SUPERSTRING IN AdS BACKGROUND WITH R-R FLUX

G/H COSET SIGMA MODEL

G : SUPERGROUP $\quad PSU(M,M)$

$$g = \left(\begin{array}{c|c} B & F \\ \hline F & B \end{array}\right)$$

$$\left.\begin{array}{l} (g^{-1}\partial g)^{c} \\ (g^{-1}\partial g)^{i} \in \mathcal{H} \end{array}\right\} \text{bosonic}$$

$c = 1 \ldots 2M^2 - 2 - \dim \mathcal{H}$

$i = 1 \ldots \dim \mathcal{H}$

$$\left.\begin{array}{l} (g^{-1}\partial g)^{\alpha} \\ (g^{-1}\partial g)^{\bar{\alpha}} \end{array}\right\} \text{fermionic}$$

$\alpha = 1 \ldots M^2$

$\bar{\alpha} = 1 \ldots M^2$

are left-invariant currents.

$$(*)\quad S = \int d^2 z \Big[(g^{-1}\partial g)^{c} (g^{-1}\bar\partial g)^{c} + (g^{-1}\partial g)^{\alpha} (g^{-1}\bar\partial g)^{\bar\alpha}$$
$$+ \tfrac{1}{2} (g^{-1} dg)^{\alpha} \wedge (g^{-1} dg)^{\bar\alpha} \Big] + S_I$$

TERMS IN BLACK BREAK K-SYMMETRY BUT ALLOW N=2 SUPERCONF. INVARIANCE AND QUANTIZATION.

$AdS_2 \times S^2$: $G/H = PSU(1,1|2) / U(1) \times U(1)$

$$S_I = \int d^2 z \, \partial\rho \, \bar\partial\rho \; + \; c=9 \;\; N=2 \;\; \text{SCFT}$$

$AdS_3 \times S^3$: $G/H = PSU(1,1|2) \times PSU(2|2) / SU(2) \times SU(2)$

$$S_I = \int d^2 z \, (\partial\rho \, \bar\partial\rho + \partial\sigma \, \bar\partial\sigma)$$
$$+ \; c=6 \;\; N=2 \;\; \text{SCFT} \; + \; \text{HARMONIC CONSTRAINTS}$$

$AdS_5 \times S^5$: $G/H = PSU(2,2|4) / SO(4,1) \times SO(5)$

$$S_I = \; ?$$

AdS ACTIONS WERE DERIVED USING "HYBRID" FORMALISM OF THE SUPERSTRING

$\boxed{\text{HYBRID}}$

GREEN-SCHWARZ VIA WARREN SIEGEL	RAMOND-NEVEU-SCHWARZ VIA FRIEDAN, MARTINEC, SHENKER
"CLASSICAL SUPERSTRING MECHANICS" NPB 263 (1985) 93.	"CONFORMAL INVARIANCE, SUPERSYMMETRY, AND STRING THEORY" NPB 271 (1986) 93.

SIEGEL (1985):

WRITE GS ACTION IN HAMILTONIAN FORM

$$S = \int d\tau\, d\sigma \left(\dot{x}^m P_m + \dot{\theta}^\alpha p_\alpha \right)$$ $d: 3, 4, 6, 10$

with constraints

$$T: \ \Pi^m \Pi_m = 0$$

$$d_\alpha: \ p_\alpha + \not{\Pi}\, \theta_\alpha = 0$$

where $\Pi_m = P_m + i X'_m + i \theta \gamma_m \theta'$

$\{ d_\alpha , d_\beta \} = \Pi_m \gamma^m_{\alpha\beta}$ ⇒ second-class constraints

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Proposal: Replace with first-class constraints built from

d_α , Π^m , θ'^α

Form closed algebra since $[d_\alpha, \Pi^m] = \gamma^m_{\alpha\beta} \theta'^\beta$

Vertex op: $V = \int d\tau \left[d_\alpha W^\alpha + y'^M A_M \right]$

$W^\alpha(x,\theta)$ = field-strength $\quad y^M = (x^m, \theta^\alpha)$

$A_M(x,\theta)$ = gauge fields

WHAT ARE THE APPROPRIATE FIRST-CLASS CONSTRAINTS?

FRIEDAN, MARTINEC SHENKER (1986): IN RNS FORMALISM,

$q_\alpha = \int d\sigma \, e^{-\varphi/2} \Sigma_\alpha$ IS SUSY GENERATOR IN $-\frac{1}{2}$ PICTURE

$\Sigma_\alpha = e^{\frac{i}{2}(\pm\sigma_1 \pm \ldots \pm \sigma_5)}$,

$\gamma = \eta e^{\varphi}$, $\beta = \partial\xi e^{-\varphi}$, $\psi^m \pm i\psi^{m+5} = e^{\pm i\sigma_m}$

THIS SUGGESTS DEFINING

$\Theta^\alpha = e^{+\varphi/2} \Sigma^\alpha$ TO GIVE AN

RNS $\leftrightarrow$ GS DICTIONARY.

FIRST-CLASS CONSTRAINTS FORM A CRITICAL (TWISTED) $N=2$ ALGEBRA

$T = T_{RNS}$

$G^+ = j_{BRST}$

$G^- = b$

$J = j_{GHOST}$

(+ HARMONIC CONSTRAINTS)

OUTLINE OF REST OF TALK

I. Hybrid formalism for compactification to $d=4$

A. Flat $d=4$ (NB, 9404162)

B. Curved $d=4$ (NB + W. Siegel, 9510106)

C. $AdS_2 \times S^2$ (NB, M. Bershadsky, T. Hauer, S. Zhukov, B. Zwiebach, 9907???)

II. Hybrid formalism for compactification to $d=6$

A. Flat $d=6$ (NB + C. Vafa, 9407190)

B. Curved $d=6$ (NB, 9907???)

C. $AdS_3 \times S^3$ (NB, C. Vafa, E. Witten, 9902098)

III. Hybrid formalism in $d=10$

A. Flat $d=10$ (NB, 9902099)

B. Speculations

I. Compactification to $d=4$

A. Flat $d=4$ ($m=0\ldots3$; $\alpha,\dot\alpha=1,2$)

RNS : $[x^m, \psi^m, b, c, \xi, \eta, \varphi] + \begin{matrix}N=2\\ c=9\\ \text{SCFT}\end{matrix}$

$$\Big\downarrow \text{FIELD REDEF.}$$

HYBRID : $[\underbrace{x^m, \theta^\alpha, \hat\theta^{\dot\alpha}}_{\substack{N=1\ D=4\\ \text{SUPERSPACE}}}, \underbrace{p_\alpha, \hat p_{\dot\alpha}}_{\substack{\text{momenta}\\ \text{for } \theta}}, \rho] + \begin{matrix}N=2\\ c=9\\ \text{SCFT}\end{matrix}$

$\uparrow$ chiral boson

$$S = \int d^2z \left[\partial x \cdot \bar\partial x + p_\alpha \bar\partial\theta^\alpha + \hat p_{\dot\alpha} \bar\partial \hat\theta^{\dot\alpha} \right] + S_\rho + S_{\text{COMPACTIFICATION}}$$

DEFINE

$$d_\alpha = p_\alpha + i(\partial \not{x}\, \hat\theta)_\alpha$$

$$\hat d_{\dot\alpha} = \hat p_{\dot\alpha} + i(\partial \not{x}\, \theta)_{\dot\alpha}$$

$$\Pi^m = \partial x^m + i\sigma^m_{\alpha\dot\alpha}(\theta^\alpha \partial\hat\theta^{\dot\alpha} + \hat\theta^{\dot\alpha}\partial\theta^\alpha)$$

$$d_\alpha(y)\,\hat d_{\dot\alpha}(z) \to (y-z)^{-1}\sigma^m_{\alpha\dot\alpha}\Pi_m$$

N=2 constraints:

$$T = \underbrace{\partial x \cdot \partial x}_{c=4} + \underbrace{p_\alpha \partial \theta^\alpha + \hat{p}_{\dot\alpha} \partial \hat{\theta}^{\dot\alpha}}_{c=-8} + \underbrace{\tfrac{1}{2} \partial \rho \partial \rho}_{c=1} + \underbrace{T_c}_{c=9}$$

$$G^+ = e^{i\rho} d_\alpha d^\alpha + G_c^+ \rightarrow j_{BRST}$$

$$G^- = e^{-i\rho} \hat{d}_{\dot\alpha} \hat{d}^{\dot\alpha} + G_c^- \rightarrow b$$

$$J = i \partial \rho + J_c \rightarrow j_{GHOST}$$

Twisting the N=2, we can eliminate the need to add N=2 ghosts.

Physical vertex op's are N=2 primaries.

Massless vertex operators

Open: $\int dz\, G^-_{-1} G^+_0 V(x, \theta, \bar\theta)$

$$= \int dz \left[d_\alpha \hat{D}^2 D^\alpha V + \hat{d}_{\dot\alpha} D^2 \hat{D}^{\dot\alpha} V + \Pi^m \sigma_m^{\alpha\dot\alpha} [D_\alpha, \hat{D}_{\dot\alpha}] V + \partial\theta^\alpha D_\alpha V - \partial\hat{\theta}^{\dot\alpha} \hat{D}_{\dot\alpha} V \right]$$

$$= \int dz \left[d_\alpha W^\alpha + \hat{d}_{\dot\alpha} \hat{W}^{\dot\alpha} + \partial y^M A_M \right]$$

as predicted by Siegel

B. Curved $d=4$

Action in flat background + Massless closed vertex op. = Action in curved background

$$S_{FLAT} = \int d^2z \left[\partial x \cdot \bar\partial x + p_\alpha \bar\partial \theta^\alpha + \hat p_{\dot\alpha} \bar\partial \hat\theta^{\dot\alpha} + \bar p_\alpha \partial \bar\theta^\alpha + \hat{\bar p}_{\dot\alpha} \partial \hat{\bar\theta}^{\dot\alpha} \right] + S_I$$

$$= \int d^2z \left[\Pi_m \bar\Pi^m + d_\alpha \bar\partial \theta^\alpha + \hat d_{\dot\alpha} \bar\partial \bar\theta^{\dot\alpha} + \bar d_\alpha \partial \bar\theta^\alpha + \hat{\bar d}_{\dot\alpha} \partial \hat{\bar\theta}^{\dot\alpha} + B^0_{MN} \partial y^M \bar\partial y^N \right] + S_I$$

where $B^0_{MN} \partial y^M \bar\partial y^N$ is the same as in flat GS action

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$$S_{CURVED} = \int d^2z \left[\Pi_c \bar\Pi^c + B_{MN} \partial y^M \bar\partial y^N + d_\alpha \bar\Pi^\alpha + \hat d_{\dot\alpha} \bar\Pi^{\dot\alpha} + \bar d_\alpha \Pi^{\bar\alpha} + \hat{\bar d}_{\dot\alpha} \Pi^{\bar{\dot\alpha}} + d_\alpha \bar d_\beta P^{\alpha\beta} + d_\alpha \hat{\bar d}_{\dot\beta} P^{\alpha\dot\beta} + c.c. \right] + S_{FT} + S_I$$

$\Pi^A = E^A{}_M \partial y^M \quad (A = c, \alpha, \dot\alpha, \bar\alpha, \bar{\dot\alpha})$

Lowest component of $P^{\alpha\beta}$ is R-R field strength

C. $AdS_2 \times S^2$ with R-R flux

$\langle \text{dilaton} \rangle = \lambda$

$F^{01} = F^{23} = N$

$\Rightarrow \quad P^{\alpha\beta} = \lambda N \delta^{\alpha\beta}$

$$\Rightarrow S = \int d^2z \, [\Pi_c \bar{\Pi}^c + B_{MN} \partial Y^M \bar{\partial} Y^N + d_\alpha \bar{\Pi}^\alpha + \ldots + \lambda N (d_\alpha \bar{d}_\alpha + \hat{d}_{\dot{\alpha}} \hat{\bar{d}}_{\dot{\alpha}})] + S_I$$

Integrating out $d_\alpha, \bar{d}_\alpha, \hat{d}_{\dot{\alpha}}, \hat{\bar{d}}_{\dot{\alpha}}$ and rescaling $E^A{}_M$,

$$S = \frac{1}{\lambda^2 N^2} \int d^2z \, [\Pi_c \bar{\Pi}^c + B_{MN} \partial Y^M \bar{\partial} Y^N + \bar{\Pi}^\alpha \Pi^{\bar{\alpha}} + \bar{\Pi}^{\dot{\alpha}} \Pi^{\bar{\dot{\alpha}}}] + S_I$$

But $\Pi^A = (g^{-1} \partial g)^A$ where $(x^m, \theta^\alpha, \hat{\theta}^{\dot{\alpha}}, \bar{\theta}^\alpha, \hat{\bar{\theta}}^{\dot{\alpha}})$ parametrizes the coset supermanifold $\dfrac{PSU(1,1|2)}{U(1) \times U(1)}$

Zhou 9906013

Metsaev + Tseytlin 9805028

and $B_{\alpha\bar{\beta}} = \delta_{\alpha\bar{\beta}} \Rightarrow AdS_2 \times S^2$ action of (*)

Checked conf. inv. to one-loop.

II. Compactification to d=6

A. Flat d=6 ($m=0\ldots5$, $\alpha=1\ldots4$)

RNS: $[x^m, \psi^m, b, c, \xi, \eta, \varphi] +$ $N=2$ $c=6$ SCFT

$$\downarrow$$

HYBRID: $[x^m, \theta^\alpha, p_\alpha, \underbrace{\rho, \sigma}_{\text{chiral bosons}}] +$ $N=2$ $c=6$ SCFT

$$S = \int d^2z \, [\partial x \cdot \bar\partial x + p_\alpha \bar\partial \theta^\alpha] + S_{\rho,\sigma} + S_{COMP}$$

N=2 constraints:

$$T = \partial x \cdot \partial x + p_\alpha \partial\theta^\alpha + \tfrac{1}{2}\partial\rho\partial\rho + \tfrac{1}{2}\partial\sigma\partial\sigma - \tfrac{1}{2}\partial^2(\rho + i\sigma) + T_c$$

$$G^+ = e^{-2\rho - i\sigma}(p)^4 + e^{-\rho} p_\alpha p_\beta \sigma_m^{\alpha\beta} \partial x^m + e^{i\sigma} T$$

$$G^- = e^{-i\sigma} + G_c^-$$

$$J = \partial(\rho + i\sigma) + J_c$$

ONLY 4 OF 8 SUPERSYMMETRIES ARE MANIFEST.

NEED 8 θ's, i.e. $\theta^{\alpha j}$ for $j=1,2$.

INTRODUCE BY HAND 4 NEW θ's AND 4 NEW CONSTRAINTS:

$$S = \int d^2z \left[\partial x \cdot \bar\partial x + p_{\alpha j} \bar\partial \theta^{\alpha j} \right] + S_{\rho,\sigma} + S_{comp}$$

WITH "HARMONIC" CONSTRAINTS

$$d_{\alpha 2} - e^{-\rho - i\sigma} d_{\alpha 1} = 0$$

AND $N=2$ CONSTRAINTS.

HARMONIC CONSTRAINTS ARE FIRST-CLASS
$\Rightarrow$ CAN USE TO GAUGE-FIX $\theta^{\alpha 2} = 0$.

IN THIS GAUGE, ACTION AND $N=2$ CONSTRAINTS RETURN TO ORIGINAL FORM.

Massless Vertex Operator:

$$\int d\tau\, G^-_{-1} G^+_0 \sum_n e^{n(\rho + i\sigma)} V_n(x^m, \theta^{\alpha 1}, \theta^{\alpha 2})$$

$$= \int d\tau \left[e^{-\rho - i\sigma} d_1 \nabla_1 \nabla_1 \nabla_2 V_0 + d_1 \nabla_2 \nabla_2 \nabla_1 V_0 + \Pi^m \sigma_m^{\alpha\beta} [\nabla_{\alpha 1}, \nabla_{\beta 2}] V_0 + \partial\theta^{\alpha 1} \nabla_{\alpha 1} V_0 - \partial\theta^{\alpha 2} \nabla_{\alpha 2} V_0 \right]$$

$$= \int d\tau \left[d_{\alpha j} W^{\alpha j} + \partial y^M A_M \right]$$

B. Curved $d=6$

$$S_{FLAT} = \int d^2z \, [\Pi_m \overline{\Pi}^m + B^0_{MN} \partial Y^M \bar\partial Y^N + d_{\alpha j} \bar\partial \theta^{\alpha j} + \bar d_{\alpha j} \partial \bar\theta^{\alpha j}] + S_I$$

$$S_{CURVED} = \int d^2z \, [\Pi_c \overline{\Pi}^c + B_{MN} \partial Y^M \bar\partial Y^N + d_{\alpha j} \overline{\Pi}^{\alpha j} + \bar d_{\alpha j} \Pi^{\alpha j} + d_{\alpha j} \bar d_{\beta k} P^{\alpha j \, \beta k}] + S_{FT} + S_I$$

$\Pi^A = E^A{}_M \partial Y^M \quad (A = c, \alpha j, \bar\alpha j)$

Lowest component of $P^{\alpha j \, \beta k}$ is R-R field-strength

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C. $AdS_3 \times S^3$ with R-R flux

$H^{012}_{jk} = H^{345}_{jk} = N \epsilon_{jk} \Rightarrow P^{\alpha j \, \beta k} = N\lambda \, \delta^{\alpha\beta} \epsilon^{jk}$

Integrating out $d_{\alpha j}, \bar d_{\alpha j}$ and rescaling,

$$S = \frac{1}{\lambda^2 N^2} \int d^2z \, [\Pi_c \overline{\Pi}^c + B_{MN} \partial Y^M \bar\partial Y^N + \epsilon_{jk} \overline{\Pi}^{\alpha j} \Pi^{\bar\alpha k}]$$

$= AdS_3 \times S^3$ action of (*)

where $(x^m, \theta^{\alpha j}, \bar\theta^{\alpha j})$ parameterizes $\dfrac{PSU(1,1|2) \times PSU(2|2)}{SU(2) \times SU(2)}$

Relation with $AdS_3 \times S^3$ action of hep-th/9902098 (NB, C. Vafa, E. Witten):

Before integrating out $(d^{\alpha j}, \bar{d}^{\alpha j})$, use 8 harmonic constraints and 6 $SU(2)\times SU(2)$ invariances to gauge fix to identity the parameters of one of the $PSU(2|2)$'s.

$$\mathcal{S}_{CURVED} = \int d^2z \, [\Pi_c \bar{\Pi}^c + B_{MN} \partial Y^M \bar{\partial} Y^N$$
$$+ d_{\alpha 1} \bar{\Pi}^{\alpha 1} + e^{-\rho - i\sigma} d_{\alpha 1} \bar{\Pi}^{\alpha 2}$$
$$+ \bar{d}_{\alpha 1} \Pi^{\bar{\alpha} 1} + e^{-\bar{\rho} - i\bar{\sigma}} \bar{d}_{\alpha 2} \Pi^{\bar{\alpha} 2}$$
$$+ d_{\alpha 1} \bar{d}_{\alpha 1} (1 - e^{-\rho - i\sigma} e^{-\bar{\rho} - i\bar{\sigma}})] + \mathcal{S}_I$$

Integrating out $(d_{\alpha 1}, \bar{d}_{\alpha 1})$ gives action based on $PSU(2|2)$ supergroup of hep-th/9902098.

Conformal inv. to all orders.

III. A. Flat D=10 ($m=0\ldots 9$, $a=1\ldots 5$)

RNS: $[x^m, \psi^m, b, c, \xi, \eta, \varphi]$

$\downarrow$

HYBRID: $[x^a, x_a, \theta^a, \hat{\theta}, p_a, \hat{p}, \rho, \sigma]$

$$S = \int d^2z\, [\partial x^a \bar{\partial} x_a + p_a \bar{\partial}\theta^a + \hat{p}\bar{\partial}\hat{\theta}] + S_{\rho,\sigma}$$

N=2 CONSTRAINTS:

$$T = \partial x^a \partial x_a + p_a \partial\theta^a + \hat{p}\,\partial\hat{\theta} + \tfrac{1}{4}(\partial\rho\partial\rho + \partial\sigma\partial\sigma) + \tfrac{1}{2}\partial^2(\rho - i\sigma)$$

$$G^+ = e^{\frac{1}{2}(\rho+i\sigma)} d_a \Pi^a + e^{\frac{1}{2}(3\rho+i\sigma)} d^5$$

$$G^- = e^{-\frac{1}{2}(\rho+i\sigma)}\hat{p}$$

$$J = i\partial\sigma$$

ONLY 6 OF 16 SUSY'S ARE MANIFEST

B. Speculations

1) ADD 10 θ's + 10 "HARMONIC" CONSTRAINTS

2) VERTEX OPERATOR = $\int dz\, [d_\alpha W^\alpha + \partial y^m A_m]$

3) IN $AdS_5 \times S^5$ BACKGROUND, $S = (*)$

ONE-LOOP CONFORMAL INVARIANT!