

Zero Energy States
in

Supersymmetric
Matrix Models

$$H = -\vec{\nabla}_1^2 - \vec{\nabla}_2^2 - \vec{\nabla}_3^2 + (\vec{q}_1 \times \vec{q}_2)^2 \\ + (\vec{q}_2 \times \vec{q}_3)^2 + (\vec{q}_1 \times \vec{q}_3)^2$$

$$H = \text{Tr} \left(P_1^2 + \dots + P_d^2 - \sum_{i < j} [X_i, X_j]^2 \right)$$

Goldstone, H.
1980

SU(2)

$$H = -\vec{\nabla}_1^2 - \dots - \vec{\nabla}_d^2 + \sum_{i < j} (\vec{t}_i \times \vec{t}_j)^2$$

SO(d)

$$\begin{aligned}
 \tilde{H} &= -\partial_1^2 - \partial_2^2 - \partial_3^2 + \overbrace{x_1 x_2 + x_2 x_3 + x_3 x_1}^{\checkmark} \\
 &\quad - \sum_{j < k} \frac{x_j^2 + x_k^2}{(x_j^2 - x_k^2)^2} \\
 &\quad + \frac{1}{4} (d-3)(d-5) \left(\frac{1}{x_1^2} + \frac{1}{x_2^2} + \frac{1}{x_3^2} \right)
 \end{aligned}$$

$$x_3 \geq x_2 \geq x_1 \geq 0$$

$$\int d^3x |\tilde{\psi}|^2 < \infty$$

$$\begin{aligned}
 \tilde{H}_{3,5} &\underbrace{\sqrt{(x_3^2 - x_2^2)(x_3^2 - x_1^2)(x_2^2 - x_1^2)}}_{=} e^{-x_1 x_2 x_3} = 0 \\
 &= \tilde{\psi} = \rho \psi
 \end{aligned}$$

$$H = -\frac{1}{\rho} \vec{\nabla} \rho \vec{\nabla} + V$$

$$\int d^3x \rho(x) |\psi|^2 < \infty$$

$$H e^{-x_1 x_2 x_3} = 0$$

$$\# e^{\pm \vec{r}_i \cdot (\vec{r}_2 \times \vec{r}_3)} = 0$$

$$\# e^{-x_1 x_2 x_3} = 0$$

$$\# e^{-|\vec{r}_i \cdot (\vec{r}_2 \times \vec{r}_3)|}$$

$$= 2V \delta(x) e^{-|x|}$$

$$\# e^{-|\vec{r}_i \cdot (\vec{r}_2 \times \vec{r}_3)|} = 0$$

$$H = p_{tA} p_{tA} + \frac{1}{2} f_{ABC} f_{ABC'} q_{sB} q_{tc} q_{sB'} q_{tc'}$$

$$+ i q_{tc} f_{ABC} \gamma_{\alpha\beta}^t \theta_{\alpha A} \theta_{\beta B}$$

$s, t = 1, \dots, d = 2, 3, 5, 9$
 $\alpha, \beta = 1, \dots, s_d = 2, 4, 8, 16$

SU(N)

$$\gamma_{\alpha\beta}^t \gamma_{\alpha\beta}^s + \gamma_{\alpha\beta}^s \gamma_{\alpha\beta}^t = 2 \delta^{st} \mathbb{1}_{s_d \times s_d}$$

$$[q_{tA}, p_{sB}] = i \delta_{AB} \delta_{st}, \quad \{\theta_{\alpha A}, \theta_{\beta B}\} = \delta_{\alpha\beta} \delta_{AB}$$

$$J_A = f_{ABC} (q_{sB} p_{sC} - \frac{i}{2} \theta_{\alpha B} \theta_{\alpha C})$$

$$J_{st} = q_{sA} p_{tA} - q_{tA} p_{sA} - \frac{i}{4} \theta_{\alpha A} \gamma_{\alpha\beta}^{st} \theta_{\beta A}$$

$$Q_{\alpha} = (p_{tA} \gamma_{\alpha\beta}^t + \frac{1}{2} f_{ABC} q_{sB} q_{tc} \gamma_{\alpha\beta}^{st}) \theta_{\beta A}$$

$$\{Q_{\alpha}, Q_{\beta}\} = \delta_{\alpha\beta} H + 2 \gamma_{\alpha\beta}^t q_{tA} J_A$$

$$J_A \psi = 0$$

$$H\psi = 0$$

$$H = -\Delta + V + H_F$$

$$\langle -\Delta \rangle = \langle V \rangle$$

$$0 = \langle \psi, [\vec{x} \cdot \vec{\nabla}, -\Delta + V + H_F] \psi \rangle$$

$$= \langle \psi, (2\Delta + 4V + H_F) \psi \rangle$$

$$= \langle \psi, (3\Delta + 3V) \psi \rangle$$

$$\langle \Delta + V \rangle = 0 \quad !$$

$$-\partial_x^2 - \partial_y^2 - \frac{x^2 + y^2}{(x^2 - y^2)^2}$$

$$x \geq y \geq 0$$

$$= -\partial_r^2 - \frac{1}{r} \partial_r + \frac{1}{r^2} \left(-4\partial_\theta^2 - \frac{1}{\cos^2 \theta} \right)$$

$$\theta = 2\varphi \in [0, \pi/2]$$

$$\ln \sqrt{\cos \theta} = (+1) \sqrt{\cos \theta}$$

$$\left\langle \frac{-1}{\cos \theta} \right\rangle = -\infty \quad \left\langle \left(\sqrt{\cos \theta} \right)^2 \right\rangle = +\infty$$

$$\langle \quad \rangle + \langle \quad \rangle = -1$$

$$\ln u = u$$

$$\sqrt{\cos \theta} \ln \frac{1 + \sin \theta}{1 - \sin \theta}$$

$$\langle \quad \rangle + \langle \quad \rangle = \infty$$

SU(2)

$$V=0: \vec{g}_1 \parallel \vec{g}_2 \parallel \dots \parallel \vec{g}_d \parallel \vec{e}$$

$$\vec{g}_s = r \vec{e} E_s + \vec{\gamma}_s$$

$$\Psi \underset{r \rightarrow \infty}{\sim} r^{-\alpha} e^{-r\gamma/2} |F^\perp\rangle \cdot |F''\rangle$$

[BCS] $\vec{\theta} \cdot \vec{e}$

| d | α |
|---|----------|
| 2 | 1 |
| 3 | 0 |
| 5 | -1 |
| 9 | 6 |

Fröhlich, H

Yan, H.

Haldane, Schwartz
Graf, H

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$$Q_{\hat{\alpha}} = Q_{\alpha}^{\dagger} \hat{=} Q_{\alpha}, Q_{\alpha}^{\dagger}$$

1...16
1...8
1...8

$$\{Q_{\alpha}, Q_{\beta}^{\dagger}\} = \delta_{\alpha\beta} H + 2 \Gamma_{\alpha\beta}^j g_{jA} J_A$$

$$\{Q_{\alpha}, Q_{\beta}\} = 2i \delta_{\alpha\beta} \bar{E}_A J_A$$

$$g_{SA} \rightarrow g_{jA} (j=1, \dots, d-2), g_{SA} \pm i g_{SA}$$

de Wit, Nicolai, H

$$J_E = -i f_{EAA'} (g_{jA} \partial_{jA'} + E_A \partial_{A'} + \bar{E}_A \partial_{A'} + \lambda_{\alpha A} \partial_{\lambda_{\alpha A'}})$$

$$\Psi \in \mathcal{X}_+ = \Psi + \frac{1}{2} \underbrace{\Psi_{\alpha A, \beta B}}_{\Psi_2} \lambda_{\alpha A} \lambda_{\beta B} + \dots \Psi^{(n)}$$

$$Q_\beta = M_{\alpha A}^{(\beta)} \lambda_{\alpha A} + D_{\alpha A}^{(\beta)} \frac{\partial}{\partial \lambda_{\alpha A}} = M \cdot \lambda + D \cdot \partial_\lambda$$

$$Q_\beta^\dagger = M_{\alpha a}^\dagger \partial_{\lambda_{\alpha a}} + D_{\alpha a}^\dagger \lambda_{\alpha a} = M^\dagger \cdot \partial_\lambda + D^\dagger \cdot \lambda$$

$$Q\Psi = 0 : D_{a_{2k}} \Psi_{[a_1 \dots a_{2k}]} = (2k-1) M_{[a_1 \dots a_{2k-1}]} \Psi$$

$$Q^\dagger \Psi = 0 : M_{a_{2k}}^\dagger \Psi_{[a_1 \dots a_{2k}]} = (2k-1) D_{[a_1}^\dagger \Psi_{[a_2 \dots a_{2k-1}]}$$

$$Q\chi = 0 : \chi_{2k} = \chi_{2k}^{[L]} \pm \chi_{2k}^{[rim]}$$

$$Q^\dagger \Psi = 0 : \Psi_{2k} = \Psi_{2k}^{(L)} \pm \Psi_{2k}^{(rim)}$$

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