IIB Spectroscopy on AdS_5 and SCFT's

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Plan of the talk:

1) Unitarity bounds and multiplet shortening for $SU(2,2/N)$
($N=0,1,2,4$)
($N=4$ relevant for "multitrace" operators)
(A.Zaffaroni, S.F.)

2) IIB on $AdS_5 \times T_{11}$, spectroscopy and $SU(2,2/1)$ superconformal primaries
(A.Ceresole, G.Dall'Agata, R.D'Auria, S.F.)
(A.Ceresole, G.Dall'Agata, D'Auria)
to appear

One of the most stringent tests of the AdS / CFT correspondence
(Maldacena, Gubser Klebanov Polyakov, Witten)
is the matching of the spectrum of K-K supergravity modes and of superconformal primary operators of the boundary SCFT.

A particular class of such states correspond to the so called "shortened" multiplets of the superconformal algebra ($SU(2,2/N)$ for the $AdS_5 \to SCFT_4$ correspondence).

These multiplets have "protected" dimensions because of supersymmetry, and corresponds to BPS states from the bulk point of view and to shortened "superfields" from the boundary point of view.

Such "shortening" is known to occur at the "threshold" of the unitarity bounds of "highest weight" representations of the superconformal algebra.

From the bulk point of view, at these "thresholds", some states become "null vectors" in the Hilbert space of the UIR.

From the superconformal point of view these thresholds correspond to the possibility that certain differential constraints on the superfield is superconformal invariant.

These constraints fix the conformal dimension E_0 (mass) of a given operator in terms of the $SL(2,C)$ quantum numbers (J_1, J_2) and of the $U(1) \times SU(N)$ labels $(r, d_1 \ldots d_{N-1})$ of the highest weight state

$$\mathcal{D}(E_0, J_1, J_2, r, d_1 \ldots d_{N-1})$$

Note that, while for $N \neq 4$

$$SU(2,2/N) \sim PU(2,2/N)$$

for $N=4$ we have $PSU(2,2/4)$ $(r=0)$ while for $PU(2,2/4)$ r is the non trivial center of the superalgebra

(Bars, Gunaydin, Dobrev, Petkova, Binegar)

Actually for $PSU(2,2/4)$, as realized in IIB supergravity on $AdS_5 \times S_5$ the outer $U(1)$ automorphism (which therefore commutes with $SU(4)$ but not with supersymmetry) can be identified with the $U(1) \subset SL(2,R)$
(Bonati, Zaffaroni, S.F.; Intriligator)
classical symmetry of IIB superg.
(Green, Schwarz).

Spin 2 $\left(\frac{1}{2}, \frac{1}{2}\right)$ $E_0 = 2 + \sqrt{4 + m_2^2}$

Spin $\frac{3}{2}$ $\left(\frac{3}{2}, 1\right)$ $E_0 = 2 + |m_{3/2} + \frac{3}{2}|$

$\left(1, \frac{3}{2}\right)$

Spin 1 $\left(\frac{1}{2}, \frac{1}{2}\right)$ $E_0 = 2 + \sqrt{1 + m_1^2}$

two form $(1, 0)$ $E_0 = 2 + |m|$

$(0, 1)$

Spin $\frac{1}{2}$ $(1/2, 0)$ $E_0 = 2 \pm |m_{1/2}|$

$(0, 1/2)$

Spin 0 $(0, 0)$ $E_0 = 2 \pm \sqrt{4 + m_0^2}$

H.W. UIR of $SU(2,2)$

$\mathcal{D}(E_0, J_1, J_2)$

1) $J_1 J_2 \geq 0$ $\quad E_0 \geq 2 + J_1 + J_2$ $\quad$ generic

2) $J_1 J_2 = 0$ $\quad E_0 \geq 1 + J$ $\quad$ not-generic

3) $J_1 = J_2 = 0$ $\quad E_0 = 0$ $\quad$ Id. rep.

Unitarity Thresholds

1) $E_0 = 2 + J_1 + J_2$ $\qquad J_1 J_2 \neq 0$

Massless bulk fields $\leftrightarrow$ conserved currents

$$\partial^{\alpha_1 \dot{\alpha}_1} O_{\alpha_1 \ldots \alpha_{2J_1}, \dot{\alpha}_1 \ldots \dot{\alpha}_{2J_2}} = 0$$

2) $E_0 = 1 + J$

"Singletons" $\rightarrow$ massless boundary conformal fields

$$\partial^{\dot{\alpha}_1 \alpha_1} O_{\alpha_1 \ldots \alpha_{2J}} = 0 \quad (J \neq 0) \qquad \Box O = 0 \quad (J = 0)$$

(Wigner Eqs.)

(Flato, Fronsdal, Mack, ...)

H.W. UIR of $SU(2,2/N)$

$N = 1, 2, 4$

Three classes of U.I.R's which generalize 1) 2) 3) of $SU(2,2)$.

(modulo conjugation i.e.

$J_1 \to J_2$, $r \to -r$, $R \xrightarrow[SU(N)]{} \overline{R}$)

(Flato, Fronsdal; Dobrev, Petkova) :

$N = 1$:

1) $J_1 J_2 \geq 0$ $\quad E_0 \geq 2 + J_1 + J_2$

$$2 + 2J_1 - E_0 \leq \frac{3}{2} r \leq E_0 - 2 - 2J_2$$

2) $J_1 J_2 = 0$ $\quad E_0 \geq 1 + J \quad \left(E_0 = \frac{3}{2} |r| \right)$

3) $J_1 = J_2 = 0$ $\quad E_0 = r = 0$

(Freedman, Gubser, Pilch, Klebanov
Ceresole, D'Auria, Dall'Agata, S.F.)

Unitarity thresholds:

1) $E_0 = 2 + J_1 + J_2$, $\frac{3}{2} r = J_1 - J_2$

(conserved superfields) → supermassless

1)' $\frac{3}{2} r = E_0 - 2 - 2 J_2$

(semiconserved superfields) → semi long

2) $E_0 = 1 + J$, $\frac{3}{2} r = 1 + J$

(massless superfields) → supersingletons

2)' $E_0 = \frac{2}{3} r$

(chiral superfields)

Since in $N=1$ SCFT's the value of "r" can change, due to renormalization effects, from free field theory, the multiplet shortening 1', 2' can occur with anomalous dimensions as it happens in the SCFT dual to supergravity on $AdS_5 \times T^{11}$

(Klebanov, Witten, Gubser, Ceresole, Dall'Agata, D'Auria, SF)

$\mathcal{N}=2$ $\mathcal{D}(E_0, J_1, J_2, r, \ell)$

1) $J_1 J_2 \geq 0$ $E_0 \geq 2 + J_1 + J_2 + 2\ell$

$2 + 2J_1 + 2\ell - E_0 \leq r \leq E_0 - 2 - 2J_2 - 2\ell$

2) $J_1 J_2 = 0$ $E_0 = r + 2\ell,\ r \geq 1 + J$ (chiral multiplets, $\ell = 0$)

3) $J_1 = J_2 = 0$ $E_0 = 2\ell$ (hypermultiplets) multilinear

Unitarity thresholds

1) $E_0 = 2 + J_1 + J_2 + 2\ell$ ($\ell = 0$ conserved currents)

1') $E_0 = 2 + 2J_2 + r + 2\ell$ (semiconserved)

2) $E_0 = 1 + J + 2\ell$ ($\ell = 0$ massless supercurrent)

(Exceptional singleton @ thr. / c.h.)

$N=4 \quad (\underline{P}SU(2,2/4)) : z=0$

1) $J_1 J_2 \geq 0 \quad E_0 \geq 2+2J+2q+k \quad (q,k,q)$
$(J_1=J_2=J)$

2) $J_1 J_2 = 0 \quad E_0 = \dfrac{3p+2k+q}{2} \quad (q,k,p)$
$p \geq 2+q$

3) $J_1=J_2=0 \quad E_0 = 2q+k \quad (q,k,q)$
$(q=1, k=0$ is not primary in $N=4$ Y.M.)

Unitary thresholds:

1) $E_0 = 2+2J+2q+k \quad (q, k=0)$ (cons. curr.)

2) $p=2+q, \; E_0 = 3+2q+k \quad (q+2, k, q)$
($(2,0,0)$ is not primary in $N=4$ Y.M.)

3) supersingleton primaries $(q=0)$ + others.
(Kim, Romans, van Nieuwenhuizen, Gunaydin Marcus)

$z \neq 0$ generic singleton $\rightarrow$ 2)
exceptional singleton $(S_M \leq 3/2)$ 3)
(Gunaydin, Minic, Zagermann)

(A. Zaffaroni L.F.)

Application to multi-trace operators:

(related to log singularity in OPE) (Freedman, Mathur, Matusis, Rastelli, D'Hoker)

$\mathrm{Tr}\,(0,k_1,0)\,\mathrm{Tr}\,(0,k_2,0)\ldots\mathrm{Tr}\,(0,k_J,0)$

$= (0,\sum_i k_i,0) + \ldots$

$\Downarrow$

Not renormalized ("Antichiral" S.F.)

follows from harmonic superspace (Lett. Math. Phys. 46, 265 (1998)) (Galperin, Ivanov, Ogievetsky, Sokatchev) Howe, West)

Other structures? (examples)

$\mathrm{Tr}(0,2,0)\,\mathrm{Tr}(0,2,0) = (0,4,0) + (2,0,2) + (0,2,0) + (0,0,0)$

$\mathrm{Tr}\, \underbrace{\phi_\alpha \phi_\beta}\ \mathrm{Tr}\, \underbrace{\phi_\gamma \phi_\delta}$, $\mathrm{Tr}\, \phi_\alpha \phi_\beta\, \mathrm{Tr}\, \phi_\gamma \phi_\delta$

$\mathbf{210} = 105 + 84 + \underbrace{20 + 1}$

$\downarrow$ short, $\downarrow$ short, long

$84 = (2,0,2)$ has protected dimension!

Confirmed by an explicit calculation (hep-th 9906188)

by ... Bianchi, Kovacs, Rossi, Stanev -

$k(84)$ not primary $\quad 84'' = 84 - k(84)$

IIB Supergravity on $AdS_5 \times T^{11}$ (Romans)

(Klebanov, Witten, Gubser)

$$T^{11} = \frac{SU(2)\times SU(2)}{U(1)}$$

T^{11}: $b_2 = b_3 = 1$!

Complete spectrum obtained by performing Harmonic Analysis.

(Ceresole, Dall'Agata, D'Auria, S.F.)
hep-th 9905226

(Jatkar, Randjbar-Daemi hepth 9904187)

All the above shortenings are realized:

$$\bar{D}^{\dot\alpha_1} J_{\dot\alpha_1 \ldots \dot\alpha_{2J_2}, \alpha_1 \ldots \alpha_{2J_1}} = 0$$

$$r = \frac{2}{3}(E_0 - 2 - 2J_2)$$

(conserved if left-right conserved, massless singleton if $J_1 = 0$ and chiral)

$$\bar{D}^{\dot\alpha} J_{\alpha_1 \ldots \alpha_{2J}} = 0 \qquad E_0 = \frac{3}{2} r$$

families: product of conserved × chiral

Laplacian

$$H_0(J, \ell, r) = 6\left[J(J+1) + \ell(\ell+1) - \frac{r^2}{8}\right]$$

$H_0^{\pm}$, $H_0^{\pm\pm}$ $r \to r \pm 1$, $r \to r \pm 2$

All masses computed. (vectors)

$$M^2(b_{\mu i}) = \begin{cases} 3 + H_0^{\pm\pm} & \ldots \\ H_0 + 4 \pm 2\sqrt{H_0 + 4} \end{cases}$$

$$M^2(g_{\mu i}, A_{\mu ijk}) = \begin{cases} H_0 + 7 \pm 4\sqrt{H_0+4} \\ H_0 + 12 \pm 6\sqrt{H_0+4} \\ H_0 + 4 \pm 2\sqrt{H_0+4} \end{cases}$$

$$M^2(a_{\mu i}) = \begin{cases} 3 + H_0^{\pm\pm} \\ H_0^{\pm} + 4 \pm 2\sqrt{H_0^{\pm}+4} \end{cases}$$

scalars

$$M^2(b_{ij}) = \begin{matrix} H_0 + 4 \pm 4\sqrt{H_0+4} \\ H_0^{\pm\pm} + 1 \pm 2\sqrt{H_0^{\pm\pm}+4} \end{matrix}$$

$$M^2(g_{ij}) = \begin{matrix} H_0 + 9 \pm 6\sqrt{H_0+4} \\ H_0 + 4 \pm 4\sqrt{H_0+4} \\ H_0 \end{matrix}$$

Shortening:

$\sqrt{H_0(j,\ell,r)+4}$ rational

$j=\ell=\left|\frac{r}{2}\right|=\frac{k}{2}$ $\quad H_0+4\to\left(\frac{3}{2}k+2\right)^2$

$j=\ell=\left|\frac{r}{2}\right|-1=\frac{k}{2}$ $\quad H_0+4\to\left(\frac{3}{2}k+1\right)^2$

$$\left\{\begin{array}{l} j=\ell-1=\left|\frac{r}{2}\right|=\frac{k}{2} \\ j-1=\ell=\left|\frac{r}{2}\right|=\frac{k}{2}\end{array}\right. \qquad H_0+4\to\left(\frac{3}{2}k+4\right)^2$$

All families with rational dimension (shortened or not) fall in this list.

Long Graviton Multiplet $\quad E_0 = 1 + \sqrt{H_0 + 4}$.

		(s_1, s_2)	$E_0^{(s)}$	R–symm.	field	Mass
$\diamond$	$\star$	(1,1)	$E_0 + 1$	r	$H_{\mu\nu}$	H_0
$\diamond$	$\star$	(1,1/2)	$E_0 + 1/2$	$r - 1$	ψ_μ^L	$-2 + \sqrt{H_0 + 4}$
$\diamond$	$\star$	(1/2,1)	$E_0 + 1/2$	$r + 1$	ψ_μ^R	$-2 + \sqrt{H_0 + 4}$
	$\star$	(1/2,1)	$E_0 + 3/2$	$r - 1$	ψ_μ^R	$-2 - \sqrt{H_0 + 4}$
		(1,1/2)	$E_0 + 3/2$	$r + 1$	ψ_μ^L	$-2 - \sqrt{H_0 + 4}$
$\diamond$	$\star$	(1/2,1/2)	E_0	r	ϕ_μ	$H_0 + 4 - 2\sqrt{H_0 + 4}$
		(1/2,1/2)	$E_0 + 1$	$r + 2$	a_μ	$H_0 + 3$
	$\star$	(1/2,1/2)	$E_0 + 1$	$r - 2$	a_μ	$H_0 + 3$
		(1/2,1/2)	$E_0 + 2$	r	B_μ	$H_0 + 4 + 2\sqrt{H_0 + 4}$
		(1,0)	$E_0 + 1$	r	$b_{\mu\nu}^+$	$\sqrt{H_0 + 4}$
	$\star$	(0,1)	$E_0 + 1$	r	$b_{\mu\nu}^-$	$-\sqrt{H_0 + 4}$
		(1/2,0)	$E_0 + 1/2$	$r + 1$	λ_L	$1/2 - \sqrt{H_0 + 4}$
	$\star$	(0,1/2)	$E_0 + 1/2$	$r - 1$	λ_R	$1/2 - \sqrt{H_0 + 4}$
		(1/2,0)	$E_0 + 3/2$	$r - 1$	λ_L	$1/2 + \sqrt{H_0 + 4}$
		(0,1/2)	$E_0 + 3/2$	$r + 1$	λ_R	$1/2 + \sqrt{H_0 + 4}$
		(0,0)	$E_0 + 1$	r	B	H_0

LONG GRAVITINO MULTIPLET I $E_0 = -\frac{1}{2} + \sqrt{H_0^- + 4}$

		(s_1, s_2)	$E_0^{(s)}$	R–symm.	field	Mass
	⋆	(1/2,1)	$E_0 + 1$	r	ψ_μ^L	$-3 + \sqrt{H_0^- + 4}$
	⋆	(1/2,1/2)	$E_0 + 1/2$	$r + 1$	ϕ_μ	$H_0^- + 7 - 4\sqrt{H_0^- + 4}$
	⋆	(1/2,1/2)	$E_0 + 3/2$	$r - 1$	a_μ	$H_0^- + 4 - 2\sqrt{H_0^- + 4}$
•	⋆	(1,0)	$E_0 + 1/2$	$r - 1$	$a_{\mu\nu}$	$2 - \sqrt{H_0^- + 4}$
		(1,0)	$E_0 + 3/2$	$r + 1$	$b_{\mu\nu}^+$	$1 - \sqrt{H_0^- + 4}$
•	⋆	(1/2,0)	E_0	r	$\psi_L^{(T)}$	$-5/2 + \sqrt{H_0^- + 4}$
•	⋆	(1/2,0)	$E_0 + 1$	$r - 2$	$\psi_L^{(T)}$	$-3/2 + \sqrt{H_0^- + 4}$
	⋆	(0,1/2)	$E_0 + 1$	r	λ_R	$3/2 - \sqrt{H_0^- + 4}$
		(1/2,0)	$E_0 + 1$	$r + 2$	$\psi_L^{(T)}$	$-3/2 + \sqrt{H_0^- + 4}$
		(1/2,0)	$E_0 + 2$	r	$\psi_L^{(T)}$	$-1/2 + \sqrt{H_0^- + 4}$
•	⋆	(0,0)	$E_0 + 1/2$	$r - 1$	a	$H_0^- + 4 - 4\sqrt{H_0^- + 4}$
		(0,0)	$E_0 + 3/2$	$r + 1$	a	$H_0^- + 1 - 2\sqrt{H_0^- + 4}$

X

Long Gravitino Multiplet II $E_0 = 5/2 + \sqrt{H_0^+ + 4}$

(s_1, s_2)	$E_0^{(s)}$	R–symm.	field	Mass
(1,1/2)	$E_0 + 1$	r	ψ_μ^L	$-3 - \sqrt{H_0^+ + 4}$
(1/2,1/2)	$E_0 + 1/2$	$r + 1$	a_μ	$H_0^+ + 4 + 2\sqrt{H_0^+ + 4}$
(1/2,1/2)	$E_0 + 3/2$	$r - 1$	B_μ	$H_0^+ + 7 + 4\sqrt{H_0^+ + 4}$
(1,0)	$E_0 + 1/2$	$r - 1$	$b_{\mu\nu}^+$	$1 + \sqrt{H_0^+ + 4}$
(1,0)	$E_0 + 3/2$	$r + 1$	$a_{\mu\nu}$	$2 + \sqrt{H_0^+ + 4}$
(1/2,0)	E_0	r	$\psi_L^{(T)}$	$-1/2 - \sqrt{H_0^+ + 4}$
(1/2,0)	$E_0 + 1$	$r - 2$	$\psi_L^{(T)}$	$-3/2 - \sqrt{H_0^+ + 4}$
(0,1/2)	$E_0 + 1$	r	λ_R	$3/2 + \sqrt{H_0^+ + 4}$
(1/2,0)	$E_0 + 1$	$r + 2$	$\psi_L^{(T)}$	$-3/2 - \sqrt{H_0^+ + 4}$
(1/2,0)	$E_0 + 2$	r	$\psi_L^{(T)}$	$-5/2 - \sqrt{H_0^+ + 4}$
(0,0)	$E_0 + 1/2$	$r - 1$	a	$H_0^+ + 1 + 2\sqrt{H_0^+ + 4}$
(0,0)	$E_0 + 3/2$	$r + 1$	a	$H_0^+ + 4 + 4\sqrt{H_0^+ + 4}$

Long Gravitino Multiplet III $E_0 = -1/2 + \sqrt{H_0^+ + 4}$

	(s_1, s_2)	$E_0^{(s)}$	R–symm.	field	Mass
$\star$	(1,1/2)	$E_0 + 1$	r	ψ_μ^R	$-3 + \sqrt{H_0^+ + 4}$
$\star$	(1/2,1/2)	$E_0 + 1/2$	$r - 1$	ϕ_μ	$H_0^+ + 7 - 4\sqrt{H_0^+ + 4}$
	(1/2,1/2)	$E_0 + 3/2$	$r + 1$	a_μ	$H_0^+ + 4 - 2\sqrt{H_0^+ + 4}$
$\star$	(0,1)	$E_0 + 1/2$	$r + 1$	$a_{\mu\nu}$	$2 - \sqrt{H_0^+ + 4}$
$\star$	(0,1)	$E_0 + 3/2$	$r - 1$	$b_{\mu\nu}^-$	$1 - \sqrt{H_0^+ + 4}$
$\star$	(0,1/2)	E_0	r	$\psi_R^{(T)}$	$-5/2 + \sqrt{H_0^+ + 4}$
$\star$	(0,1/2)	$E_0 + 1$	$r + 2$	$\psi_R^{(T)}$	$-3/2 + \sqrt{H_0^+ + 4}$
	(1/2,0)	$E_0 + 1$	r	λ_L	$3/2 - \sqrt{H_0^+ + 4}$
	(0,1/2)	$E_0 + 1$	$r + 2$	$\psi_R^{(T)}$	$-3/2 + \sqrt{H_0^+ + 4}$
	(0,1/2)	$E_0 + 2$	r	$\psi_R^{(T)}$	$-1/2 + \sqrt{H_0^+ + 4}$
	(0,0)	$E_0 + 1/2$	$r + 1$	a	$H_0^+ + 4 - 4\sqrt{H_0^+ + 4}$
	(0,0)	$E_0 + 3/2$	$r - 1$	a	$H_0^+ + 1 - 2\sqrt{H_0^+ + 4}$

Long Gravitino Multiplet IV $E_0 = 5/2 + \sqrt{H_0^- + 4}$

	(s_1, s_2)	$E_0^{(s)}$	R–symm.	field	Mass
$\star$	(1/2,1)	$E_0 + 1$	r	ψ_μ^R	$-3 - \sqrt{H_0^- + 4}$
$\star$	(1/2,1/2)	$E_0 + 1/2$	$r - 1$	a_μ	$H_0^- + 4 + 2\sqrt{H_0^- + 4}$
	(1/2,1/2)	$E_0 + 3/2$	$r + 1$	B_μ	$H_0^- + 7 + 4\sqrt{H_0^- + 4}$
$\star$	(0,1)	$E_0 + 1/2$	$r + 1$	$b_{\mu\nu}^-$	$1 + \sqrt{H_0^- + 4}$
$\star$	(0,1)	$E_0 + 3/2$	$r - 1$	$a_{\mu\nu}$	$2 + \sqrt{H_0^- + 4}$
$\star$	(0,1/2)	E_0	r	$\psi_R^{(T)}$	$-1/2 - \sqrt{H_0^- + 4}$
$\star$	(0,1/2)	$E_0 + 1$	$r + 2$	$\psi_R^{(T)}$	$-3/2 - \sqrt{H_0^- + 4}$
	(1/2,0)	$E_0 + 1$	r	λ_L	$3/2 + \sqrt{H_0^- + 4}$
	(0,1/2)	$E_0 + 1$	$r - 2$	$\psi_R^{(T)}$	$-3/2 - \sqrt{H_0^- + 4}$
	(0,1/2)	$E_0 + 2$	r	$\psi_R^{(T)}$	$-5/2 - \sqrt{H_0^- + 4}$
	(0,0)	$E_0 + 1/2$	$r + 1$	a	$H_0^- + 1 + 2\sqrt{H_0^- + 4}$
	(0,0)	$E_0 + 3/2$	$r - 1$	a	$H_0^- + 4 + 4\sqrt{H_0^- + 4}$

Vector Multiplet I $E_0 = \sqrt{H_0 + 4} - 2$

			(s_1, s_2)	$E_0^{(s)}$	R–symm.	field	Mass
$\diamond$		$\star$	(1/2,1/2)	$E_0 + 1$	r	ϕ_μ	$H_0 + 12 - 6\sqrt{H_0 + 4}$
$\diamond$	$\bullet$	$\star$	(1/2,0)	$E_0 + 1/2$	$r - 1$	$\psi_L^{(L)}$	$7/2 - \sqrt{H_0 + 4}$
$\diamond$		$\star$	(0,1/2)	$E_0 + 1/2$	$r + 1$	$\psi_R^{(L)}$	$7/2 - \sqrt{H_0 + 4}$
		$\star$	(0,1/2)	$E_0 + 3/2$	$r - 1$	$\psi_R^{(L)}$	$5/2 - \sqrt{H_0 + 4}$
			(1/2,0)	$E_0 + 3/2$	$r + 1$	$\psi_L^{(L)}$	$5/2 - \sqrt{H_0 + 4}$
$\diamond$	$\bullet$	$\star$	(0,0)	E_0	r	b	$H_0 + 16 - 8\sqrt{H_0 + 4}$
	$\bullet$	$\star$	(0,0)	$E_0 + 1$	$r - 2$	ϕ	$H_0 + 9 - 6\sqrt{H_0 + 4}$
			(0,0)	$E_0 + 1$	$r + 2$	ϕ	$H_0 + 9 - 6\sqrt{H_0 + 4}$
			[illegible]	[illegible]			

$e^{V_1} \to e^{i\Lambda_1} e^{V_1} e^{-i\bar{\Lambda}_1}$ $\qquad SU(N)\times SU(N)$

$e^{V_2} \to e^{i\Lambda_2} e^{V_2} e^{-i\bar{\Lambda}_2}$

$A \to e^{i\Lambda_1} A e^{-i\Lambda_2}$ $\qquad (N_1, \bar{N}_2)$

$B \to e^{i\Lambda_2} B e^{-i\Lambda_1}$ $\qquad (\bar{N}_2, N_1)$

$AB \,,\ BA \,,\ A e^{V_2} \bar{A} e^{-V_1} \,,\ B e^{V_1} \bar{B} e^{-V_2}$

(Zumino, SF, Wess)

$\Delta(A) = \frac{3}{4} \qquad R = \frac{1}{2}$

Chiral Primary Towers:

$$S^k = \mathrm{Tr}\,(AB)^k \qquad \Delta^k = \tfrac{3}{2} k$$

$$T^k = \mathrm{Tr}\,(W_\alpha (AB)^k) \qquad \Delta^k = \tfrac{3}{2}(k+1)$$

$$\Phi^k = \mathrm{Tr}\,(W^\alpha W_\alpha (AB)^k) \qquad \Delta^k = 3 + \tfrac{3}{2} k$$

fermionic multiplets

$$L^{1k}_{\dot\alpha} = \mathrm{Tr}\,(e^V \bar W_{\dot\alpha} e^{-V} (AB)^k) \qquad j = l$$

$$L^{2k}_{\dot\alpha} = \mathrm{Tr}\,(e^V W_{\dot\alpha} e^{-V} W^2 (AB)^k) \qquad j = l$$

$$L^{3k}_{\alpha} = \mathrm{Tr}\, W_\alpha (A e^V \bar A e^{-V})(AB)^k \qquad j = l+1$$

$$J^k = \mathrm{Tr}\; A e^V \bar A e^{-V} (AB)^k \qquad j = l+1$$

$$I^k = \mathrm{Tr}\; A e^V \bar A\, e^{-V}\, W^2 (AB)^k \qquad j = l+1$$

$$J^k_{\alpha\dot\alpha} = \mathrm{Tr}\; J_{\alpha\dot\alpha} (AB)^k \qquad j = l$$

$$J_{\alpha\dot\alpha} \to W_\alpha e^V \overline{W}_{\dot\alpha} e^{-V}$$

The above multiplets come from multiplet shortening of some otherwise be long-generic multiplet.

Massless multiplets:

Graviton, U(1) 2 current $\quad J_{\alpha\dot\alpha} \quad D^{\alpha} J_{\alpha\dot\alpha} = \bar D^{\dot\alpha} J_{\alpha\dot\alpha} = 0$

$SU(2)\times SU(2)$ vector bosons $\quad D^2 J = \bar D^2 J = 0$

Baryon multiplet $\quad b_3 = 1 \quad D^2 I = \bar D^2 I = 0$

(Baryonic current, Klebanov, Witten, Klebanov, Gubser)

Other "long" multiplets with rational dimension:
$(J=0)$

$\mathrm{Tr}\, W^2 e^{V} \bar W^2 e^{-V} (AB)^k \qquad (F^2)^2 (AB)^k$

$\mathrm{Tr}\left(W_\alpha e^{V} \bar W^2 e^{-V} (AB)^k\right) \qquad F^3 (AB)^k$

$\mathrm{Tr}\left(e^{V} \bar W^2 e^{-V} (AB)^k\right) \qquad F (AB)^k$

(: families occurring in the Born-Infeld action: (Liccardo, Zaffaroni, S.F.)

Flavor + Betti multiplets:

$$f = \mathrm{Tr}\,(A_i e^V \bar{A}^j e^{-V}) - \frac{1}{2}\delta_i{}^j\,\mathrm{Tr}\,(A_i e^V \bar{A}^i e^{-V})$$

$$(A \to B)$$

$$b = \mathrm{Tr}\,(A_i e^V \bar{A}^i e^{-V}) - \mathrm{Tr}\,(B_e e^V \bar{B}^e e^{-V})$$

Massive chiral multiplets: (Klebanov Witten)

$\mathrm{Tr}(W_1^2 \pm W_2^2)$ $E_0 = 3$ $R = 2$

(dilaton $E_0 = 4$, b_{ij} $E_0 = 4$) $b_2 = 1$

partner partner

b_{ij} $(m^2 = -3)$ g_{ij} $(m^2 = -3)$

$(b^c_{ij}\ \phi^c)$ $(g_{ij}\ b^c_{ij})$

The effective lagrangian for the massless (vector multiplet) sector can be obtained from the knowledge of the Chern-Simons couplings

$$d_{\Lambda\Sigma\Delta} \int A^\Lambda \wedge F^\Sigma \wedge F^\Delta$$

$\downarrow$

$O(N^2)$ (Gunaydin, Sierra, Townsend)

AdS-CFT correspondence · $d_{\Lambda\Sigma\Delta}$ connected to the Flavor and R-symmetry anomalies of the boundary SCFT (Witten)

N large

In our case $\Lambda = 1 \ldots 8$

$U(1)$ R sym.
$U(1)$ b baryonic
$SU(2)_A$, $SU(2)_B$

$d_{\Lambda AA} = d_{\Lambda BB}$

$d_{bAA} = -d_{bBB}$

$d_{\Lambda\Lambda\Lambda}$, $d_{\Lambda bb}$

$U_\Lambda(1)$, $U_b(1)$ traceless.

$c_\Lambda \int A^\Lambda \wedge R \wedge R$ absent

(Herrington, Skenderis, Gubser) (Anselmi, Rehagias)