Zero Energy States

in

Supersymmetric Matrix Models

$$H = -\vec{\nabla}_1^2 - \vec{\nabla}_2^2 - \vec{\nabla}_3^2 + \left(\vec{\underset{t}{q}}^1 \times \vec{\underset{t}{q}}^2\right)^2$$

$$+ \left(\vec{\underset{t}{q}}^2 \times \vec{\underset{t}{q}}^3\right)^2 + \left(\vec{\underset{t}{q}}^1 \times \vec{\underset{t}{q}}^3\right)^2$$

$$H = \mathrm{Tr}\left(P_1^2 + \ldots + P_d^2 - \sum_{i<j} [X_i, X_j]^2\right)$$

Goldstone, H.

1980

$SU(2)$

$$H = -\vec{\nabla}_1^2 - \ldots - \vec{\nabla}_d^2 + \sum_{i<j} \left(\vec{\underset{t}{q}}^i \times \vec{\underset{t}{q}}^j\right)^2$$

$SO(d)$

$$\tilde{H}_{d\geq 3} = -\partial_1^2 - \partial_2^2 - \partial_3^2 + \overbrace{x_1^2 x_3^2 + x_2^2 x_3^2 + x_3^2 x_1^2}^{V}$$

$$- \sum_{j<k} \frac{x_j^2 + x_k^2}{(x_j^2 - x_k^2)^2}$$

$$+ \frac{1}{4}(d-3)(d-5)\left(\frac{1}{x_1^2} + \frac{1}{x_2^2} + \frac{1}{x_3^2}\right)$$

$x_3 \geq x_2 \geq x_1 \geq 0$ $\quad$ $\int d^3x\, |\tilde{\psi}|^2 < \infty$

$$\tilde{H}_{3,5} \underbrace{\sqrt{(x_3^2 - x_2^2)(x_3^2 - x_1^2)(x_2^2 - x_1^2)}\, e^{-x_1 x_2 x_3}}_{= \tilde{\psi} = \rho_3 \psi} \equiv 0$$

$$H = -\frac{1}{\rho} \vec{\nabla} \rho \vec{\nabla} + V$$

$$\int d^3x\, \rho(x) |\psi|^2 < \infty$$

$$H e^{-x_1 x_2 x_3} = 0$$

$$H\, e^{\pm \vec{e}_1\cdot(\vec{e}_2\times\vec{e}_3)} = 0$$

$$H\, \underbrace{e^{-x_1 x_2 x_3}}_{\shortparallel} = 0$$

$$H\, e^{-|\vec{e}_1\cdot(\vec{e}_2\times\vec{e}_3)|}$$

$$= 2V\delta(x)\, e^{-|x|}$$

$$H'\, e^{-|\vec{e}_1\cdot(\vec{e}_2\times\vec{e}_3)|} = 0$$

9609232

$$H = p_{tA} p_{tA} + \frac{1}{2} f_{ABC} f_{AB'C'} q_{sB} q_{tC} q_{sB'} q_{tC'}$$

$$+ i\, q_{tC} f_{ABC} \gamma^{t}_{\alpha\beta} \theta_{\alpha A} \theta_{\beta B}$$

$SU(N)$

$s, t = 1, \dots, d = 2, 3, 5, 9$

$\alpha, \beta = 1, \dots, s_d = 2, 4, 8, 16$

$\gamma^t \gamma^s + \gamma^s \gamma^t = 2 \delta^{st} \mathbb{1}_{s_d \times s_d}$

$$[q_{tA}, p_{sB}] = i \delta_{AB} \delta_{st}, \quad \{\theta_{\alpha A}, \theta_{\beta B}\} = \delta_{\alpha\beta} \delta_{AB}$$

$$J_A = f_{ABC} \left(q_{sB} p_{sC} - \frac{i}{2} \theta_{\alpha B} \theta_{\alpha C} \right)$$

$$J_{st} = q_{sA} p_{tA} - q_{tA} p_{sA} - \frac{i}{4} \theta_{\alpha A} \gamma^{st}_{\alpha\beta} \theta_{\beta A}$$

$$Q_\alpha = \left(p_{tA} \gamma^t_{\alpha\beta} + \frac{1}{2} f_{ABC} q_{sB} q_{tC} \gamma^{st}_{\alpha\beta} \right) \theta_{\beta A}$$

$$\{Q_\alpha, Q_\beta\} = \delta_{\alpha\beta} H + 2 \gamma^t_{\alpha\beta} q_{tA} J_A$$

$$J_A \psi \overset{!}{=} 0$$

$$H\psi = 0$$

$$H = -\Delta + V + H_F$$

$$\text{"}\langle -\Delta \rangle = \langle V \rangle\text{"}$$

$$0 = \langle \psi, [\vec{x}\cdot\vec{\nabla}, -\Delta + V + H_F]\psi \rangle$$

$$= \langle \psi, (2\Delta + 4V + H_F)\psi \rangle$$

$$= \langle \psi, (3\Delta + 3V)\psi \rangle$$

$$\langle \Delta + V \rangle = 0 \quad !$$

$$-\partial_x^2 - \partial_y^2 - \frac{x^2+y^2}{(x^2-y^2)^2} \qquad x \geq y \geq 0$$

$$= -\partial_r^2 - \frac{1}{r}\partial_r + \frac{1}{r^2}\left(-4\partial_\theta^2 - \frac{1}{\cos^2\theta}\right)$$

$$\theta = 2\varphi \in [0, \pi/2]$$

$$h\sqrt{\cos\theta} = (+1)\sqrt{\cos\theta}$$

$$\left\langle \frac{-1}{\cos^2\theta} \right\rangle = -\infty \qquad \left\langle \left(\partial\sqrt{\cos\theta}\right)^2 \right\rangle = +\infty$$

$$\text{"}\langle\quad\rangle + \langle\quad\rangle\text{"} = -1$$

$$h\,u = u$$

$$\uparrow$$

$$\sqrt{\cos\theta}\,\ln\frac{1+\sin\theta}{1-\sin\theta}$$

$$\text{"}\langle\quad\rangle + \langle\quad\rangle\text{"} = \infty$$

SU(2)

$$V=0: \vec{q}_{t_1} \parallel \vec{q}_{t_2} \parallel \ldots \parallel \vec{q}_{t_d} \parallel \vec{e}$$

$$\vec{q}_{t_s} = r\vec{e}\,E_s + \vec{y}_s$$

$$\Psi \underset{r\to\infty}{\sim} r^{-\varkappa} e^{-r y^2/2} \, |F^{\perp}\rangle \cdot |F^{\parallel}\rangle$$

[BCS] $\quad \vec{\theta}_\alpha \cdot \vec{e}$

d	$\varkappa$
2	/
3	0
5	-1
9	6

Fröhlich, H

Yau, H.

Halpern, Schwartz

Graf, H

Smilga

hep-th/9904182

$$Q_{\hat{\alpha}} = Q_{\alpha}^{\dagger} \;\hat{=}\; Q_{\alpha} \;,\; Q_{\alpha}^{\dagger}$$

1...16   1...8   1...8

$$\{Q_\alpha, Q_\beta^\dagger\} = \delta_{\alpha\beta} H + 2\Gamma^{j}_{\alpha\beta}\, \mathfrak{z}_{jA} J_A$$

$$\{Q_\alpha, Q_\beta\} = 2i\,\delta_{\alpha\beta}\, \bar{z}_A J_A$$

$$\mathfrak{z}_{sA} \to \mathfrak{z}_{jA}\ (j=1,\dots,d-2),\ \mathfrak{z}_{8A} \pm i\,\mathfrak{z}_{9A}$$

de Wit, Nicolai, H

$$J_E = -i f_{EAA'}\big(\mathfrak{z}_{jA}\partial_{jA'} + z_A \partial_{A'} + \bar{z}_A \partial_{A'} + \lambda_{\alpha A}\partial_{\lambda_{\alpha A'}}\big)$$

$$\Psi = \psi + \tfrac{1}{2}\underbrace{\psi_{\alpha A,\beta B}}_{\psi_2}\lambda_{\alpha A}\lambda_{\beta B} + \dots \psi^{(n)}_{\dots}$$

$\in \mathcal{H}_+$

$$Q_\beta = M^{(\beta)}_{\underbrace{\alpha A}_{a}} \lambda_{\alpha A} + \mathcal{D}^{(\beta)}_{\alpha A} \frac{\partial}{\partial \lambda_{\alpha A}} = M\cdot\lambda + \mathcal{D}\cdot\partial_\lambda$$

$$Q^\dagger_\beta = M^\dagger_a \partial_{\lambda_a} + \mathcal{D}^\dagger_a \lambda_a = M^\dagger\cdot\partial_\lambda + \mathcal{D}^\dagger\cdot\lambda$$

$$Q\Psi = 0 : \quad \mathcal{D}_{a_{2k}} \psi_{a_1\ldots a_{2k}} = (2k-1) M_{[a_1} \psi_{a_2\ldots a_{2k-1}]}$$

$$Q^\dagger\Psi = 0 : \quad M^\dagger_{a_{2k}} \psi_{a_1\ldots a_{2k}} = (2k-1) \mathcal{D}^\dagger_{[a_1} \psi_{a_2\ldots a_{2k-1}]}$$

$$Q\chi = 0 : \quad \chi_{2k} = \chi^{[\mathrm{r}]}_{2k} \oplus \chi^{[\mathrm{im}]}_{2k}$$

$$Q^\dagger\Psi = 0 : \quad \psi_{2k} = \psi^{(\mathrm{r})}_{2k} \oplus \psi^{(\mathrm{im})}_{2k}$$

hep-th/9709132 / 9709217 / 9711033