Non-conformal examples of AdS/CFT

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based on work with D. Freedman, K. Pilch, N. Warner, hep-th/9904017, 9906194

- Asymptotically AdS geometries:
  vacuum states and RG flows
- Proof of gravitational c-theorem
- Vacuum states on Coulomb branch of $\mathcal{N}=4$ SYM:
  D-brane geometries, consistent truncation
- Two-point functions, energy spectra
- Interpretation: brane ensembles?

References and related work:

Girardello, Petrini, Porrati, Zaffaroni, 9810126

Distler-Zamora, 9810206

Khavaer-Pilch-Warner, 9812035

Klebanov-Witten, 9807080

Henningson-Skenderis, 9806087

Gubser, 9807164

Karch-Lust-Miemiec, 9901041

Leigh-Strassler, 9503121

Peet-Polchinski, 9809022

Kraus-Larsen-Trivedi, 9811120

Cvetic-Gubser, 9903132

Brandhuber-Sfetsos, 9906201

Chepeler-Roiban, 99

Giddings-Ross, to appear

Kabat-Lifshytz, to appear

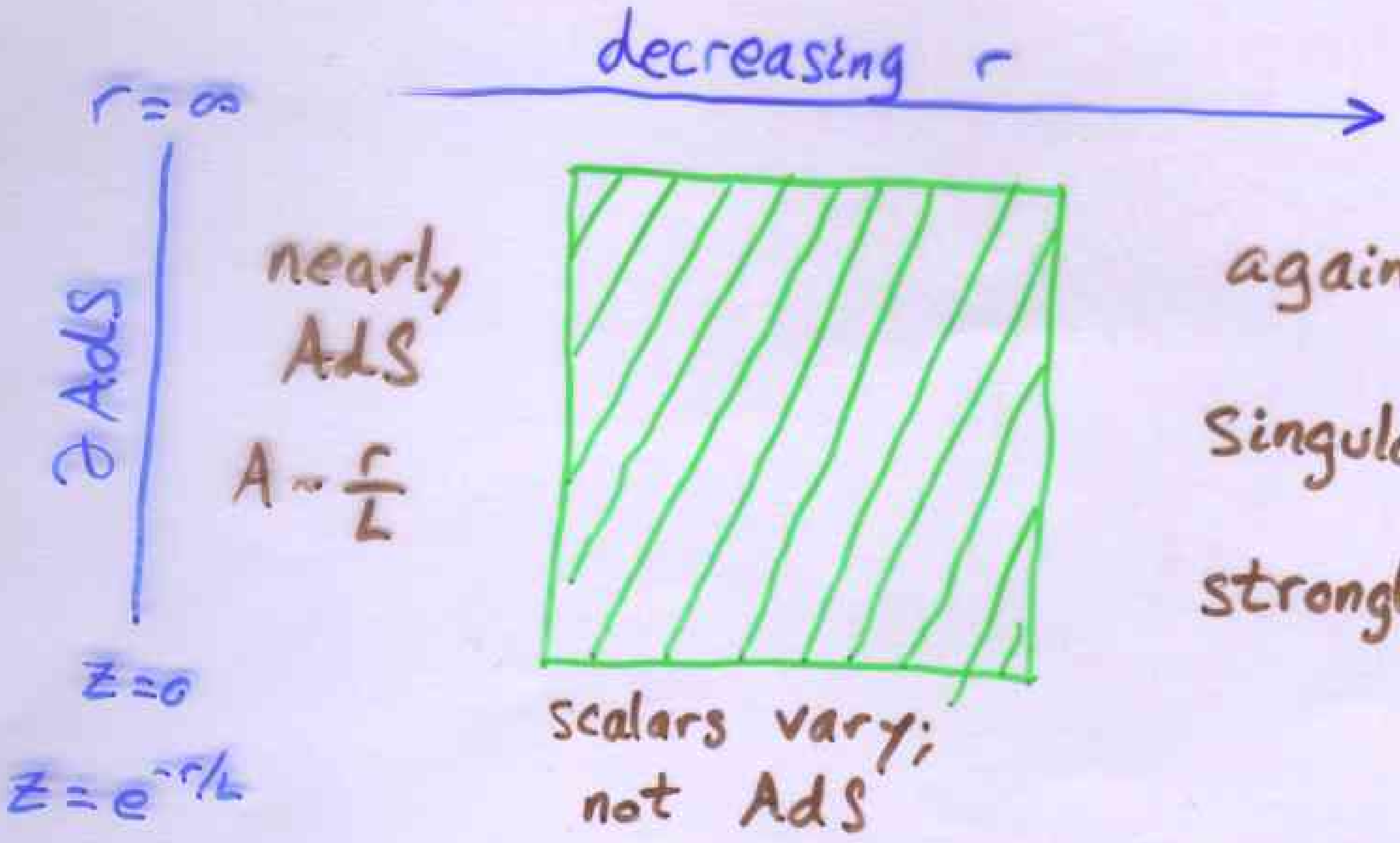
M A G O O, 9905111

M	A	G	O	O
a	h	u	o	z
l	a	b	g	
d	r	s	u	
a	o	e	r	
c	n	r	i	
e	y			
n				
a				

Asymptotically AdS metrics with 3+1-dim Poincaré invariance:

$$ds^2 = e^{2A(r)}\left(dt^2 - dx_1^2 - dx_2^2 - dx_3^2\right) - dr^2$$

plus some scalars $\varphi(r)$. $A = \frac{r}{L} \longleftrightarrow AdS_5$



again AdS?

singular at finite r?

strongly curved as $r \to -\infty$?

Is this geometry dual to a (non-conformal) vacuum state of $\mathcal{N}=4$ SYM, or to a RG flow due to soft mass terms added to $\mathcal{L}_{\mathcal{N}=4}$?

Depends on asymptotics of scalar fields:

$$(\Box + m^2)\varphi = 0 \qquad \varphi \sim z^{\Delta_\pm} \qquad \Delta_\pm = 2 \pm \sqrt{4 + (mL)^2}$$

$$\varphi \sim z^{\Delta_-} \longleftrightarrow \mathcal{L} \to \mathcal{L} + \frac{1}{z^{\Delta_-}}\phi\,\mathcal{O}$$

$$\varphi \sim z^{\Delta_+} \longleftrightarrow \langle \mathcal{O} \rangle = \frac{1}{z^{\Delta_+}}\varphi$$

A "c-theorem" from gravity:

In AdS_D, it is straightforward to show that

$$-(D-2)A'' \underset{\text{(computation)}}{=} R^t_t - R^r_r = G^t_t - G^r_r \underset{\text{(Einstein)}}{=} \kappa_D^2 (T^t_t - T^r_r) \geq 0$$

where $'$ means $\frac{d}{dr}$.

(Null energy: $T_{\alpha\beta}\xi^\alpha\xi^\beta \geq 0$ for null ξ)

Provided $D > 2$, $A'(r)$ increases as r decreases. Recall that radius in AdS $\longleftrightarrow$ energy scale in QFT.

Suggestion: $C(r) \equiv \frac{1}{A'(r)^{D-2}}$ is a count of the number of degrees of freedom in the QFT available at the energy scale dual to r.

If D is odd, then Henningson-Skenderis have shown that CFT responds to a curved <u>boundary</u> metric as follows: $\langle T^\mu_\mu \rangle_{g_{\mu\nu}} = \frac{(\text{universal})}{A'(r)^{D-2}}$, provided the bulk metric is AdS.

So $C(r)$ is an anomaly coefficient at fixed pts. of RG.

A non-trivial check: $\mathcal{N}=4$ SYM in $d=4$, deformedy by a mass for one chiral field.

$$W = \mathrm{tr}\,\phi_3[\phi_1,\phi_2] + \tfrac{1}{2} m\,\mathrm{tr}\,\phi_3^2$$

$\mathcal{N}=1$ SUSY preserved

$\mathcal{N}=4$ term   soft mass term

$\xrightarrow{\text{RG}}$ non-trivial interacting fixed point,

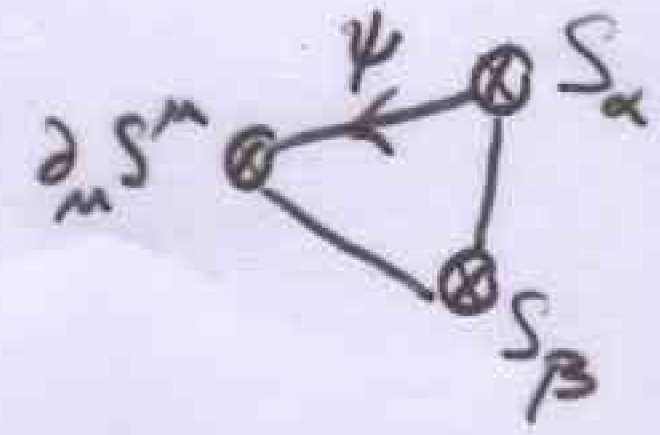
$$W \sim \mathrm{tr}\,[\phi_1,\phi_2]^2$$

$\mathcal{N}=1$ SUSY relates T^μ_μ and $\partial_\mu R^\mu$.
't Hooft anomaly matching can be used to establish $c_{IR} = \frac{27}{32} c_{UV}$.

$$S_\mu = R_\mu + \frac{2}{3} \sum_{\substack{\text{chirals} \\ \phi_i}} (\gamma^i_{IR} - \gamma^i) K^i_\mu$$

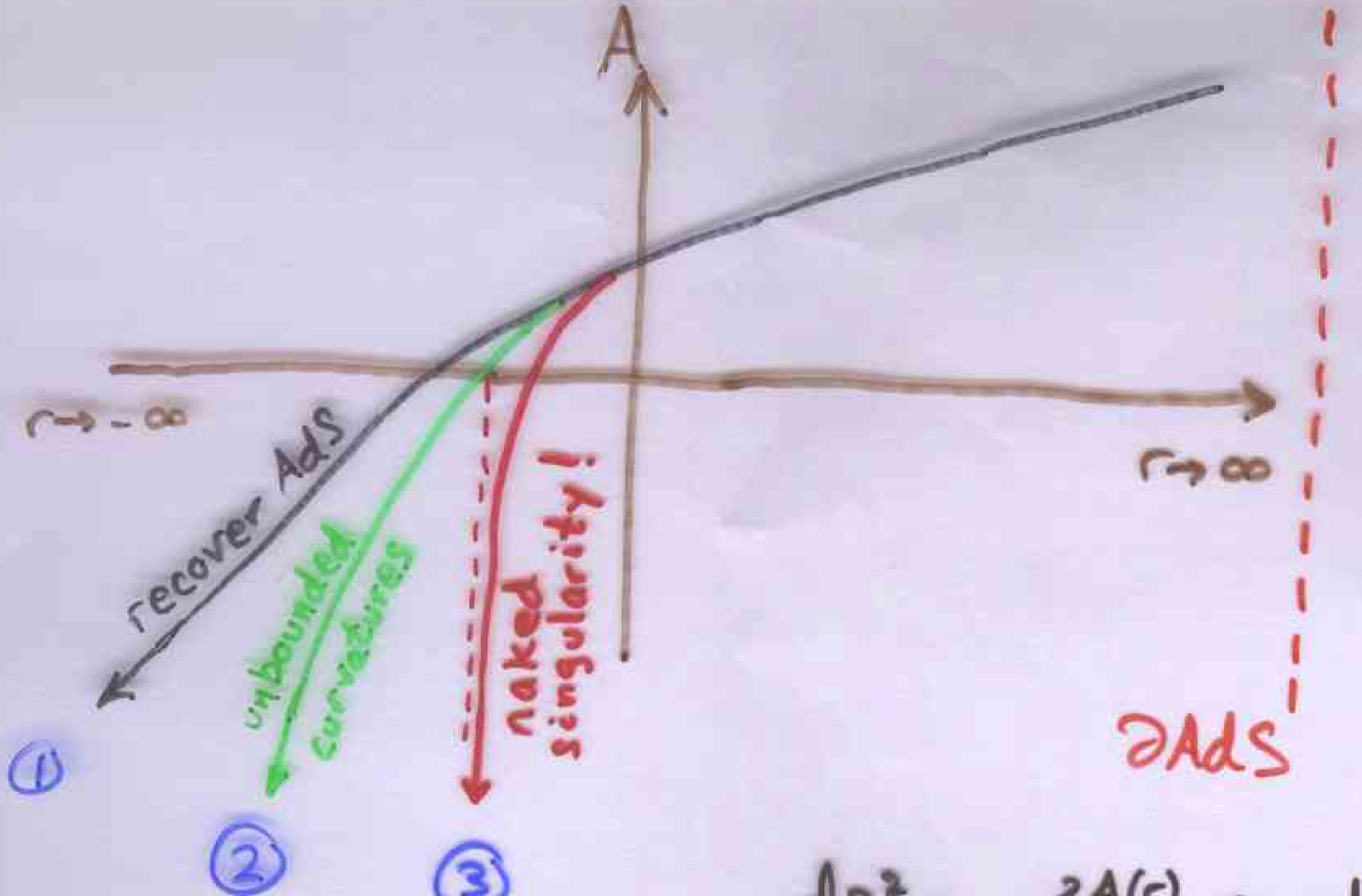
Konishi current assigns charge 1 to fields in ϕ_i.

$$\langle \partial_\mu R^\mu R_\alpha R_\beta \rangle_{IR} = \langle \partial_\mu S^\mu S_\alpha S_\beta \rangle_{UV}$$



Supergravity description of this RG flow (see talk by N. Warner) gives a $C(r)$ with

$$\lim_{r\to-\infty} C(r) = \frac{27}{32} \lim_{r\to+\infty} C(r) \qquad (!)$$



possible behaviors of $A(r)$

$$ds^2 = e^{2A(r)} \eta_{\mu\nu} dx^\mu dx^\nu - dr^2$$

Behaviors 2 and 3 are actually more generic than 1. Are they unphysical? 10-dim geometry can help us answer.

Five examples from N. Warner's talk:

Asymptotically AdS_5 geometries in $\mathcal{N}=8$ gauged SUGRA, involving only $m^2 = -4/L^2$ scalars, preserving 16 supercharges, and breaking $SO(6) \to SO(n) \times SO(6-n)$ for $n = 1, 2, 3, 4, 5$.

For $n = 1$ or 2, get behavior 2. For $n = 3, 4, 5$, get behavior 3.

The 10-dimensional geometries can only be multi-center D3-brane solutions preserving $SO(n) \times SO(6-n)$:

$$ds^2 = \frac{1}{\sqrt{H}}\left(dt^2 - dx_1^2 - dx_2^2 - dx_3^2\right) - \sqrt{H}\left(dy_1^2 + dy_2^2 + \dots + dy_6^2\right)$$

$$H = \frac{L^4}{N}\sum_{i=1}^{N}\frac{1}{|\vec{y}-\vec{y}_i|^4} \approx \frac{L^4}{N}\int d^6y' \, \frac{\sigma(\vec{y}')}{|\vec{y}-\vec{y}'|^4}$$

$n=1$

a line:

$\sigma \sim \sqrt{\ell^2 - x^2}$

2ℓ

$n=2$

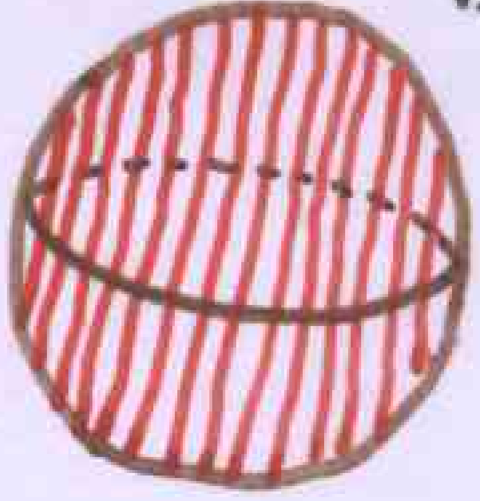
a uniform disk:

$\sigma \sim 1$

$n=3$

a solid ball B^3:

$\sigma \sim 1/\sqrt{\ell^2 - r^2}$



$n=4$

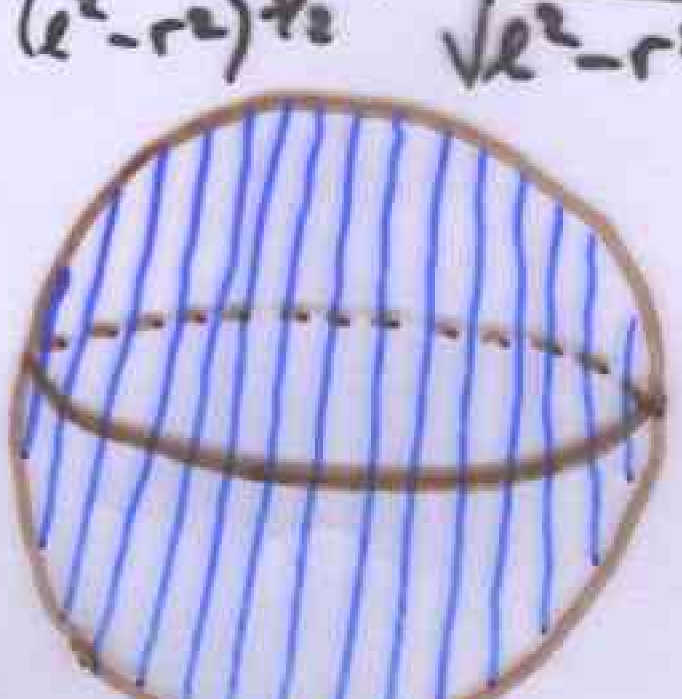
a spherical shell, S^3:

$\sigma \sim \delta(\ell^2 - r^2)$

$n=5$

a ball, B^5, with both + and − "charges" — unphysical!

$$\sigma \sim -\frac{1/2}{(\ell^2-r^2)^{3/2}} + \frac{1}{\sqrt{\ell^2-r^2}}\,\delta(\ell^2-r^2)$$



"Consistent truncation" in $d=5$:

(de Wit - Nicolai '84, de Wit - Nicolai - Warner '85 for $d=4$)

$$\begin{array}{c} \text{gauged} \rightarrow SO(6) \\ \cap \\ E_{6(6)} \supset SL(6,\mathbb{R}) \times \underset{\text{axion-dilaton}}{SL(2,\mathbb{R})} \end{array} \qquad 27 = \underset{z^{AB}\ \ z^{I\alpha}}{(\bar{6},2)} + \underset{z_{[IJ]}}{(15,1)}$$

$$E_{6(6)} \supset USp(8) \qquad 27 = 27 \quad z^{AB} \quad z^{[ab]}, \ \Omega\text{-traceless}$$

The 27-bein $\mathcal{V}_{AB}{}^{ab}$ parametrizes the 42 scalars

Say $K^{m\,IJ}$ are 15 $SO(6)$ Killing vector fields on S^5. Define $K^m{}_{ab} = K^{m\,IJ} (\mathcal{V}^{-1})_{IJ\,ab}$.

$$\left.\begin{array}{l} \Delta^{-2/3} \tilde{g}^{mn} = K^m{}_{ab} K^n{}_{cd} \, \Omega^{ac} \Omega^{bd} \\ \Delta = \sqrt{\det(\tilde{g}_{mn} \mathring{g}^{np})} \quad (\text{round } S^5) \end{array}\right\} \text{Solve simultaneously for } \Delta \text{ and } \tilde{g}^{mn}$$

$$\boxed{ds^2_{10} = \Delta^{-2/3} ds^2_5 + \tilde{g}_{mn} dy^m dy^n}$$

(Simpler in practice when $\mathcal{V} \in SL(6,\mathbb{R})/SO(6)$: then $\Delta^{-2/3} \tilde{g}_{mn}$ is metric on an ellipsoid.)

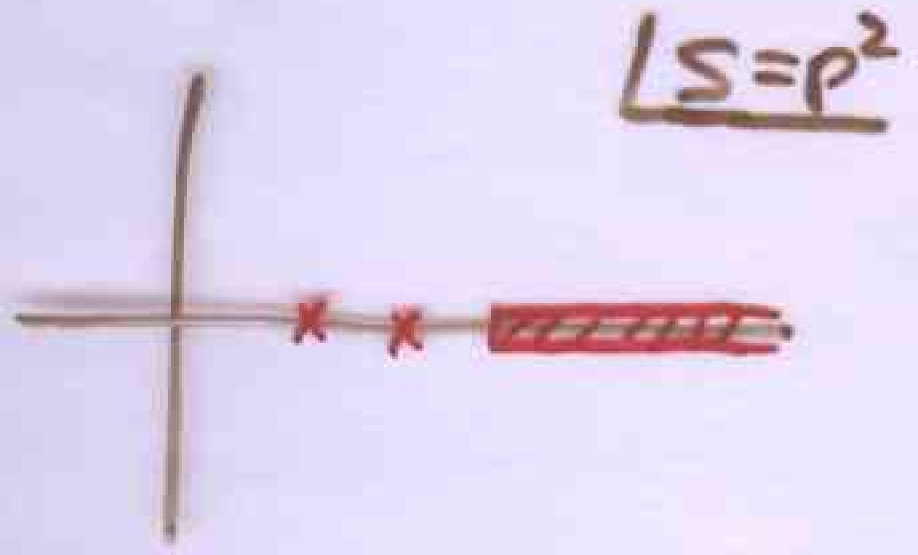
Two-point functions, energy spectra of operators:

We can use the prescription

$$W[\phi_0] = \log \left\langle e^{\int d^4x\, \phi_0 \mathcal{O}} \right\rangle_{CFT}$$

$$= \underset{\phi \xrightarrow[\partial AdS]{} \phi_0}{\text{extremum}} \; S[\phi]$$

to compute a Green's function $\langle \mathcal{O}\mathcal{O} \rangle$.



$$\Pi(s) = \int d^4x \; e^{ip\cdot x} \langle \mathcal{O}(x)\, \mathcal{O}(0) \rangle$$

is analytic in the complex s-plane, except perhaps along the real axis, and

$$\mathcal{D}\mathit{isc}\, \Pi(s) = \Pi(s+i\varepsilon) - \Pi(s-i\varepsilon)$$

for $s \in \mathbb{R}$ tells you the spectral measure of energies of states $\mathcal{O}(x)$ can create from $|0\rangle$.

This spectrum is precisely that of the linearized equation for ϕ in the bulk.

Example: the dilaton s-wave

ϕ is coupled to $\mathcal{O}_4 = \mathrm{tr}(F^2 + \bar{\Psi}\not{D}\Psi + X\Box X + \ldots)$

$$= (Q^4 + \bar{Q}^4)\,\mathrm{tr}\,X^2$$

and its 5-dim wave equation is $\Box\phi = 0$.

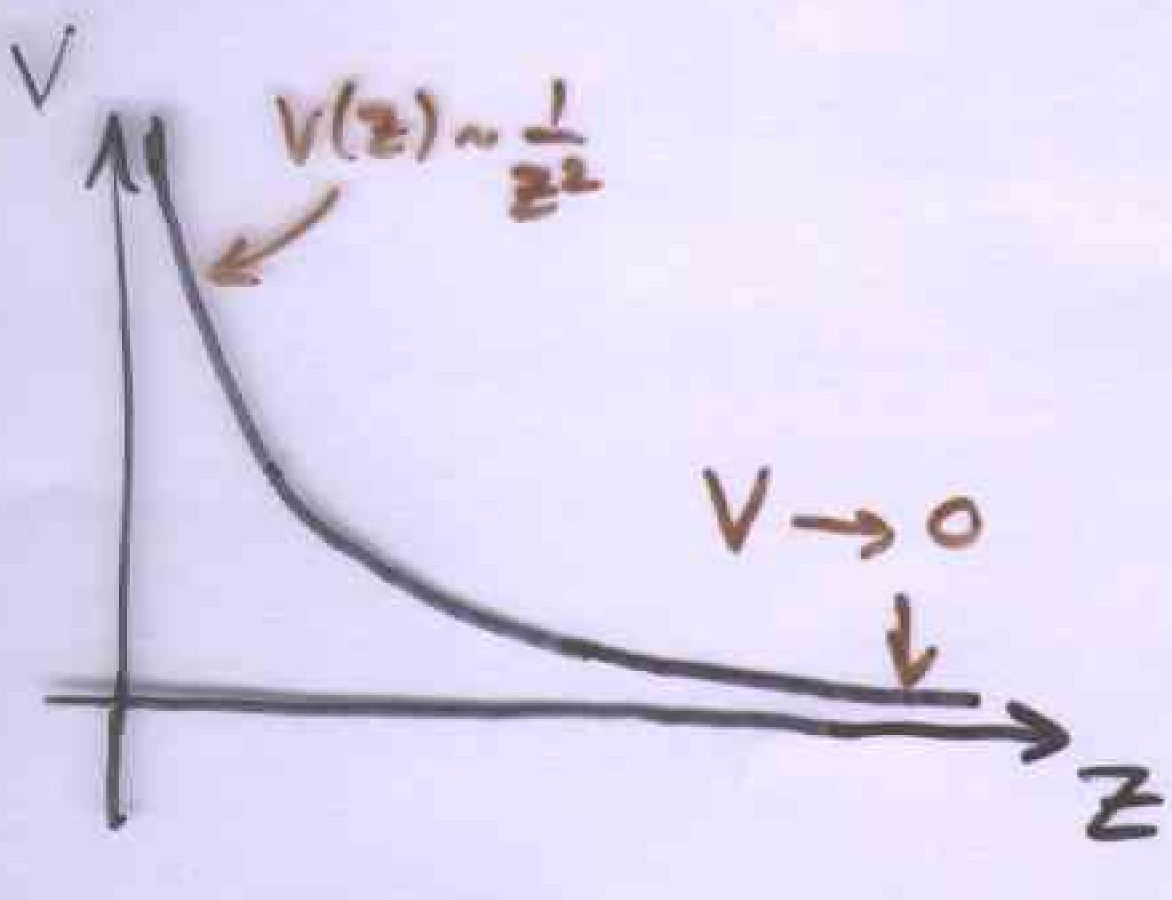
Solve via separation of variables:

$$\phi = e^{-ip\cdot x}\, e^{-3A(z)/2}\, R(z)$$

$$\left[-\partial_z^2 + V(z)\right] R(z) = p^2 R(z)$$

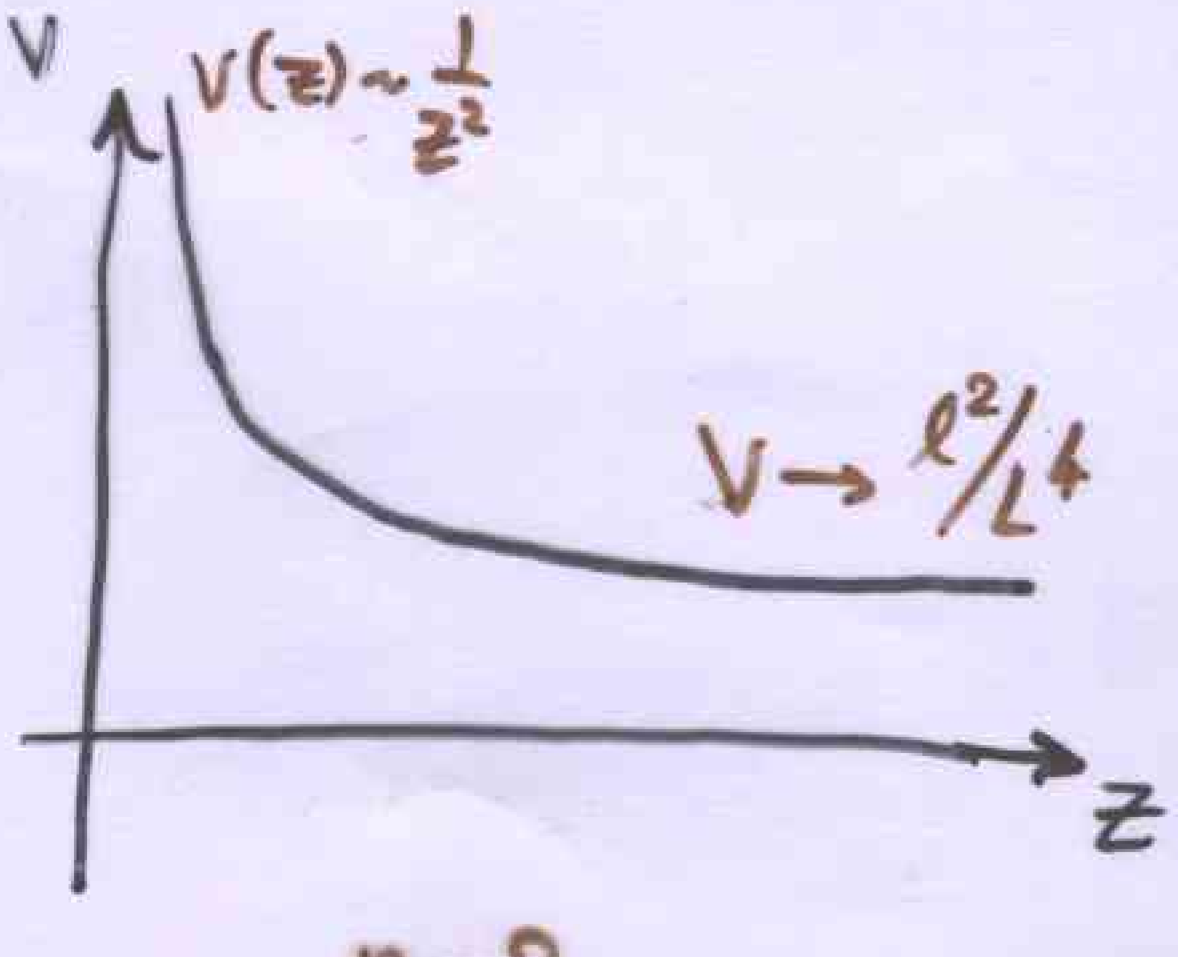
$$V(z) = \frac{3}{2} A''(z) + \frac{9}{4} A'(z)^2$$

We're looking for the spectrum of possible p^2.



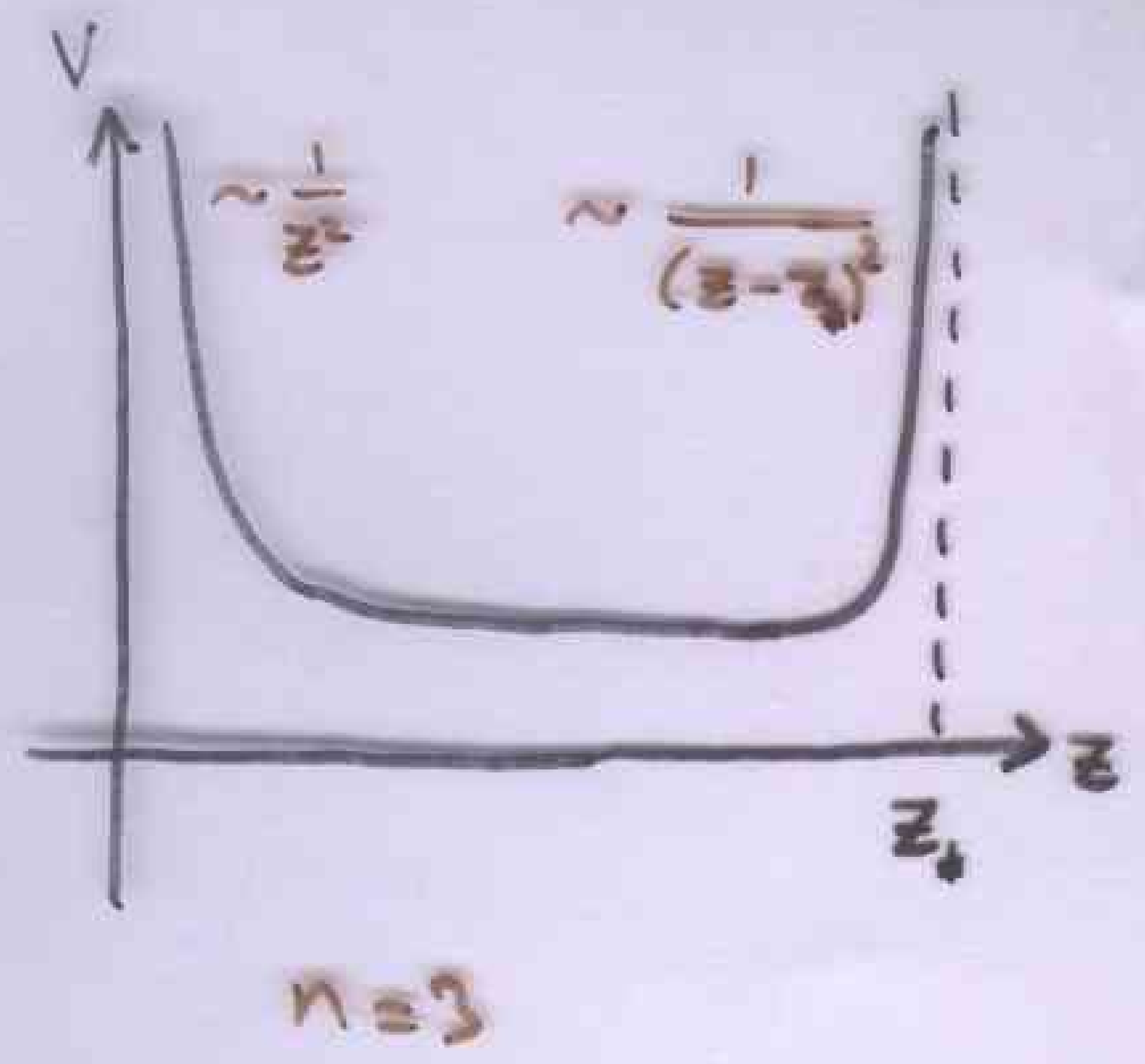
$n = 0, 1$

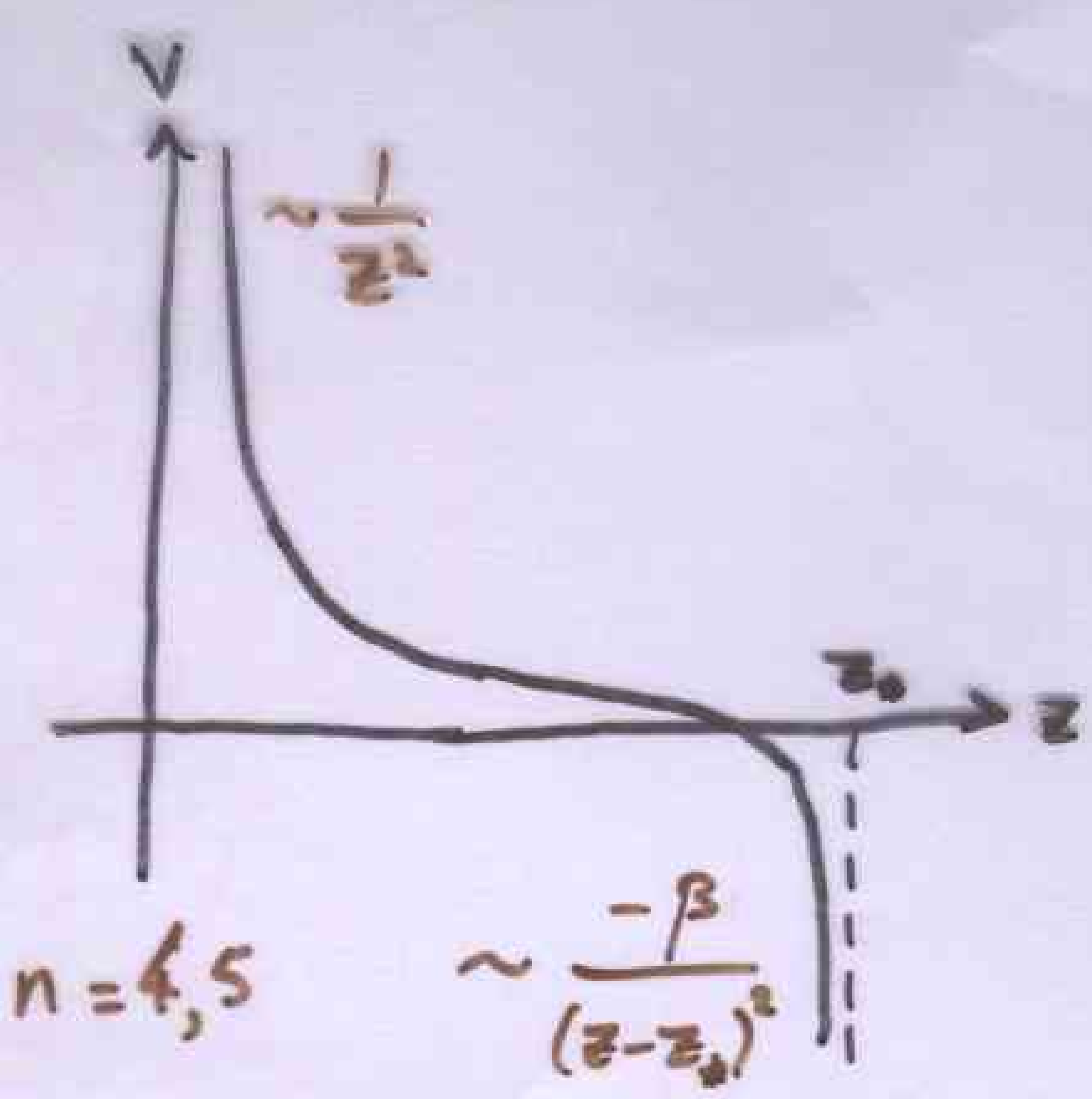
spectrum: $(0, \infty)$



$n = 2$

spectrum: $\left(\frac{\ell^2}{L^4}, \infty\right)$





Spectrum: discrete and positive

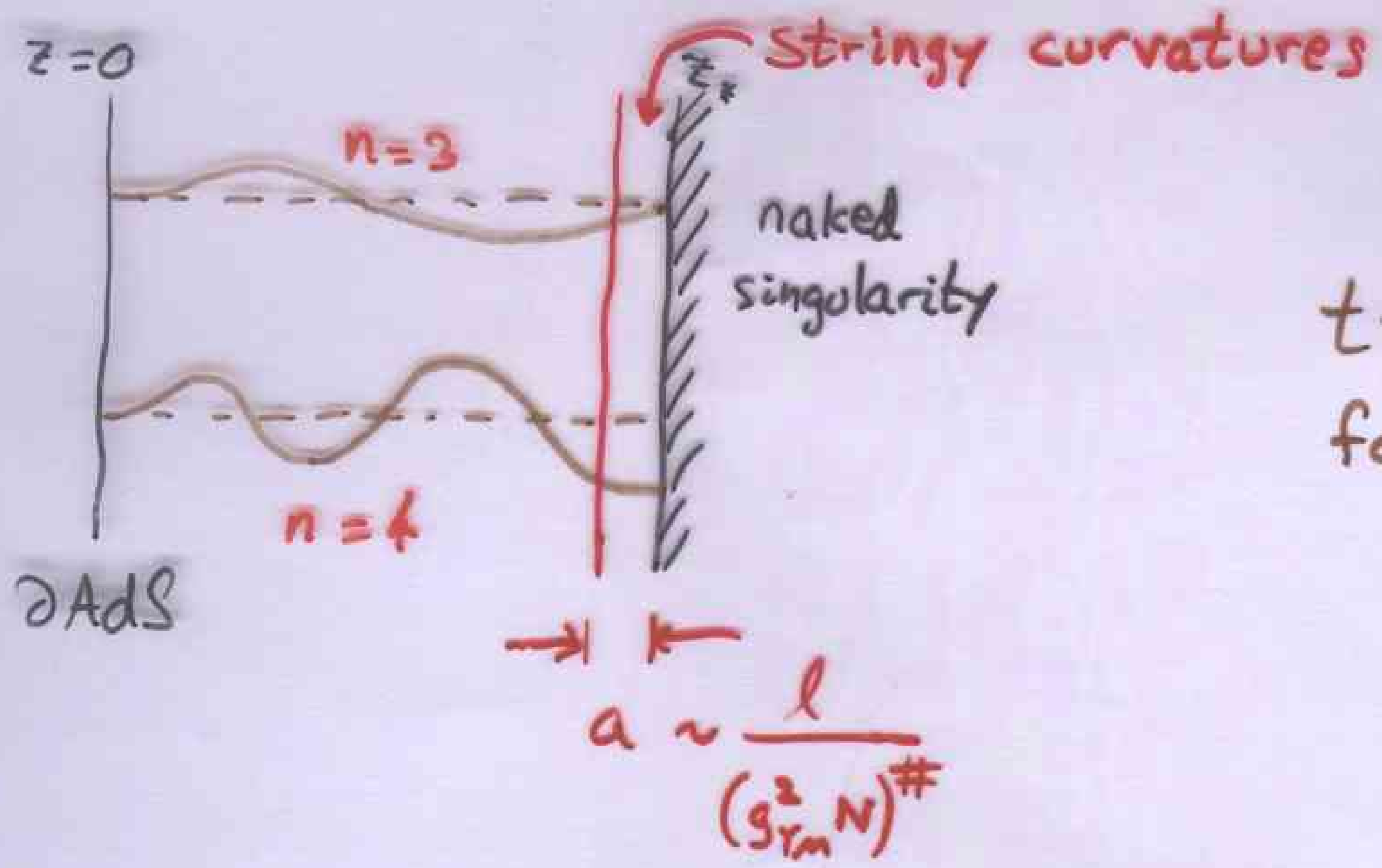
spectrum: depends on β!

For $n=4$, $\beta=\frac{1}{4}$, and kinetic energy keeps spectrum positive (and discrete): $m^2 = \frac{4\ell^2}{L^4} j(j+1)$ for $j=1,2,3,4,\ldots$

For $n=5$, $\beta=\frac{3}{4}$, and potential energy wins: spectrum is unbounded below.

Discrete spectra are very weird for a superconformal gauge theory on its Coulomb branch!

Do we trust supergravity?

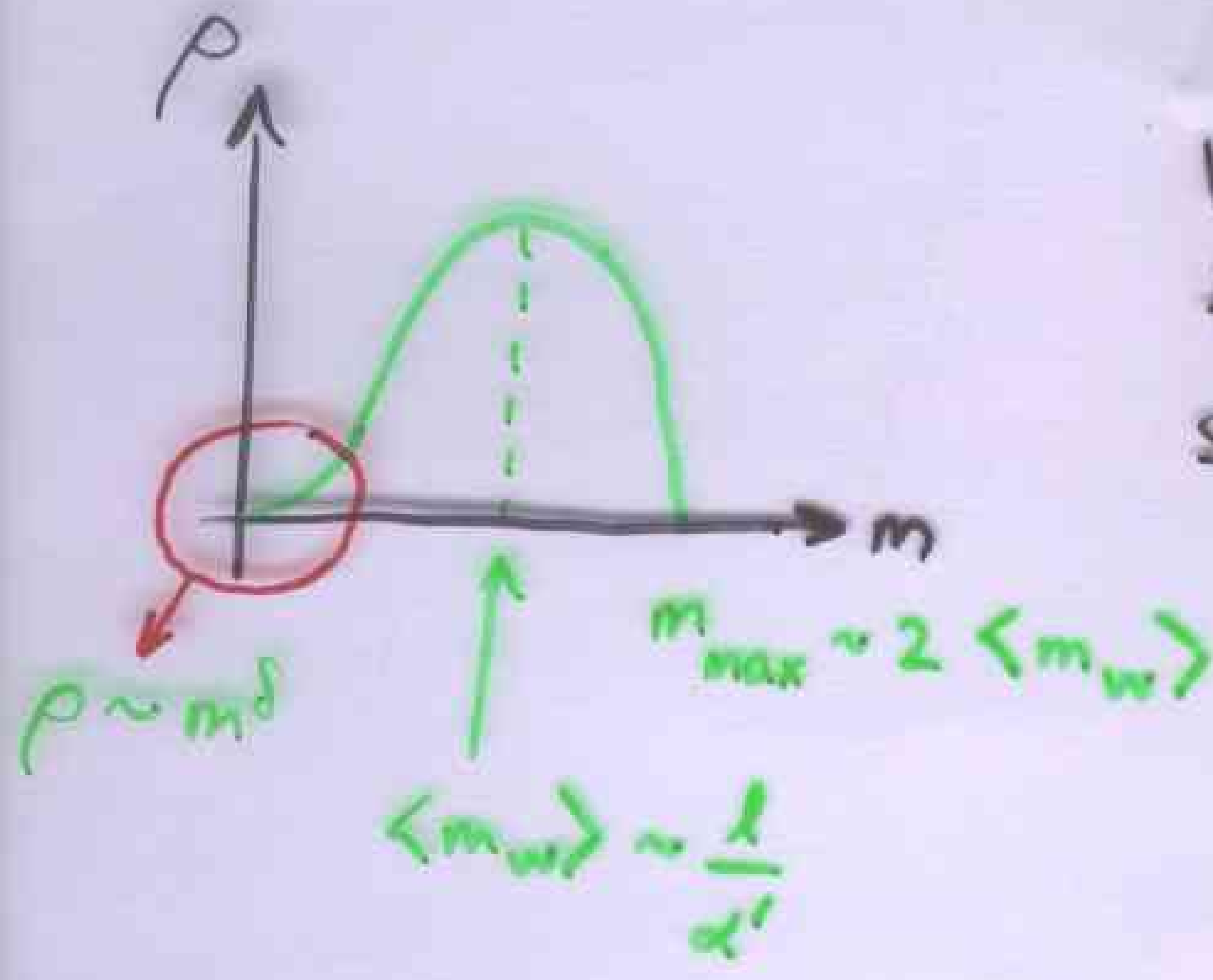


typical eigenfunctions for discrete energies

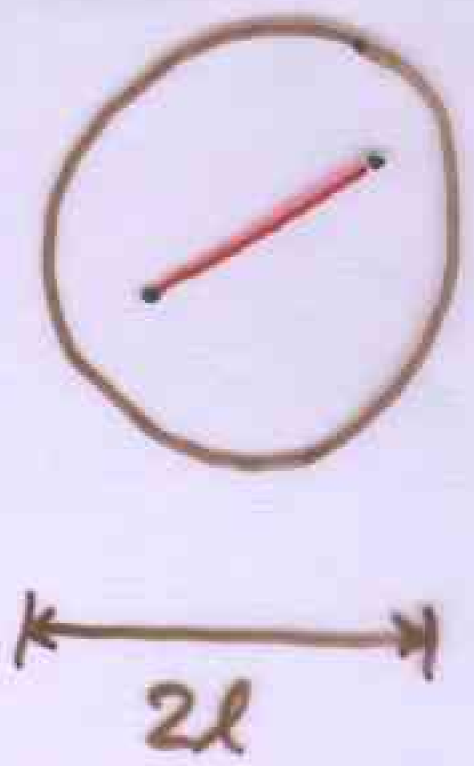
Supergravity is a bad description in the region of stringy curvatures near the singularity. But this region can be made as small as we wish by taking $g_{YM}^2 N$ large.

Presumably a low-energy gauge theory takes over in this region (includes massless $U(1)^N$), and discrete energy states can slowly decay.

BPS mass spectrum:



W-bosons come from stetched strings: BPS

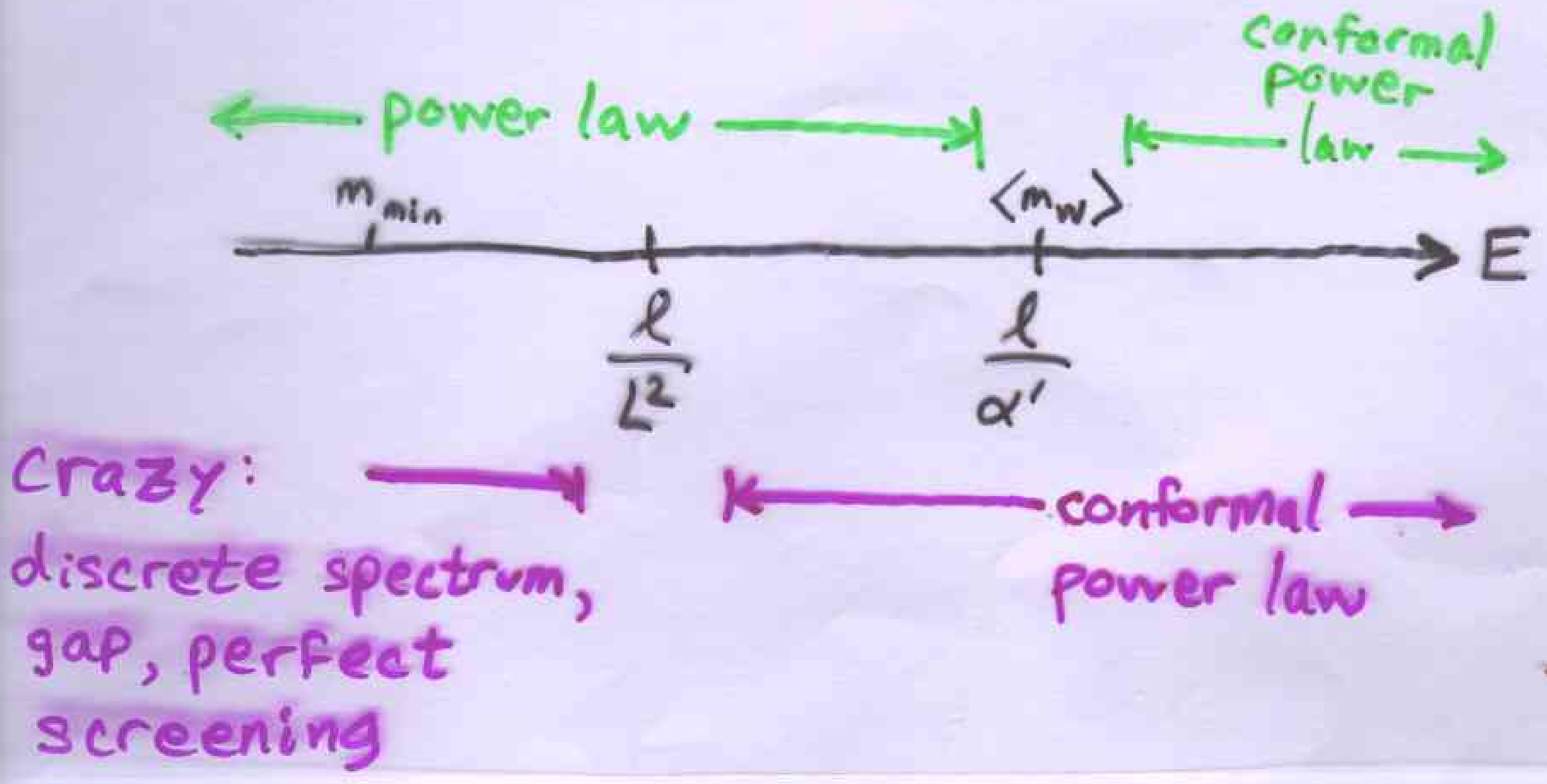


In perturbative gauge theory, expect something like this:

$$\mathrm{Disc}\,\Pi_4(s) \approx N^2 s^2 .$$

$$\int_0^{\sqrt{s}/2} dm\, \rho(m) \sqrt{1 - \frac{4m^2}{s}}$$

Upshot:



perturbative gauge theory: $g_{YM}^2 N \ll 1$

SUGRA: $1 \ll g_{YM}^2 N \ll N$

$$\frac{\ell}{L^2} = \frac{\ell/\alpha'}{\sqrt{g_{YM}^2 N}}$$

What's going on ?

Non-renormalization theorems for 

don't apply out on the Coulomb branch, so it is conceivable that the mass gap etc. are novel leading $O(N)$, leading $O(g_{YM}^2 N)$ effects in SUSY gauge dynamics. A stretch ... why the sensitivity to n?

Ensemble of brane distributions:

A p-dimensional distribution of N branes:
Branes are allowed to "wiggle" by a distance $\sim \frac{\ell}{N^{1/p}} \sim \begin{pmatrix}\text{nearest neighbor}\\ \text{separation}\end{pmatrix}$

$$Z_\sigma = \int_{\delta M} \mathcal{D}\lambda_i \int \mathcal{D}X\, \mathcal{D}\psi\, \mathcal{D}A_\mu\; e^{-S_{N=4}(X,\psi,A)}$$

What happens if we do $\int_{\delta M}$ first?

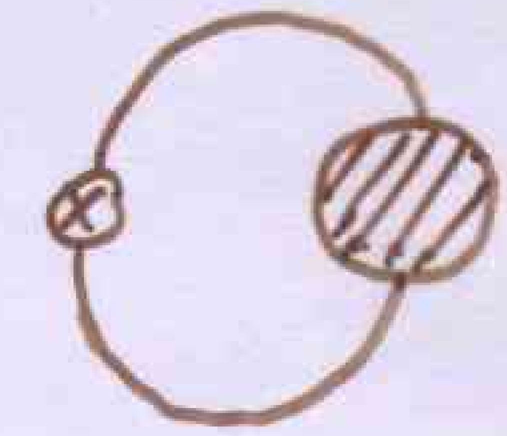
Remember $O(N)$ vector model: $\mathcal{L} = \frac{1}{2}(\partial\vec{\phi})^2 + \frac{1}{2}m^2\vec{\phi}^2 + \frac{\lambda}{4}(\vec{\phi}^2)^2$
could be obtained from $\mathcal{L} = \frac{1}{2}(\partial\vec{\phi})^2 + \frac{1}{2}m^2\vec{\phi}^2$
through some $\int dm^2$

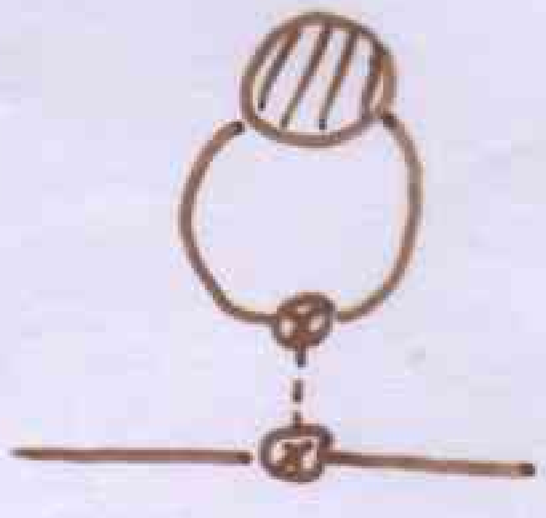
The λ_i control the Higgs masses $\propto \lambda_i - \lambda_j$ for the fields X, Ψ, A, so expect something similar:

$$\mathcal{L} = \underbrace{\mathrm{tr}\left((\partial X_i)^2 + [X_i, X_j]^2\right)}_{\mathcal{L}_{\mathcal{N}=4}} + \underbrace{\lambda\left(\mathrm{tr}\left(X_1^2 + X_2^2 + X_3^2 + X_4^2 - 2X_5^2 - 2X_6^2\right)\right)^2}_{\lambda \mathcal{O}_{20'}^2} + \ldots$$

where the size of λ reflects the amount of "disorder" in the brane distribution.

$\lambda \mathcal{O}_{20'}^2$ could be good, because it induces color-independent mass corrections in loops:

$\langle \mathcal{O}_{20'} \rangle =$ 

 $=$  $\mp$  $+ \cdots$

$$\frac{1}{p^2 + m_{ab}^2} = \frac{1}{p^2 + m_{(0)ab}^2} \mp \frac{\lambda \langle \mathcal{O}_{20'} \rangle}{(p^2 + m_{(0)ab}^2)(p^2 + m_{ab}^2)} + \ldots$$

$$m_{ab}^2 = m_{(0)ab}^2 \pm \lambda \langle \mathcal{O}_{20'} \rangle + \ldots$$

and $\langle \mathcal{O}_{20'} \rangle \sim \rho^2$

Conclusions

- $C(r) = \frac{1}{A'(r)^{D-2}}$ is a guide to # of degrees of freedom at large N.

  Radius $\longleftrightarrow$ energy relation has an ambiguity comparable to scheme dependence.

- The ½ SUSY 5-d flows of N. Warner's talk lift to parallel D-brane distributions via "consistent oxidation." Coulomb branch of $\mathcal{N}=4$.

- Two-point function calculations in supergravity reveal surprising energy spectra, for $1 \ll g_{YM}^2 N \ll N$. Interpretation?