

STRINGS ON ADS BACKGROUNDS

BASED ON HEP-TH/9905032
JDB, S. SHATASHVILI

OUTLINE :

- STRINGS ON ADS
- FROM ADS_3 TO ADS_{2d+1}
- GENERALIZED G/H COSETS
- SUPER COSETS
- SPECULATIONS

• STRINGS ON ADS

- GO BEYOND SUGRA APPROXIMATION
- COVARIANT FORMULATION OF STRING THEORY
- UNDERSTAND STRING THEORY WITH RR BACKGROUNDS
- STRING FIELD THEORY IN ADS BACKGROUNDS

APPROACHES:

NSR FORMALISM

NOT KNOWN HOW TO DESCRIBE A CONDENSATE
OF RR VERTEX OPERATORS

BERENSTEIN, LEIGH, 9904104

D. POLYAKOV, 9812044/9907021

GS FORMALISM

CLASSICAL DESCRIPTION KNOWN FOR $AdS_5 \times S^5$

AND $AdS_3 \times S^3$

METSAEV, TSEYTLIN, 9805028

KALOSH, RAJARAMAN, RAHMFELD, 9805217

PESANDO, 9808020

KALOSH, RAHMFELD, 9808038

PESANDO, 9809145

RAHMFELD, RAJARAMAN, 9809164

PARK, REY, 9812062

CLASSICAL GS STRING IN FLAT SPACE CAN
BE QUANTIZED. PROBLEMATIC IN CURVED SPACE

YU, ZHANG, 9812216

RAJARAMAN, ROZALI, 9902046

GS TYPE VARIABLES FOR NSR STRINGS

(A LA BERKOVITS)

EXISTS IN

$$D=4$$

BERKOVITS, 9404162

$$D=6$$

BERKOVITS, VAFA, 9407190

$D=10$, WITH $U(5) \subset SO(10)$
UNBROKEN

BERKOVITS, 9902099

HAVE BEEN USED TO DESCRIBE

$AdS_2 \times S^2$ WITH RR FLUX

→ SATURDAY

$AdS_3 \times S^3$ WITH RR FLUX

BERKOVITS, VAFA, WITTEN, 9902098

$AdS_3 \times S^3$ IS DESCRIBED BY A SIGMA MODEL

WITH TARGET SPACE THE GROUP MANIFOLD $PSL(2|2)$

COUPLED TO GHOSTS

DEFORMING THE RR FLUX TO NS FLUX YIELDS

THE WZW MODEL FOR $PSL(2|2)$

$PSL(2|2)$ WZW



CHANGE OF VARIABLES

$SL(2) \times SU(2)$ $N=1$ WZW : STANDARD NSR FORMULATION
OF STRINGS ON $AdS_3 \times S^3$
WITH ONLY NS FLUX

IDEA: FIND GENERALIZATIONS OF THESE THEORIES
INVOLVING OTHER AdS SPACES.

WE WILL FIND EXACTLY SOLVABLE CFT'S,
WHOSE GEOMETRY IS AdS , AND WHICH
ARE HOLOGRAPHIC.

TO OBTAIN STRINGS WITH RR BACKGROUNDS,
COUPLE TO APPROPRIATE GHOSTS & DEFORM??

USE BOTH GROUPS AND SUPERGROUPS

FROM AdS₃ TO AdS_{2d+1}

$$\text{AdS}_3 \sim \text{SL}(2)$$

$$g = \begin{pmatrix} 1 & \bar{\gamma} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-\phi} & 0 \\ 0 & e^{\phi} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \gamma & 1 \end{pmatrix} = g_+ g_0 g_-$$

GAUSS
DECOMPOSITION

$$\begin{aligned} S_{\text{WZW}}[g] &= S_{\text{WZW}}[g_0] + \frac{k}{2\pi} \int \text{tr}(g_+^{-1} \partial g_+ g_0 \bar{\partial} g_- g_-^{-1} g_0^{-1}) \\ &= \frac{k}{2\pi} \int (\partial \phi \bar{\partial} \phi + e^{2\phi} \partial \bar{\gamma} \bar{\partial} \gamma) \end{aligned}$$

↓ -INTRODUCE AUXILIARY FIELDS
-RESCALE

$$= \frac{1}{4\pi} \int (\partial \phi \bar{\partial} \phi + \beta \bar{\partial} \gamma + \bar{\beta} \partial \bar{\gamma} - \beta \bar{\beta} e^{-2\phi/\alpha_+} - \frac{2}{\alpha_+} \phi \sqrt{g} R)$$

$\alpha_+ = \sqrt{2(k-2)}$

FREE FIELD THEORY PERTURBED BY THIS OPERATOR

OF THE FORM $S\bar{S}$ WITH $S = \beta e^{-2\phi/\alpha_+}$

-
- $S\bar{S}$ IS EXACTLY MARGINAL WITH CONFORMAL WEIGHT (1,1)
 - ONLY THINGS THAT COMMUTE WITH $\oint S$ ARE SL_2 CURRENTS
 - CORRELATORS / CONFORMAL BLOCKS FROM FREE FIELDS WITH $\oint S$ INSERTIONS

GENERALIZE BY TAKING MORE FIELDS

USING FREE FIELD FORM GUARANTEES CONFORMAL

INVARIANCE

$$S = \frac{1}{4\pi} \int \left(\partial\phi\bar{\partial}\phi + \beta_r \bar{\partial}\gamma_r + \bar{\beta}_r \partial\bar{\gamma}_r - \beta_r \bar{\beta}_r e^{-2\phi/\alpha_+} - \frac{2}{\alpha_+} \phi \sqrt{g} R \right)$$

INTEGRATE
OUT $\beta_r, \bar{\beta}_r$

$$r = 1 \dots d$$
$$\alpha_+ = \sqrt{2(k-2d)}$$

$$S = \frac{k}{2\pi} \int \left(\partial\phi\bar{\partial}\phi + e^{2\phi} \partial\bar{\gamma}_r \bar{\partial}\gamma_r \right) + \frac{1}{2\pi} \int (d-1) \phi \sqrt{g} R$$

$$\left\{ \begin{array}{l} G: \quad ds^2 = k (d\phi^2 + e^{2\phi} d\gamma_r d\bar{\gamma}_r) \\ B: \quad B = k e^{2\phi} d\gamma_r \wedge d\bar{\gamma}_r \\ \Phi: \quad \Phi = (d-1)\phi \end{array} \right. \quad \text{AdS}_{2d+1}$$

ONE LOOP β -FUNCTIONS ARE SATISFIED

ONE LOOP CENTRAL CHARGE: $c = (2d+1) + \frac{6}{k}$

EXACT: $c = (2d+1) + \frac{6}{k-2d}$

* EINSTEIN FRAME:

$$ds_E^2 = e^{2\phi/(2d-1)} (e^{-2\phi} d\phi^2 + d\bar{r}_r d\bar{r}_r)$$

DISTANCE ON BOUNDARY DIVERGES ~ HOLOGRAPHY

* LESS GLOBAL SYMMETRIES: \mathcal{B} DETERMINES

COMPLEX STRUCTURE ON BOUNDARY

$$SO(2d) \longrightarrow U(d)$$

* THEORY HAS $d+1$ HOLOMORPHIC CURRENTS

* CONSTANT \mathcal{B} -FIELD \Leftrightarrow NON COMMUTATIVE
GEOMETRY?

AdS_{2d+1} IS AN EXAMPLE OF A

NOVEL TYPE OF COSET CONSTRUCTION

G/H HOMOGENEOUS
SPACE; USUALLY NOT
CONFORMAL

NOVEL G/H
COSET

STANDARD G/H
COSET

→ LESS ISOMETRIES →

HINT OF EXISTENCE OF SUCH NOVEL COSETS - FEIGIN FRENKEL

- GERASIMOV, MOROZOV, OLSHANETSKY, MARSHAKOV,
SHATASHVILI

- BARS

IDEA: $G \supset H$

$H =$ PRODUCT OF SIMPLE FACTORS

DO A PARTIAL BOSONIZATION

- JOB, FEHER

$$\mathcal{J}_G = \mathcal{J}_G[\mathcal{J}_H, \phi_i, \beta_r, \tilde{\tau}_r]$$

REMOVE THE DEGREES OF FREEDOM IN H

RECIPE :

① WRITE $g = g_+ h g_0 g_-$ $h \in H$

e.g. $\frac{SL(3)}{SL(2)}$ $g = \begin{pmatrix} 1 & \bar{\sigma}_1 & \bar{\sigma}_2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{pmatrix} \begin{pmatrix} e^{-2\phi} & 0 & 0 \\ 0 & e^\phi & 0 \\ 0 & 0 & e^\phi \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ \sigma_1 & 1 & 0 \\ \sigma_2 & 0 & 1 \end{pmatrix}$

② DROP h

③ EVALUATE $S_{WZW}[g_+ g_0 g_-]$

e.g. $S = \frac{1}{2\pi} \int (3\partial\phi\bar{\partial}\phi + e^{2\phi}(\partial\bar{\sigma}_1\bar{\partial}\sigma_1 + \partial\bar{\sigma}_2\bar{\partial}\sigma_2))$

④ Introduce AUXILIARY VARIABLES β

e.g. $S = \frac{1}{4\pi} \int (\partial\phi\bar{\partial}\phi + \beta_1\bar{\partial}\sigma_1 + \beta_2\bar{\partial}\sigma_2 + \bar{\beta}_1\partial\bar{\sigma}_1 + \bar{\beta}_2\partial\bar{\sigma}_2 - \beta_1\bar{\beta}_1 e^{-2\phi/\alpha_+} - \frac{4}{\alpha_+}\phi\sqrt{g}R)$

⑤ ADJUST BACKGROUND CHARGES SO THAT SCREENERS (MARGINAL PERTURBATIONS) HAVE THE RIGHT CONFORMAL WEIGHT

e.g. $\frac{4}{\alpha_+}\phi\sqrt{g}R \Rightarrow \frac{-2}{\alpha_+}\phi\sqrt{g}R$

⑥ (INTEGRATE OUT THE AUXILIARY FIELDS)

e.g. \rightarrow THE PREVIOUS AdS_5 SOLUTION

FEATURES:

- HOLOMORPHIC CURRENTS $\sim n^+ g_0$
- ANTI HOLOMORPHIC CURRENTS $\sim g_0 n^-$
- # ISOMETRIES $\sim \dim(G) + \dim(G_0)$
- ISOMETRIES $\not\subseteq G$, BUT CONTAIN H

QUESTION!

- IS THERE A BRST-TYPE DEFINITION OF THESE COSETS?

BUILD STRING THEORIES?

S^{2d+1} IS ANALYTIC CONTINUATION OF AdS_{2d+1} ??

$$d_+ \Rightarrow i\alpha_+$$

WORKS FOR $d=9$

$$S = S(AdS_5; \alpha_+) + S(AdS_5; i\alpha_+)$$

BACKGROUND WITH $C=10$

INTRODUCE $N=1$ WORLD-SHEET SUSY VIA SUPERFIELDS

RESULT:

$$\begin{aligned}
S(AdS_5; \alpha_+)_{N=1} = \frac{1}{4\pi} \int & \left(-\lambda^L \bar{\partial} \lambda^L + \partial \lambda^R \lambda^R - \sigma_r \bar{\partial} \psi_r^L - \bar{\sigma}_r \partial \bar{\psi}_r^R \right. \\
& + \partial \phi \bar{\partial} \phi + \beta_r \bar{\partial} \gamma_r + \bar{\beta}_r \partial \bar{\gamma}_r \\
& - e^{-2\phi/d_+} \left(\bar{\beta}_r - \frac{2}{d_+} \lambda^R \sigma_r \right) \left(\beta_r - \frac{2}{d_+} \lambda^L \sigma_r \right) \\
& \left. - e^{-4\phi/d_+} \sigma_r \bar{\sigma}_r \sigma_s \bar{\sigma}_s \right)
\end{aligned}$$

$$S = S(AdS_5; \alpha_+)_{N=1} + S(AdS_5; i\alpha_+)_{N=1}$$

IS A CRITICAL NSR BACKGROUND WITH $C=15$

THEORY HAS $(9,9)$ SUPERSYMMETRIES

(OK BECAUSE $SO(4) \rightarrow U(2)$)

Susy:

+	+	+	+	+
+	-	+	+	-
-	+	+	+	-
+	-	+	-	+
-	+	+	-	+
+	+	-	+	-
+	+	-	-	+
+	-	-	+	+
-	+	-	+	+

SUPERLOSETS

$$\frac{SL(3|3)}{SL(2) \times SL(2)}$$

HAS $AdS_5 \times S^5$ TARGET SPACE

18 ANTI COMMUTING SCALARS

$$C = -8$$

FERMIONIC/BOSONIC B-FIELDS TURNED ON

$$\frac{SL(4|4)}{Sp(2) \times Sp(2)}$$

HAS $AdS_5 \times S^5$ TARGET SPACE

32 ANTI COMMUTING SCALARS (RELATION TO GS?)

$$C = -22$$

FERMIONIC/BOSONIC B-FIELDS TURNED ON

QUESTIONS / SPECULATIONS

- ARE ALL THESE THEORIES HOLOGRAPHIC?
- TO WHAT EXTENT ARE THESE CFT'S EXACTLY SOLVABLE? (ITERATED SCREENERS...)
- ARE THESE STRING THEORIES UNITARY?
- IS THERE A BRANE INTERPRETATION? NONCOMMUTATIVE FIELD THEORIES?
("GAS" OF NS5/F1!)
- DO EVEN AdS SPACES

(e.g. $AdS_2 \times S^2 \times S^1$ WITH

$$dB = d\varphi_{S^1} \wedge (d\text{vol}(S^2) + d\text{vol}(AdS_2))$$

EXACTLY CONFORMAL?)

- WHAT IS THE SPACETIME INTERPRETATION OF THE BACKGROUND FIELDS IN THE SUPERCOSETS? DO SOME ALREADY HAVE RR FIELDS TURNED ON?
- WHICH DEFORMATIONS DO THE THEORIES HAVE, AWAY FROM "WZ" POINT?
- USE $AdS_5 \times S^5$ THEORY + U(1) COVARIANT FORMULATION OF $d=10$ STRING THEORY TO FIND FORMULATION OF $AdS_5 \times S^5$ WITH RR FLUX
CONJECTURE: IT IS A DEFORMATION OF $\frac{GL(3|3)}{GL(2|2)}$

WORK IN PROGRESS WITH N. BERKOVITS