AREA LAW CORRECTIONS FROM STATE COUNTING AND SUPERGRAVITY

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String theory → counting of microstates accounts for black hole entropy

(Strominger, Vafa)

Extremal $(d=4)$ black holes (BPS)

$$S_{micro} \sim \sqrt{Q_1 Q_2 Q_3 Q_4}$$

for large charges

Succesfully confronted with macroscopic calculations:

- corresponding (effective) field theory
- find black hole solutions
- work out the entropy (through area law)

microscopically:

count # string / Dbrane configurations

Here:

IIA string theory compactified on CY three folds

$\leftrightarrow$ M-theory on $CY \times S^1$

Wrap

D4 brane on a holomorphic smooth CY four cycle $\mathcal{P}$

$\leftrightarrow$

M5 brane on $\mathcal{P} \times S^1$

[large CY, large g°]

Maldacena, Strominger, Witten
Vafa

massless fluctuations around wrapped brane define a (0,4) conformal field theory
(L, R)

counting their state degeneracy yields the entropy

This remains true when CY is replaced by

$K3 \times T^2 \rightarrow N=4$ black hole

$T^6 \rightarrow N=8$ black hole

{ asymptotic spacetime susy

ENTROPY

$$S_{micro} = 2\pi \sqrt{\frac{1}{6}|q_0|\ c_L}$$

$$= 2\pi \sqrt{\frac{1}{6}|q_0|\left(C_{ABC}\ p^A p^B p^C + c_{2A}\, p^A \right)}$$

triple intersection
CY four cycles

second Chern class

SUBLEADING !

Note: CFT encoded in topological data
of the four cycle $\mathcal{P}$

related to → topological data of CY

MSW: subleading corrections
are associated with higher-order
derivative terms (i.e. R^2-terms)
in the effective field theory

macroscopically:

How can the macroscopic theory (usually matter coupled supergravity) give rise to such simple/systematic answers?

⇒ SUPERSYMMETRY ENHANCEMENT AT THE HORIZON

BLACK HOLE: solitonic interpolation between two different supersymmetric vacua i.e. $N=2$, $d=4$ Gibbons

$r=\infty$ flat Minkowski spacetime

$r=0$ $AdS_2 \times S^2$ Bertotti Robinson horizon

global solution

RESIDUAL $N=1$ SUPERSYMMETRY

→ BPS

generically the solution depends on the E/M charges and the values of the moduli fields at $r=\infty$

the entropy depends only on the charges

FIXED POINT BEHAVIOUR

the moduli take fixed values at the horizon, determined exclusively in terms of E/M charges

⇒ determined by E/M duality caused by susy enhancement

Ferrara, Kallosh, Strominger, Gibbons
Behrndt, Cardoso, dW, Kallosh, Lüst, Mohaupt
Moore

THEORY: $N=2$ supergravity + a number of abelian vector multiplets

→ E/M charges q_I, p^I

Subleading entropy corrections are related to R^2-terms. So the supergravity theory should incorporate both R and R^2 !

$N=2$ multiplet calculus

Van Proeyen, van Holten, dV

recall $N=2$ vector multiplets

$\Rightarrow$ field strengths are chiral superfields

$$W^I(x,\theta) = X^I(x) + \cdots \quad F^{-I}_{\mu\nu}(x)\,\bar\theta^i\gamma^{\mu\nu}\theta^j\,\varepsilon_{ij} + \cdots$$
$$+ \cdots \ \Box\bar X^I(x)\ \left(\bar\theta^i\gamma^{\mu\nu}\theta^j\,\varepsilon_{ij}\right)^2$$

general gauge invariant Lagrangians encoded in:

holomorphic function $F(X^I)$

[subject to E/M duality]

this can be formulated in an off-shell supergravity background

$\Rightarrow$ $F(X)$ holomorphic + homogeneous of 2-nd degree

Auxiliary tensor field $T^{ab\,ij}$

$$\delta\psi_\mu{}^i = 2\,\mathcal{D}_\mu\epsilon^i - \tfrac{1}{8}\gamma_{ab}T^{abij}\gamma_\mu\epsilon_j + \cdots$$

antiselfdual; couples to the graviphoton field strength

$T^{ab\,ij}$ lowest component of a (tensor) chiral superfield

$$T_{ab}{}^{ij} + \cdots \quad R^{-}_{ab}{}^{cd}\,\theta^i \gamma_{cd}\,\theta^j + \cdots \quad DD\,T_{ab\,ij}\,(\bar\theta\,\theta)^2$$

$\hat{A} = \left(T^{ab\,ij}\,\varepsilon_{ij}\right)^2$ scalar chiral superfield
can straightforwardly be incorporated into $F(X)$

$\Rightarrow$ $F(X, \hat{A})$ holomorphic + homogeneous

$$\underbrace{X^I \frac{\partial F}{\partial X^I}}_{F_I} + \underbrace{w}_{=2}\,\hat{A}\,\underbrace{\frac{\partial F}{\partial \hat{A}}}_{F_{\hat{A}}} = 2F$$

$\Rightarrow$ FIND ALL $N=2$ SUPERSYMMETRIC CONFIGURATIONS (off-shell!) THAT ARE CONSISTENT WITH THE STATIC SPHERICALLY SYMMETRIC BLACK HOLE GEOMETRY
(not using action/field equations, only $F(X,\hat{A})$)

$$ds^2 = -e^{2g(r)}\,dt^2 + e^{2f(r)}\,(d\vec{x})^2$$

$\Rightarrow$ Bertotti Robinson $AdS_2 \times S^2$

$$e^{2g(r)} = e^{-2f(r)} = \frac{r^2}{16}\left|T^{01ij}\varepsilon_{ij}\right|^2 = r^2\,\frac{e^{-K}}{|Z|^2}$$

$$e^{-K} \equiv i\left[\bar{X}^I F_I(X,\hat{A}) - \bar{F}_I(\bar{X},\bar{\hat{A}})\,X^I\right]$$

$$Z \equiv e^{K/2}\left[p^I F_I(X,\hat{A}) - q_I X^I\right]$$

only dependence on the (constant) moduli $\sim X^I$

and the E/M charges q_I, p^I

invoke E/M duality

p^I, q_I and $X^I, F_I(X,\hat{A})$

must satisfy a proportionality relation !

$\Rightarrow$ only dependence on the charges

p^I, q_I

What remains ?

How to determine the entropy ?

BH AREA LAW ?

disagrees with the microscopic result !

Behrndt, Cardoso, dW, Lüst, Mohaupt, Sabra

INSTEAD use Wald's prescription based on a constant surface charge

This ensures the validity of the First Law of Black Hole Mechanics

in this case

$$S_{macro} = \frac{1}{16} \oint_{S^2} \epsilon_{ab}\, \epsilon_{cd}\, \frac{\delta (8\pi \mathcal{L})}{\delta R_{abcd}}$$

$a, b, c, d = 0, 1$

surface charge
constant for a continuous variety of solutions

$$\Rightarrow \delta\left[\oint_{horizon} Q\right] = \delta\left[\oint_{spatial\ \infty} Q\right]$$

$\downarrow$ entropy S_{macro}

$\downarrow$ mass, angular momentum ....

$\longleftrightarrow$ First Law

Toy Example:

3D Abelian gauge theory:

$$\mathcal{L} = \mathcal{L}^{inv}(F_{\mu\nu}, \partial_\rho F_{\mu\nu}, \psi, D_\mu \psi) + c\, \varepsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho$$

"Noether potential"

$$Q^{\mu\nu}(\phi, \xi) = 2 \frac{\partial \mathcal{L}}{\partial F_{\mu\nu}} \xi + (-2\xi \partial_\rho + \partial_\rho \xi) \frac{\partial \mathcal{L}}{\partial(\partial_\rho F_{\mu\nu})}$$

$$+ 2c\, \varepsilon^{\mu\nu\rho} A_\rho \xi$$

$$\partial_\nu Q^{\mu\nu} = J^\mu(\phi, \delta\phi) \qquad \text{Noether current} \propto \delta\phi$$

$$\delta \int_C d\Omega_\mu J^\mu(\phi, \delta\phi) = \delta \int_{\partial C} d\Sigma_{\mu\nu}\, Q^{\mu\nu}(\phi, \xi)$$

$\equiv 0$ when varying over a continuous variety of solutions with a residual gauge symmetry

RESULT: rescaled variables (homogeneity)

$$X^I,\ \hat{A} \longrightarrow Y^I,\ \Upsilon$$

$F(Y, \Upsilon)$ yields

$$S_{macro} = \pi \Big[\underbrace{|Z|^2}_{\text{Area Law}} - \underbrace{256\ \mathrm{Im}\, F_\Upsilon(Y, \Upsilon)}_{\substack{\text{Wald's correction} \\ \text{(due to } R T^2\text{- terms)}}} \Big]$$

with $\Upsilon = -64$

$$|Z|^2 = p^I F_I(Y, \Upsilon) - q_I Y^I$$

$$\left.\begin{aligned} Y^I - \bar{Y}^I &= i\, p^I \\ F_I(Y, \Upsilon) - \bar{F}_I(\bar{Y}, \bar{\Upsilon}) &= i\, q_I \end{aligned}\right\} \text{fixed point behaviour}$$

Note: both the area & the correction are affected by the presence of the higher-derivative interactions

and consistent with E/M duality

COMPARISON

Effective field theory Lagrangian based on

$$F(Y,\Upsilon) = -\frac{1}{6}\,\frac{C_{ABC}\,Y^A Y^B Y^C}{Y^0} - \frac{1}{24}\,\frac{1}{64}\,c_{2A}\,\frac{Y^A}{Y^0}\,\Upsilon$$

gives $\quad c_{2A}\,\mathrm{Im}\,\frac{Y^A}{Y^0}\;C^2_{\mu\nu\rho\sigma}$

leads to

$$S_{macro} = 2\pi\sqrt{\frac{1}{6}\,q_0\left(C_{ABC}\,p^A p^B p^C + c_{2A}\,p^A\right)}$$

identical to S_{micro}

subleading corrections (homogeneity)

$$S_{macro} = \pi\sum_{g=0}^{\infty} a_g\,Q^{2-2g}$$

even powers!

Return to microstate counting (MSW)

Right movers

\# bosons $\quad 4h_{2,0}(\mathcal{P}) + 4 - 2h_{1,0}(\mathcal{P})$

\# fermions $\quad 4(h_{2,0}(\mathcal{P}) + h_{0,0}(\mathcal{P}))$

$$c_R = 6h_{2,0}(\mathcal{P}) + 6 - 2h_{1,0}(\mathcal{P})$$
$$= \mathcal{P}^3 + \tfrac{1}{2}c_2\cdot\mathcal{P} + 4h_{1,0}(\mathcal{P})$$

Left movers

\# bosons $\quad 2h_{2,0}(\mathcal{P}) + h_{1,1}(\mathcal{P}) + 2 - 2h_{1,0}(\mathcal{P})$

\# fermions $\quad 4h_{1,0}(\mathcal{P})$

$$c_L = 2h_{2,0}(\mathcal{P}) + h_{1,1}(\mathcal{P}) + 2$$
$$= \mathcal{P}^3 + c_2\cdot\mathcal{P} + 4h_{1,0}(\mathcal{P})$$

$\Longrightarrow$ entropy $\quad S_{micro} = 2\pi\sqrt{\tfrac{1}{6}q_0\left(C_{ABC}p^Ap^Bp^C + c_{2A}p^A + 4h_{1,0}(\mathcal{P})\right)}$

Vafa: $\quad \left(C_{ABC}p^Ap^Bp^C + c_{2A}p^A + 6h_{1,0}(\mathcal{P})\right)$

CY: $h_{1,0} = 0$
$K3\times T^2$: $h_{1,0} = 1$
T^6: $h_{1,0} = 3$

puzzle:

- no $\mathcal{R}$ supersymmetry /hyper kähler geometry
- wrong subleading entropy correction
- inconsistent with anomaly cancellation for M5 brane

Return to microstate counting (MSW)

Right movers $\qquad$ b_1 nondynamical gauge fields

$\#$ bosons $\quad 4h_{2,0}(\mathcal{P}) + 4 - 2h_{1,0}(\mathcal{P}) - b_1$

$\#$ fermions $\quad 4(h_{2,0}(\mathcal{P}) + h_{0,0}(\mathcal{P})) - 2b_1$

$$c_R = 6h_{2,0}(\mathcal{P}) + 6 - 2h_{1,0}(\mathcal{P})$$
$$= \mathcal{P}^3 + \tfrac{1}{2}c_2\cdot\mathcal{P} + 4h_{1,0}(\mathcal{P}) - 2b_1$$

Left movers

$\#$ bosons $\quad 2h_{2,0}(\mathcal{P}) + h_{1,1}(\mathcal{P}) + 2 - 2h_{1,0}(\mathcal{P}) - b_1$

$\#$ fermions $\quad 4h_{1,0}(\mathcal{P}) - 2b_1$

$$c_L = 2h_{2,0}(\mathcal{P}) + h_{1,1}(\mathcal{P}) + 2$$
$$= \mathcal{P}^3 + c_2\cdot\mathcal{P} + 4h_{1,0}(\mathcal{P}) - 2b_1$$

$\Longrightarrow$ entropy $\quad S_{micro} = 2\pi\sqrt{\tfrac{1}{6}q_0\left(C_{ABC}p^Ap^Bp^C + c_{2A}p^A + 4h_{1,0}(\mathcal{P})\right)}$

Vafa: $\quad \left(C_{ABC}p^Ap^Bp^C + c_{2A}p^A + 6h_{1,0}(\mathcal{P})\right)$

CY: $h_{1,0} = 0$

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Conclusion

agreement between microscopic & macroscopic results extends also to subleading corrections

subtlety in counting for $N = 4, 8$?

→ microstate counting

→ full supersymmetry enhancement at the horizon (leads to fixed-point behaviour)

→ E/M duality

→ modifications of the Area Law

→ First Law of black hole mechanics

→ higher-derivative terms in the effective action are crucial.