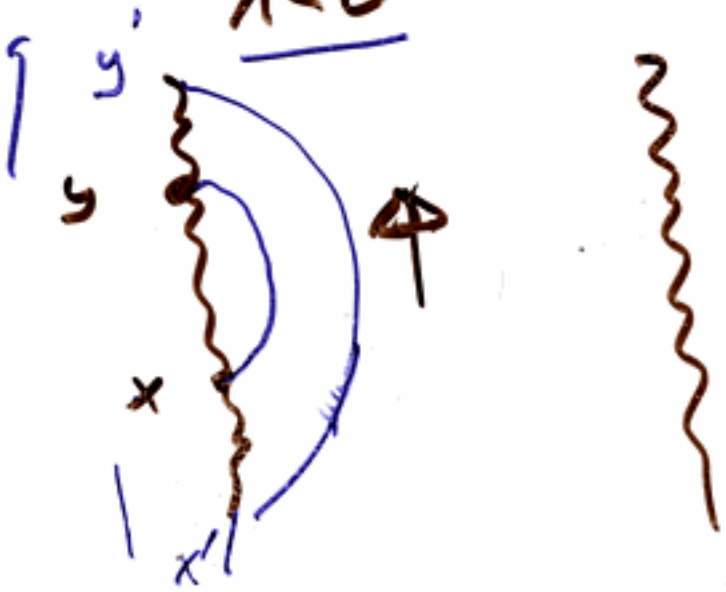
IN THIS COMPARISON

$\Lambda < 0 \Rightarrow$ CONFORMAL FIELD THEORY

$\Lambda = 0 \Rightarrow$ S MATRIX

IT SEEMS THAT SOMETHING IS REALLY LOST IN GOING TO $\Lambda = 0$ .... I THINK THIS IS RELATED TO OUR INTUITION THAT THE $\Lambda < 0$ ANSWER IS A MICROSCOPIC DEFINITION AND THE $\Lambda = 0$ ANSWER ISN'T....

$\Lambda < 0$



LET x = POINT OF EMISSION

y = POINT OF ABSORPTION

AS $\Lambda \to 0$ IF ONE KEEPS x, y NEAR EACH OTHER, THE WHOLE PROCESS MOVES OFF TO INFINITY.

AS $\Lambda \to 0$, IF WE WISH TO PROBE THINGS THAT HAPPEN "INSIDE" MINKOWSKI SPACE, WE MUST TAKE $|x-y| \to \infty$ AND WE ARE LEFT WITH THE S-MATRIX

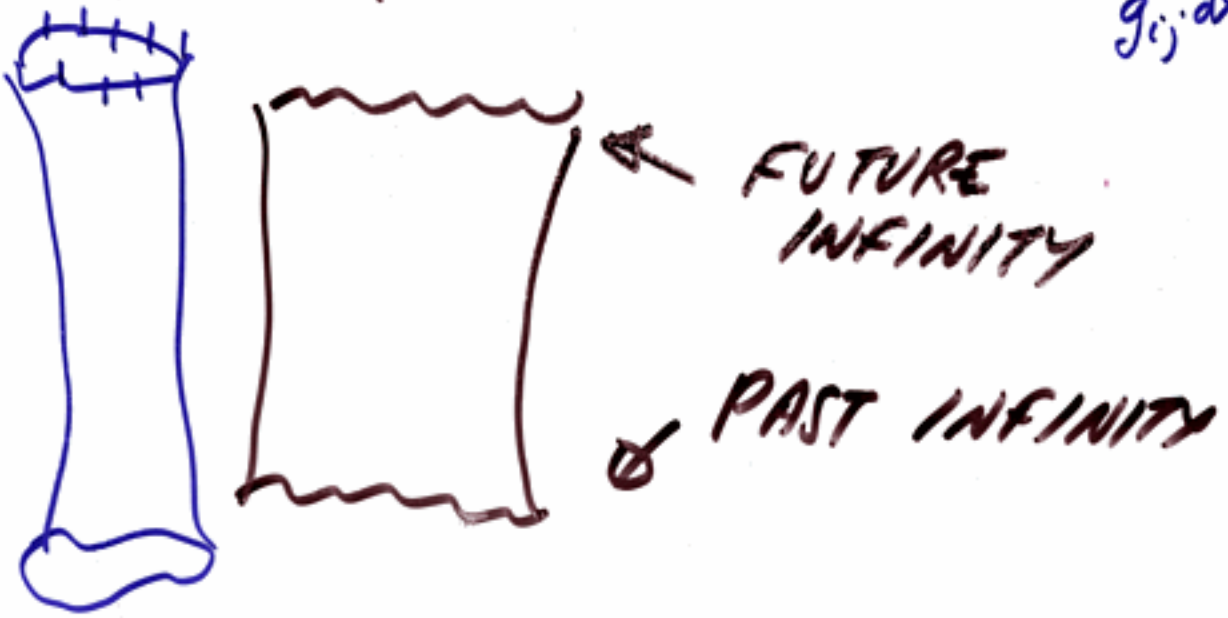
$x \to$ PAST

$y \to$ FUTURE.

IF $\Lambda > 0$, THERE IS A KIND OF S-MATRIX AS FOR $\Lambda = 0$:

$dt^2 + \cosh^2 t \, d\Omega^2$

$g_{ij} dx^i dx^j$



THOUGH TECHNICAL DEFINITION IS DIFFERENT. THE S-MATRIX IS THE ONLY OBVIOUS GLOBAL OBSERVABLE AND DOESN'T SEEM TO GIVE

A NATURAL "MICROSCOPIC DEFINITION" OF THE THEORY, JUST AS IT DOESN'T AT $\Lambda = 0$.

THE DIFFICULTY IN UNDERSTANDING HOLOGRAPHY FOR $\Lambda > 0$ MIGHT NOT SURPRISE US, AS MORE NAIVELY

"WHAT COULD BE, IN A CLOSED UNIVERSE, A DESCRIPTION BY BOUNDARY DEGREES OF FREEDOM?"

GOING BACK TO THE CASES

$\Lambda < 0, \quad \Lambda = 0 \quad \ldots$

IN HINDSIGHT THE GEOMETRY OF AdS SPACE MAKES HOLOGRAPHY SEEM INEVITABLE FOR $\Lambda < 0$.

FOR $\Lambda = 0$ WE ARE GOING TO HAVE A MUCH TOUGHER TIME - FROM THE

POINT OF VIEW – JUST SINCE MINKOWSKI SPACE DOESN'T HAVE A NATURAL BOUNDARY AT SPATIAL INFINITY.

I NOTE THAT THE ONLY PROPOSAL FOR A HOLOGRAPHIC THEORY AT $\Lambda = 0$ IS THE MATRIX MODEL. IT ISN'T REALLY CLEAR THAT IT IS HOLOGRAPHIC, BUT IF IT IS, IT CERTAINLY ACHIEVES

ITS STATUS IN A VERY DIFFERENT WAY FROM WHAT WE HAVE AT $\Lambda < 0$.

ONE THING WHICH SHOWS THAT IT MUST BE HARD TO UNDERSTAND HOLOGRAPHY FOR $\Lambda = 0$ IS THAT, APPARENTLY, SUCCESS WOULD HAVE AN EXPLANATION OF WHY $\Lambda = 0$ BUILT IN

A CONFORMAL FIELD THEORY IN n DIMENSIONS - IF UNDERSTOOD AS A HOLOGRAPHIC DESCRIPTION OF A GRAVITATIONAL THEORY IN $n+1$ DIMENSIONS - IS AUTOMATICALLY TIED TO $\Lambda < 0$, JUST FROM THE SYMMETRY GROUP.

A HOLOGRAPHIC DESCRIPTION FOR $\Lambda = 0$, IF THERE REALLY IS SUCH A THING, MUST INVOLVE NOT C.F.T. BUT SOMETHING ELSE - CALL IT "STRUCTURE X" AS WE DON'T KNOW WHAT IT IS. WHATEVER IT IS, AN EXAMPLE OF THIS STRUCTURE MUST CORRESPOND

TO A STABLE (??)

VACUUM WITH

$\Lambda = 0$.

(IN ASSUMING THAT

STRUCTURE X IS RELATED

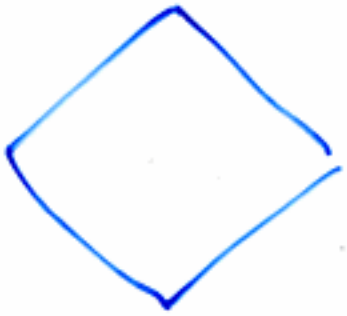
TO A STABLE VACUUM,

I AM ASSUMING THAT TO

DO HOLOGRAPHY AT $\Lambda = 0$

ONE NEEDN'T ALSO

DO COSMOLOGY.)



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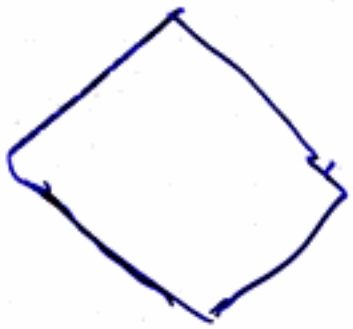
A TOO-NAIVE VERSION OF STRUCTURE X WOULD BE A FIELD THEORY ON NULL INFINITY WITH ITS PECULIAR DIFFERENTIAL GEOMETRY.

NOTE THAT THE MATRIX MODEL ISN'T A FIELD THEORY ANYWHERE



$0++++$

$SO(2,n)$

$$\mathcal{O}_i(x)\,\mathcal{O}_j(y) = \sum_k c_{ijk}(x-y)\,\mathcal{O}_k(y)$$

$\mathbb{R}^3$