Raphael Bousso
Stanford University

The Holographic Principle for General Backgrounds

hep-th/9905177
hep-th/9906022

What is N_{dof} in nature?

- QFT + Planck cutoff $\rightarrow N_{\text{dof}} \sim V$

- thermodynamic entropy cannot exceed number of degrees of freedom: $S \leq N_{\text{dof}}$

- therefore $S_{\text{max}} \sim V$? No!

- with gravity, most states collapse to black holes

- Bekenstein (1981): $S \leq A/4$

- 't Hooft (1993): $N_{\text{dof}} \leq A/4$

't Hooft, gr-qc/9310026

Susskind, hep-th/9409089

Holographic Principle: $N_{\text{dof}} \leq A/4$

"the world is two-dimensional"

- required by quantum gravity
- should be manifest in the fundamental theory
- implies that space and time are derived concepts?
- strongly supported by AdS/CFT correspondence

  Maldacena; Gubser, Klebanov+Polyakov; Witten;

  Susskind+Witten; ...

The Problem

- **if truly fundamental, H.P. should apply to all solutions of Einstein's equations**

- but Bekenstein's bound is valid only for "regions of limited self-gravity."

- holographic principle violated in

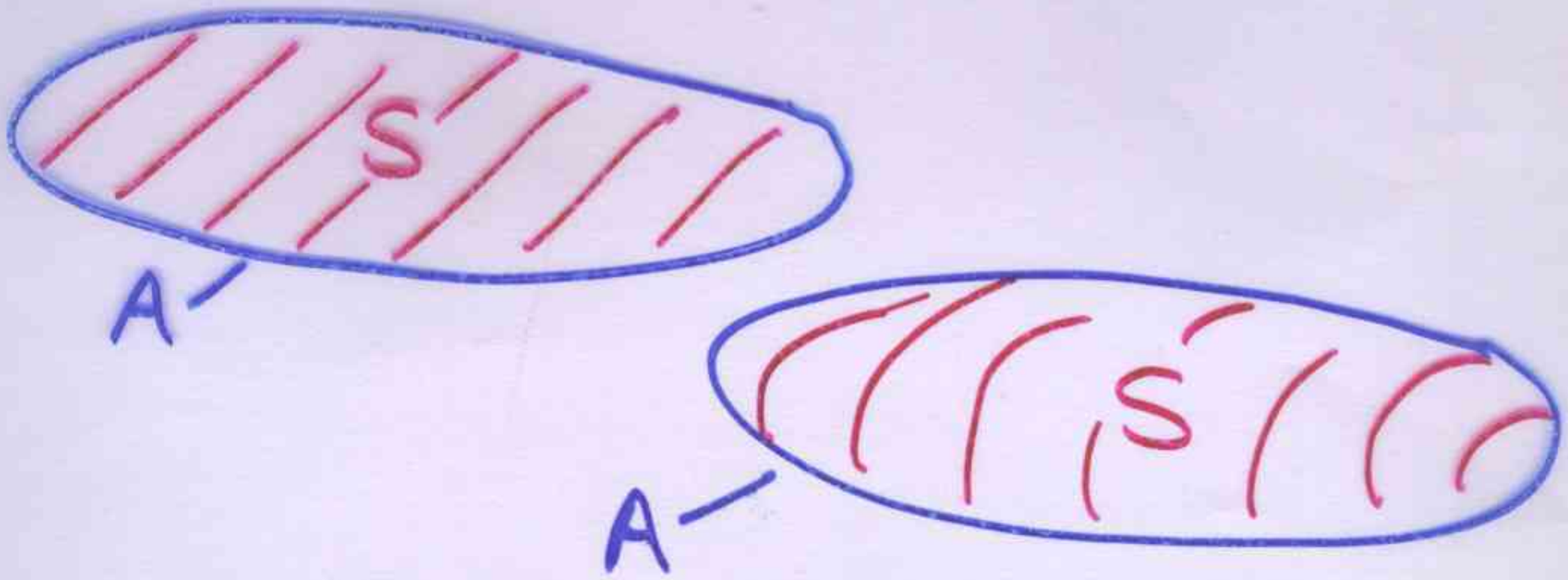
  – gravitationally collapsing systems

  – large cosmological regions

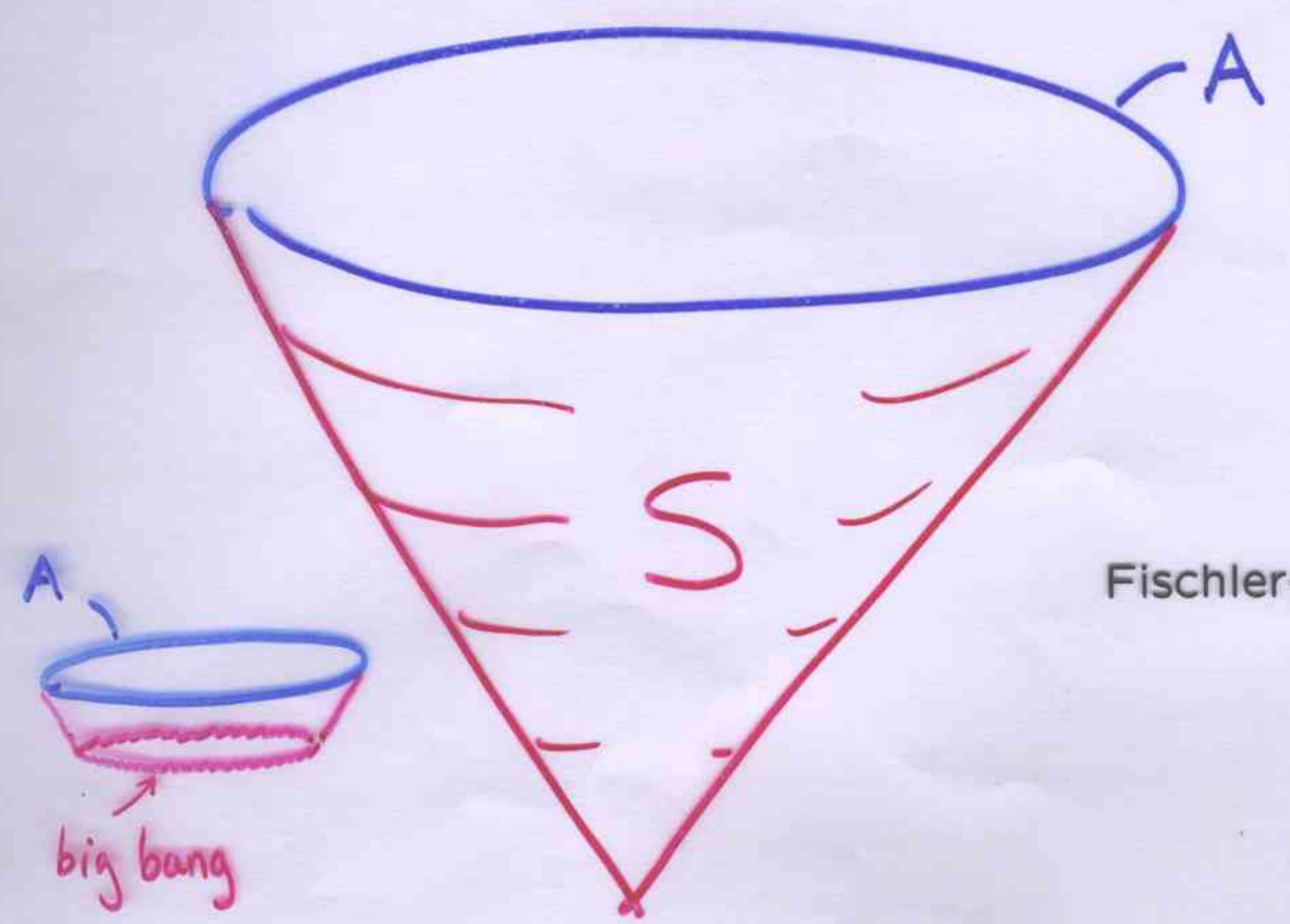
  Fischler+Susskind, hep-th/9806039

- **⇒ a more general formulation is needed**

Demand Covariance!

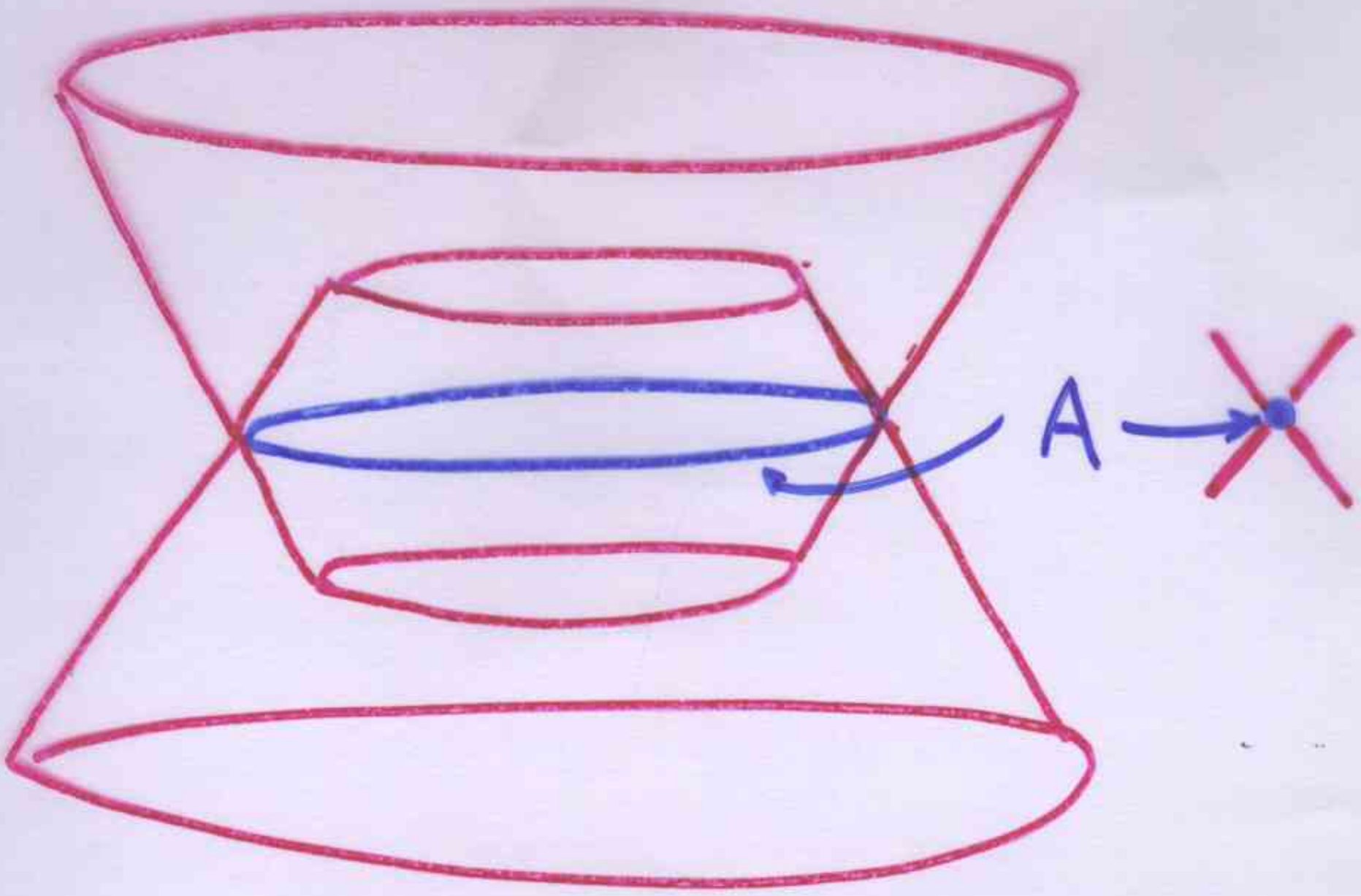


- spacelike hypersurfaces excluded
- try light-like hypersurface



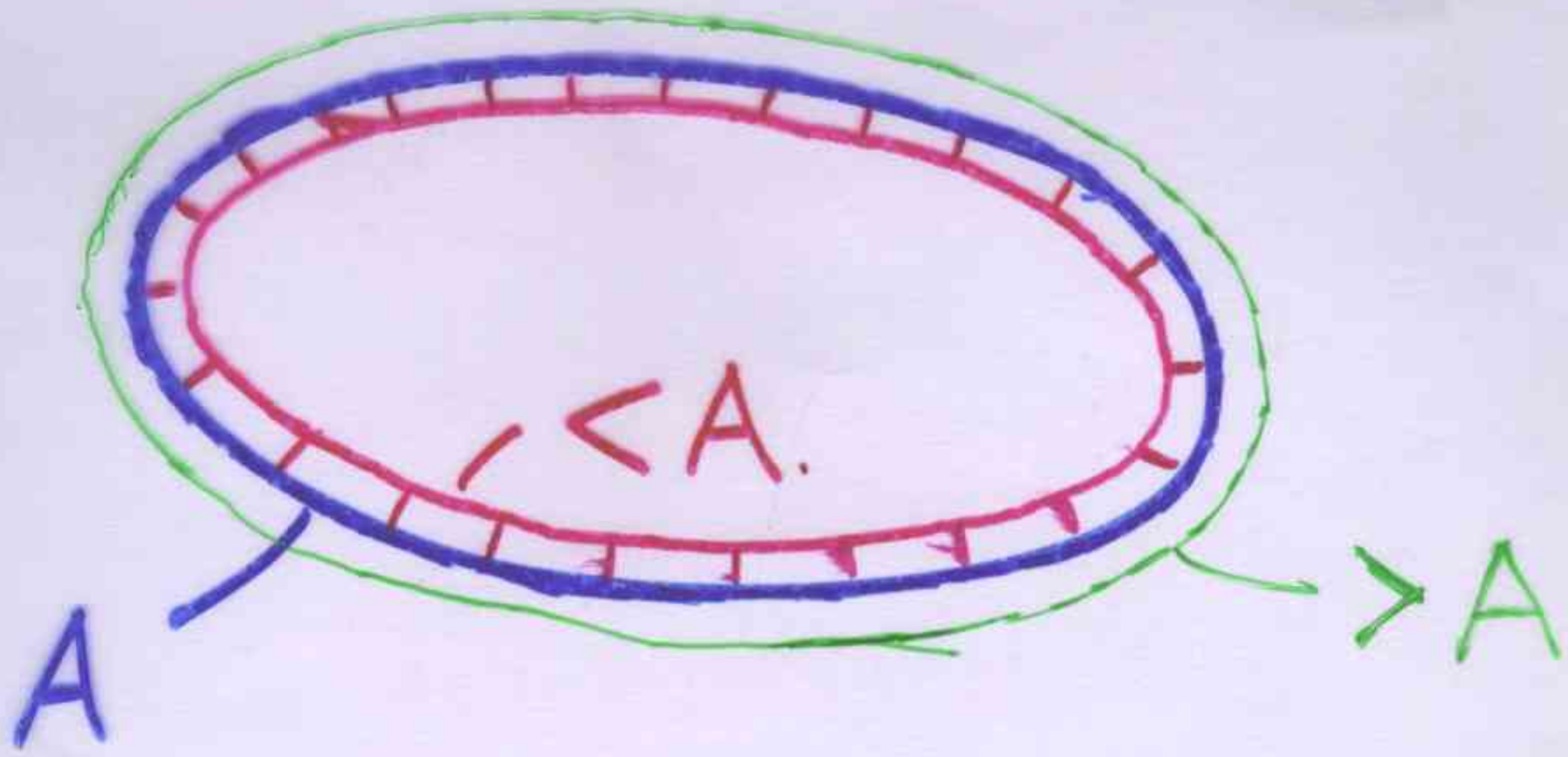
Fischler+Susskind

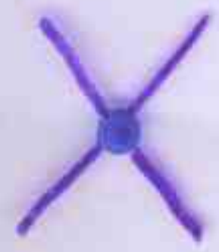
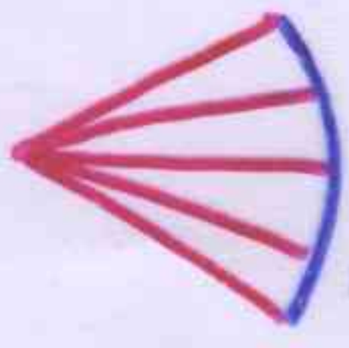
4 choices



- orthogonal light-rays generate 4 null hypersurfaces
- select a region *inside* the area
- need covariant definition of "inside"

Where is "inside"?



- cross-sectional area must decrease: $\theta \leq 0$
-  trapped:  anti-trapped:  normal: 
- $\theta \leq 0 \Rightarrow$ must stop at caustics 
- surface need not be closed 

Covariant Entropy Bound

- pick a surface, area A

- find the "allowed" families of light-rays (at least 2 out of 4)

- pick an allowed family

- construct **light-sheet** by following each light-ray until $\theta > 0$

$$S \text{ (light-sheet)} \leq A/4$$

Properties of the Bound

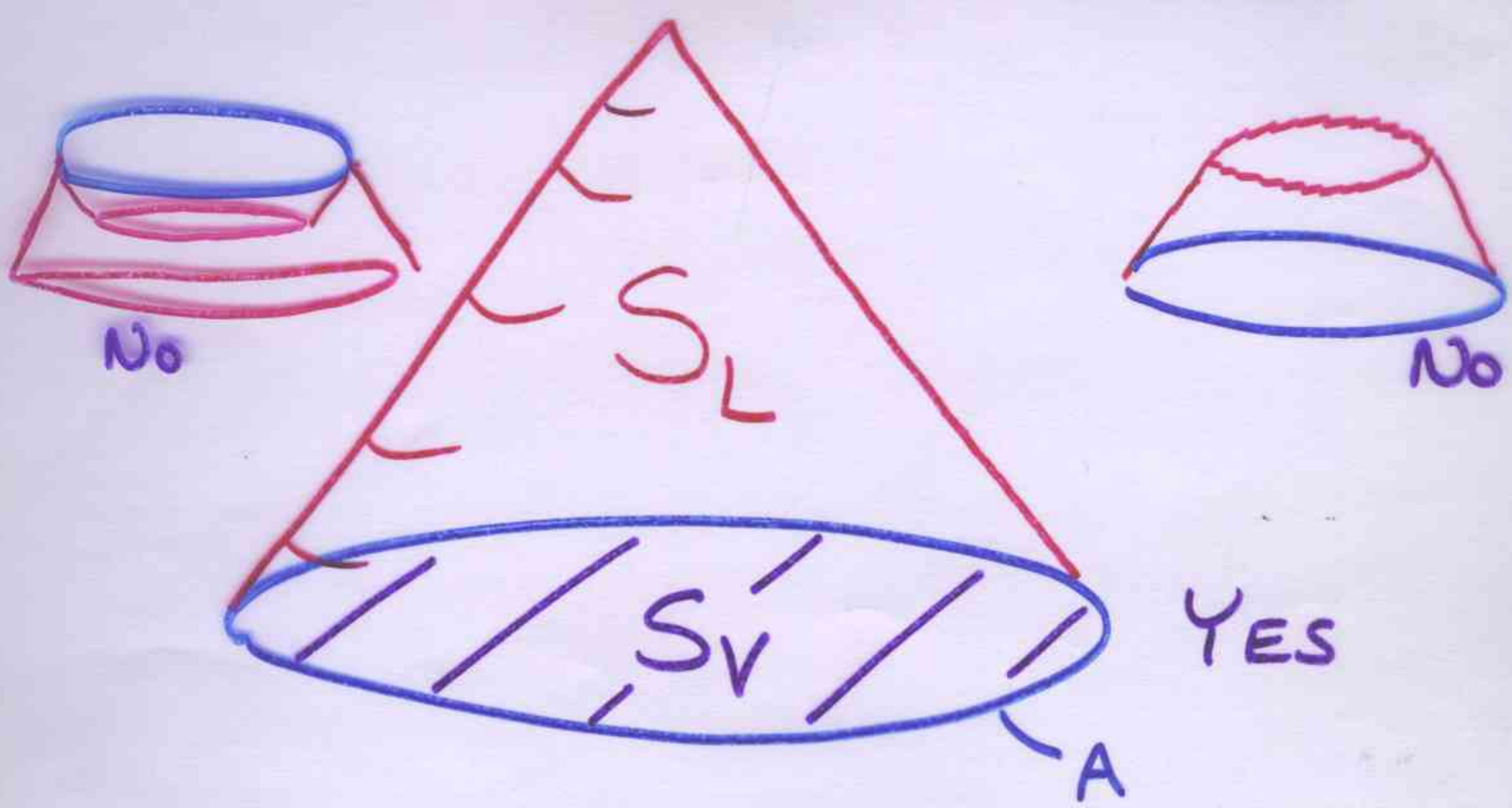
- democratic treatment of all 4 null directions
- no reference to inside/outside
- no reference to past/future:
  time reversal invariant

The Holographic Principle for General Spacetimes

$$N_{\text{dof}} \text{ (light-sheet)} \leq A/4$$

The Entropy in Spatial Regions

- When are spacelike hypersurfaces permitted?



- causality + second law $\Rightarrow S_V \leq S_L \leq A/4$

- $\Rightarrow$ **recover Bekenstein's bound**

Does it work?

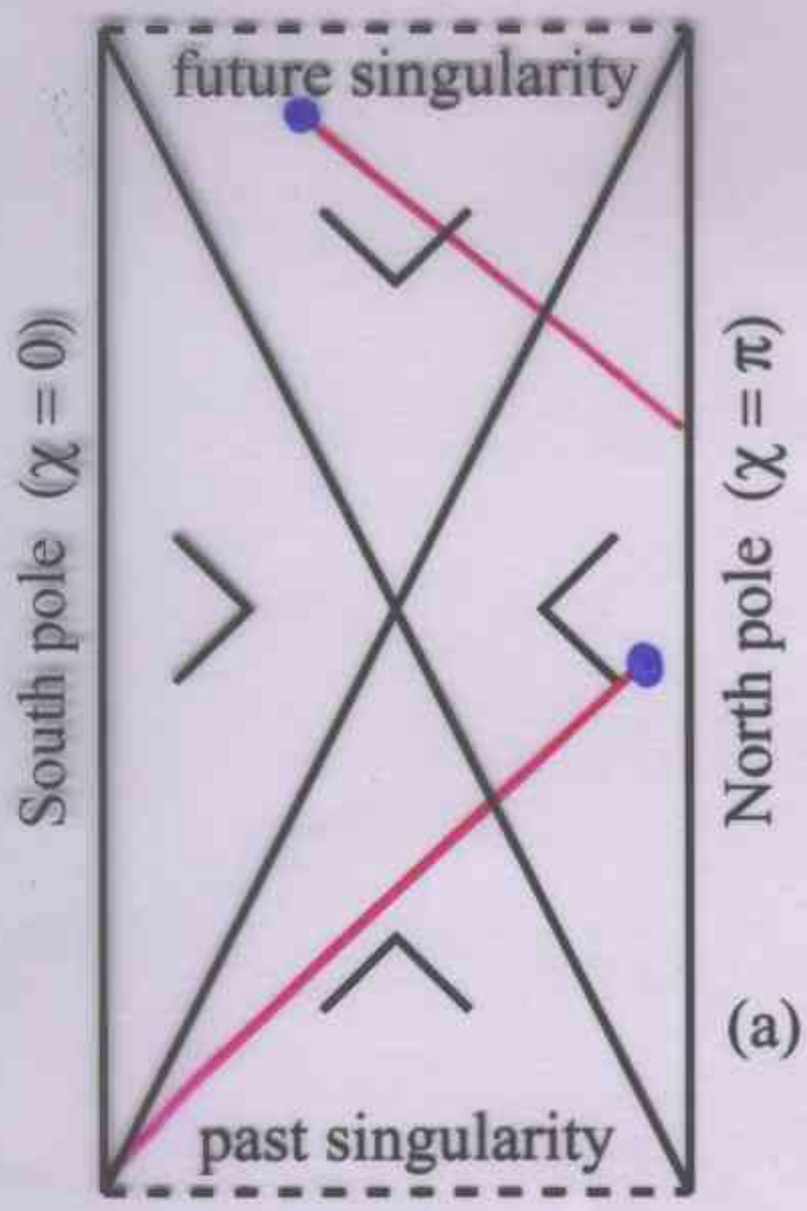
- no straightforward proof

- need comprehensive, non-trivial tests, especially for gravitational collapse

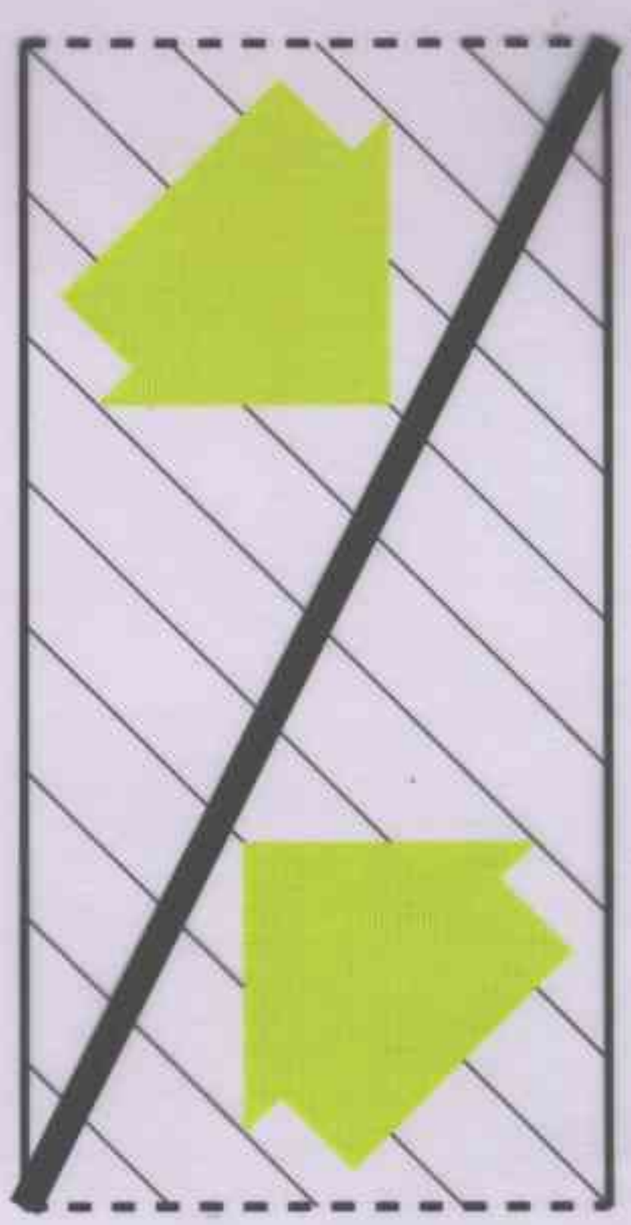
R.B., hep-th/9905177

- worst case assumptions → saturation

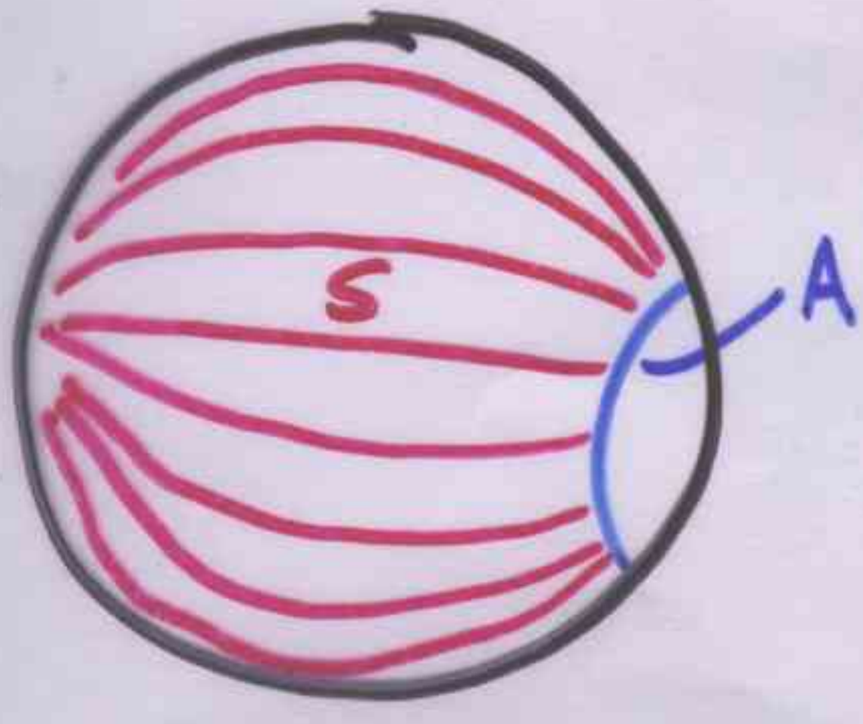
Closed FRW Universe



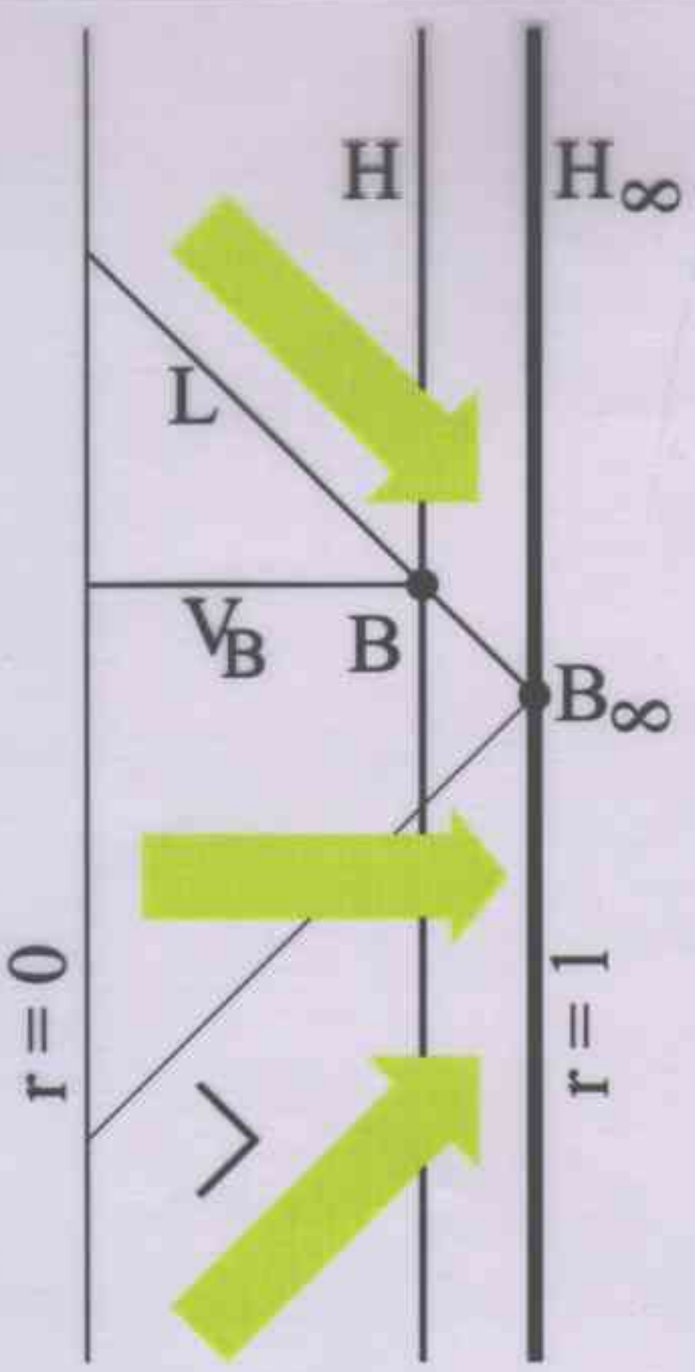
(a)



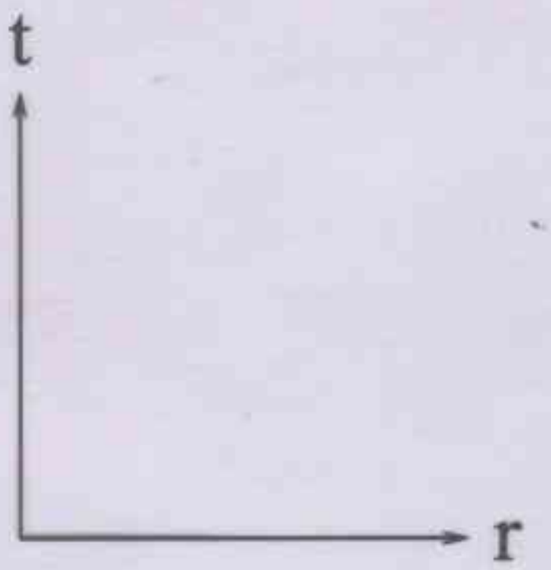
(b)



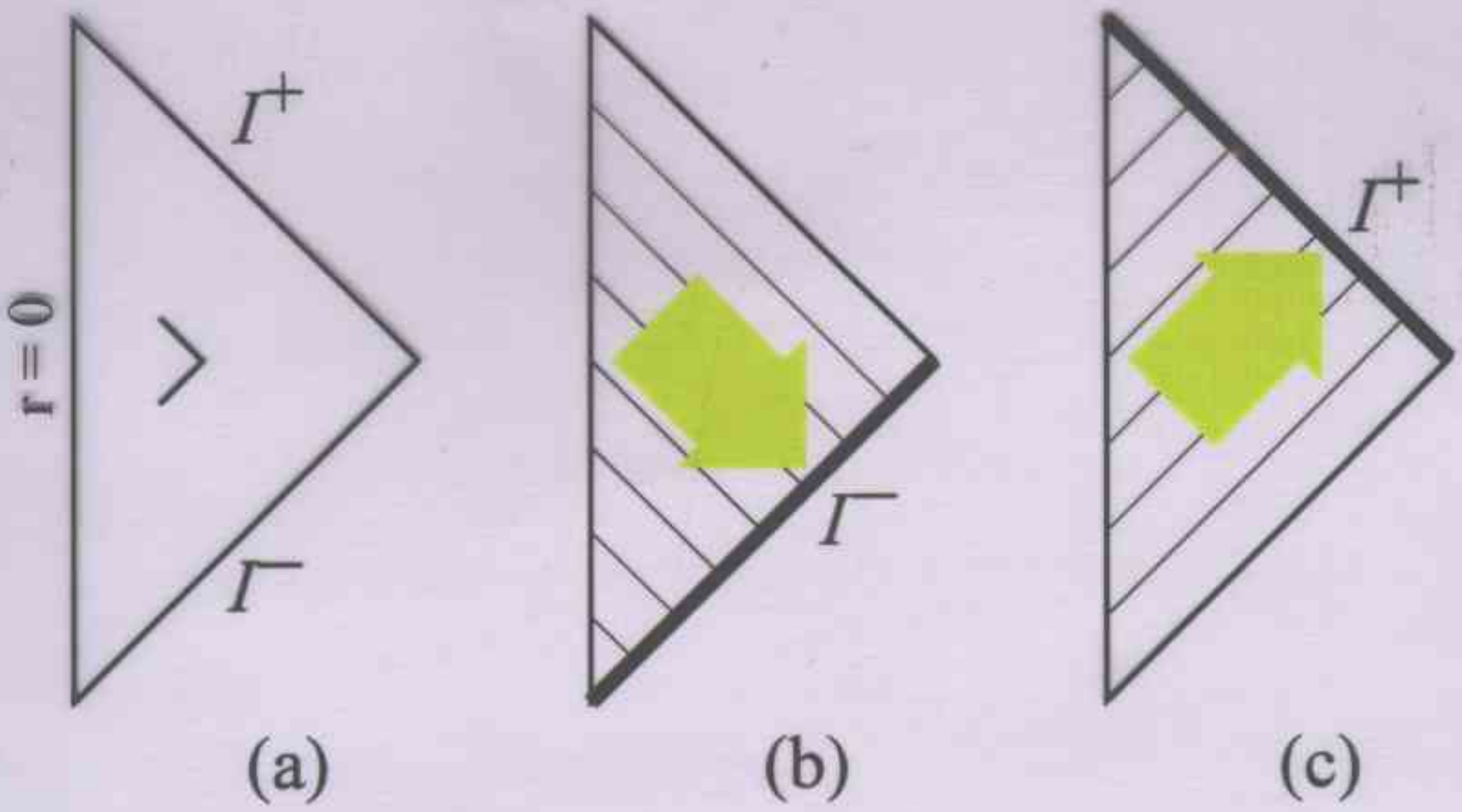
Anti-de Sitter Space



AdS

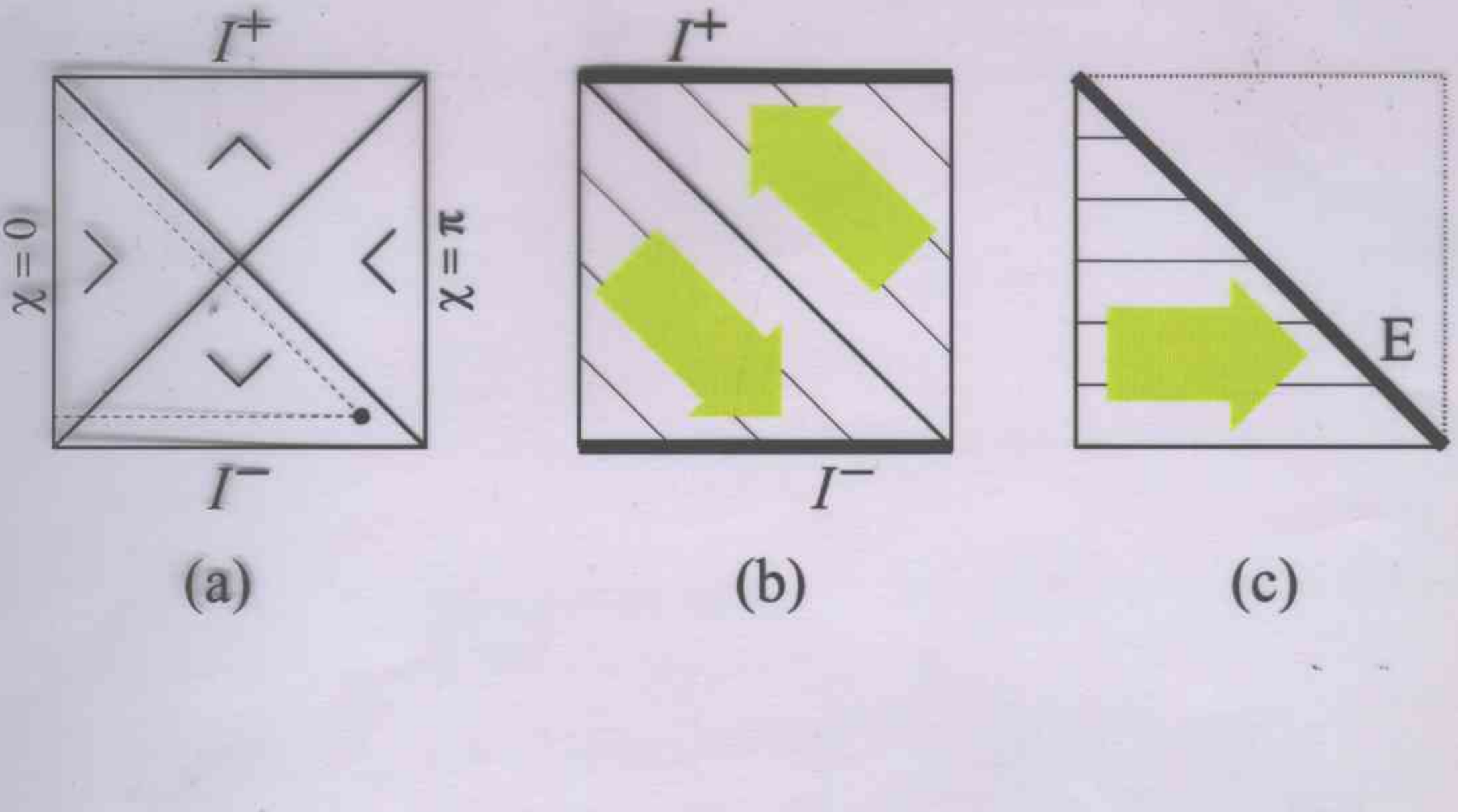


Minkowski Space



(a) (b) (c)

de Sitter Space



(a) (b) (c)

Properties of Screens

- can be timelike, null, spacelike, or mixed
- area can be time-dependent
- there may be several disconnected screens
- spacelike projection not normally possible
- AdS very special

What is the Holographic Theory?

- a quantum field theory on the screen, as in AdS/CFT?

- generically, this possibility excluded by the screen properties

- unconventional "theory" on screen?

  = has varying N_{dof}

  = lives variously in Euclidean/Lorentzian time

Speculations

- screen-theory approach cannot be fundamental
  (presupposes geometric structure)

- turn holographic principle around?

  = hol. principle: "geometry → entropy"

  = hol. theory: "entropy → geometry"

- two tasks:

  = construct theory without gravity
  with varying N_{dof}

  = reconstruct space-time from N_{dof}