M-Theory, Mach's Principle & Tachyon Condensation on Branes

Petr Hořava

Strings '99, Potsdam

Based on:

hep-th/9812135, and to appear;

also:

O. Bergman, E. Gimon & P.H., hep-th/9902160.

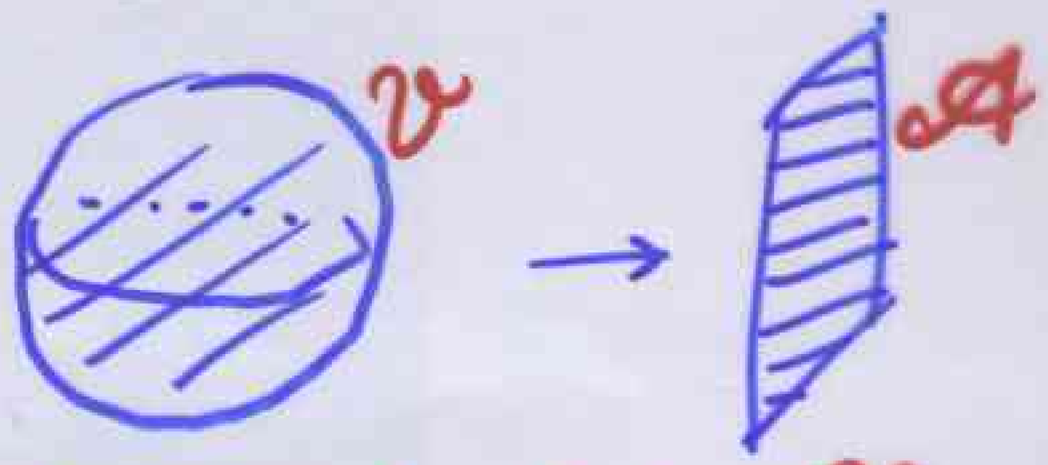
A. Sen,
E. Witten,
...

Basic question:

What is M-theory?

"MICROSCOPIC" M-THEORY:

- 11d supergravity as a low-energy limit near flat eleven-dimensional super Poincaré vacuum;
  (apparent locality at low energies)
- 11d covariance ? background independence ?
  nice, but optional
- M-theory is supposed to be (as an extension of string theory) a consistent quantum theory of gravity; therefore, its d.o.f. should satisfy the
  holographic principle ← 't Hooft Susskind
  (manifestly ?)



\# of phys. d.o.f. in $\mathcal{V}$ should scale like $\mathcal{A}$, with finite density in Planck units

(In my opinion,)

$\boxed{\text{We still do not have a satisfactory formulation of the theory that satisfies these criteria.}}$

however:
- Matrix thy reproduces sugra, supposedly holographic
- AdS/CFT a working example of holography in non-flat spacetime

PHENOMENOLOGICAL APPROACH TO HOLOGRAPHY

hep-th/9712130

Perhaps M-theory is a local gauge field theory?

(= enough gauge invariance to lead to the holographic reduction of physical degrees of freedom, despite locality)

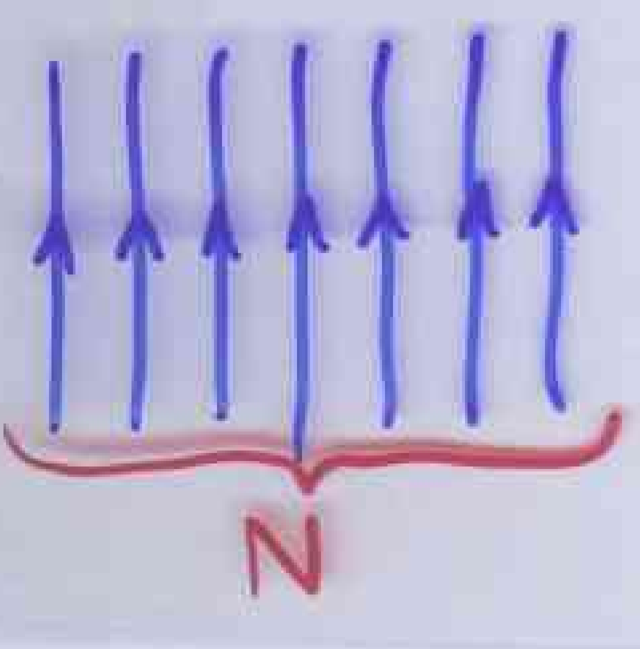
<u>idea</u>: Write down a local Yang-Mills theory in 11d, with diffeomorphism invariance, whereby the metric (=vielbein) appears as a component of the gauge field:

$$A_\mu = M e_\mu{}^\alpha P_\alpha + \ldots$$

in close analogy with the way 2+1 gravity can be rewritten as a Chern-Simons gauge theory (Witten, 1989). Achúcarro & Townsend;

- <u>The Lagrangian:</u> $\mathcal{L} = \Sigma \int \omega_{11}(A)$ ← Chern-Simons forms
- <u>The gauge group:</u> 11d Anti-de Sitter: $OSp(1|32) \times OSp(1|32)$
- <u>Consequences of this setup:</u>

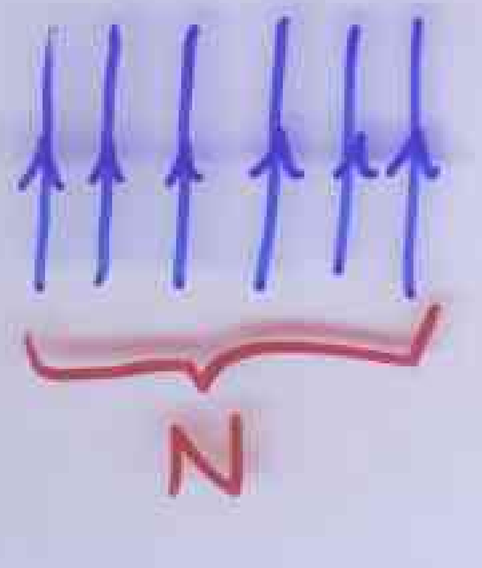
in the presence of mysterious first-quantized partons, and in the mean field approx. for large N,



- supergravity follows at very low energies
- sugra valid until Bekenstein bound reached
- holography ($N \sim A$ in M_{Pl} units)
- cosmological constant naturally small

ALL TIED TO MACH'S PRINCIPLE!

1. One cannot have a macroscopically large spacetime for free:



> Large macroscopic spacetime is generated by a large number of microscopic partons
> (= no geometry without matter distribution)

It is this (Machian) property of the theory that is responsible for:

- holography (in flat space)

$$N = \mathcal{A} \text{ in Planck units;}$$

- naturalness of small cosmological constant

$$\Lambda \sim M_P^2 / R^{d-2} \quad \text{(if non-zero)}$$

2. Another formulation of Mach's principle is valid: $\boxed{\text{inertia (i.e., kinetic term of elem. quanta) generated by distribution of matter (= partons).}}$

another connection to Mach's principle: cf. Einstein; Brans & Dicke

However:

Some aspects of this phenomenological approach still murky,

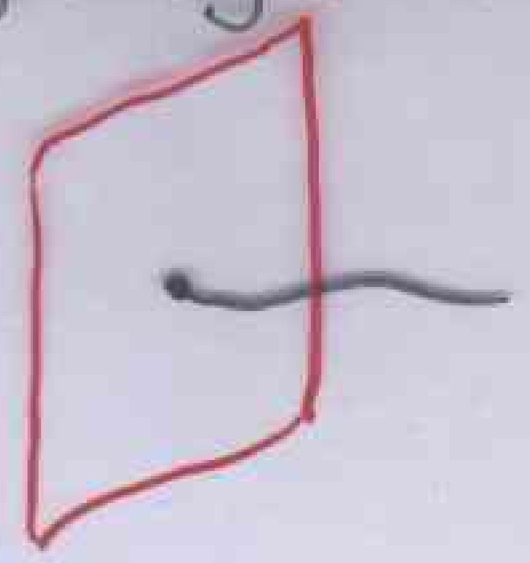
search for a microscopic implementation.

hint: partons to be compared to D0-branes in Matrix theory.

UNSTABLE CONFIGURATIONS OF D-BRANES

D-branes are string-theory solitons on which open strings can end:

Feb. 1989, P.H.
Dai, Leigh, Polchinski



Typically, we require them to preserve some supersymmetry, and to carry a unit of D-brane (= Ramond-Ramond) charge...

This means that D-branes couple to RR fields (which are diff. forms in spacetime); one expects then that D-brane charges are classified by cohomology. (This turns out not to be exactly correct; D-brane charges are classified by K-theory groups, closely related to cohomology)

The open string spectrum leads to worldvolume field theory, containing a $U(N)$ Yang-Mills gauge field for a system of N coincident D-branes. } Chan-Paton group
For D-branes carrying a unit of RR charge, the worldvolume theory is supersymmetric (on $\mathbb{R}^{10}$). $\Rightarrow$ Type IIA: 2p-branes
Type IIB: (2p+1)-branes

We can relax the condition of supersymmetry & the requirement of RR charge; once we do that, we can construct D-branes of any dimension. In particular, we can construct spacetime-filling D9-branes in Type IIA theory.

Such D-branes do not carry RR charge, and are in fact highly unstable: The worldvolume theory contains, in addition to the YM gauge field, a tachyon (coming from the lowest mode of the open string ending with both ends on the same D-brane).

$V(T_0) = -\varepsilon$

Therefore, we expect these unstable, non-supersymmetric excitations of Type II string theory to quickly decay into the susy vacuum. Sen, Witten, ...

CONFIGURATIONS OF 9-BRANE $\overline{9}$-BRANE PAIRS IN TYPE IIB

Consider N 9-branes wrapping a spacetime manifold X, together with N' $\overline{9}$-branes. (One could consider $(2k+1)$-branes wrapping a submanifold $Y \subset X$)

Chan-Paton factors of open strings ending on these branes:
$U(N)$ Yang-Mills symmetry on 9-branes, $U(N')$ YM symm. on $\overline{9}$-branes.

The open strings connecting 9-branes and $\overline{9}$-branes have the opposite GSO projection, and therefore yield a tachyon T instead of a gauge field.

Thus, the bosonic low-energy field content on worldvolume is

$$\begin{pmatrix} A_\mu dx^\mu & T \\ \overline{T} & A'_\mu dx^\mu \end{pmatrix},$$

where A_μ and A'_μ are the YM gauge fields of $U(N) \times U(N')$, and the tachyon T is in $(N, \overline{N'})$.

Now, set $N=N'$. We expect the tachyon to develop a vev, with negative energy density of the condensate that exactly cancels the positive energy density from the tension of the branes; in the process, the state decays into susy vacuum of Type IIB.

However, this annihilation can only occur if the YM gauge bundles of $U(N)$ carried by 9-branes (E) and by $\overline{9}$-branes (F) are exactly the same. Only such configurations can be annihilated or created to/from vacuum.

This defines an equivalence relation on pairs of bundles (E,F):

$$(E_1, F_1) \cong (E_2, F_2) \text{ if } \quad E_1 \oplus H = E_2 \oplus H', \; F_1 \oplus H = F_2 \oplus H' \quad \boxed{\text{defines group } K(X)!}$$

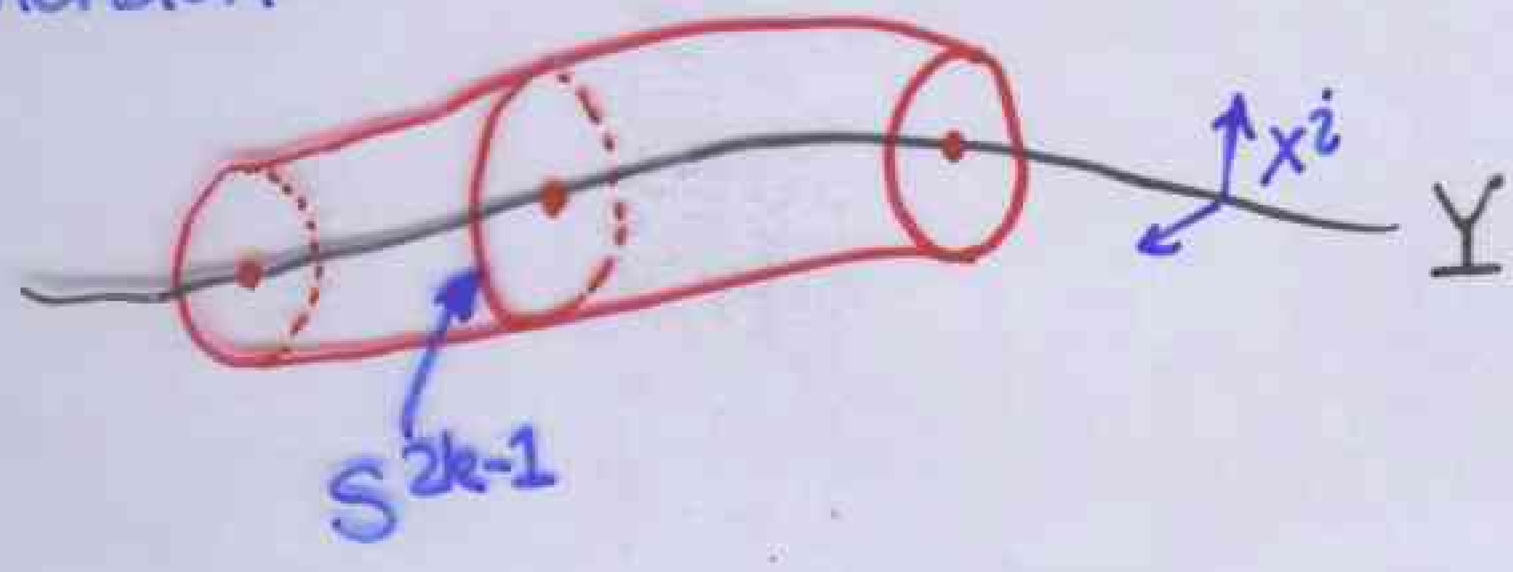
CONSTRUCTION OF ALL TYPE IIB D-BRANES AS BOUND STATES

Everything about Type IIB D-branes can be done using only 9-branes and $\bar{9}$-branes:

Start with N pairs, gauge symmetry $U(N)\times U(N)$, tachyon in $(N,\bar{N})$.
T develops a vev that breaks $U(N)\times U(N)$ to the diagonal $U(N)$.
Thus, the vacuum manifold is

$$\mathcal{V}ac_0 = \big(U(N)\times U(N)\big)/U(N) \cong U(N).$$

$\mathcal{V}ac_0$ has homotopy groups $\pi_{2k}(\mathcal{V}ac_0)=0$, $\pi_{2k+1}(\mathcal{V}ac_0)=\mathbb{Z}$; therefore, the tachyon can form stable vortices in each even codimension:



K-theory suggests a particular # of 9-brane pairs N for given codimension 2k, and gives a recipe for the vortex configuration:

Consider x^i, $i=1,..2k$ coordinates in the normal dimensions to Y; the rotation group $SO(2k)$ has two inequivalent spinor reps S_+, S_- of dimension 2^{k-1}. Extend S_+, S_- to a neighborhood of Y in X, and choose the 9-branes and $\bar{9}$-branes such that their Chan-Paton bundles are S_+ and S_-, respectively. (i.e., $N=2^{k-1}$)

The tachyon vortex representing the D-brane wrapping Y as a bound state of 2^{k-1} pairs:

$$T = \Gamma_i x^i$$

this is all exactly the Atiyah-Bott-Shapiro construction of K-theory, discovered in 1963!

Here Γ_i are the Γ-matrices of $SO(2k)$, convergence factor omitted.

TYPE IIA D-BRANES AS BOUND STATES

Let's try to do this for Type IIA D-branes.
There is an object in K-theory that has the right properties to possibly classify D-brane charges of Type IIA: $K^{-1}(X)$.

This group can be easily calculated for spheres, giving the results complementary to $\tilde{K}(S^m)$:

	$\tilde{K}(S^m)$	$K^{-1}(S^m)$
$m = 2k$	$\mathbb{Z}$	0
$m = 2k+1$	0	$\mathbb{Z}$

(2k-1)-branes of Type IIB

2k-branes of Type IIA

However, this higher K-theory group $K^{-1}(X)$ is usually defined using an eleven-dimensional extension of X, roughly $X \times S^1$!
($K^{-1}(X)$ is defined as a certain subgroup in $\tilde{K}(X \times S^1)$)

This is very interesting, because it is suggestive of M-theory; nevertheless, we don't have any understanding of "D-branes" in M-theory, in particular, there is - very likely - no spacetime filling "D10-brane" in M-theory, and therefore this 11d definition of $K^{-1}(X)$ is at this point useless.

Luckily, there is an alternative definition of $K^{-1}(X)$ using properties of X only; if it weren't already known in math literature, we would discover it in string theory! → unstable Type IIA 9-branes

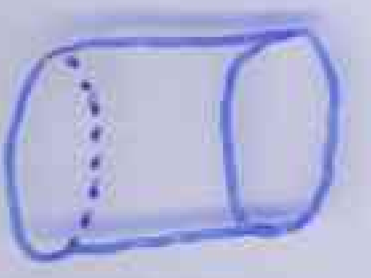
UNSTABLE TYPE IIA 9-BRANES

We can discover $K^{-1}(X)$ in string theory as follows.
Consider the tachyonic D9-brane (carrying no RR charge).
It is described by a boundary state

$$|B\rangle = |B,+\rangle_{NS} - |B,-\rangle_{NS}$$

which has no RR component, as none is invariant under the Type IIA GSO projection in closed-string channel.
No RR boundary state $\Rightarrow$ no tadpole cancellation requirement, and we can have an arbitrary number of these 9-branes w/o generating anomalies.

Spectrum of open strings ending on 9-brane is inferred from



– since there is no RR boundary state, there is no GSO projection in the open-string channel, and in particular the tachyon of the NS open string does not get projected out.

N such 9-branes will have a $U(N)$ gauge field A_μ, and an adjoint tachyon T; in the massless fermion sector, we get (χ, χ') - two fermions w/ opposite chiralities.

This spectrum can be thought of as coming from KK reduction of 11d vector A_M, and 11d spinor Ψ.

To compare:

	Type IIB	Type IIA
A_μ	$U(N) \times U(N)$	$U(N)$
T	$(N, \bar{N})$	adjoint

TYPE IIA D-BRANES AS BOUND STATES OF 9-BRANES

Configurations of these unstable 9-branes, modulo creation and annihilation of "elementary" 9-brane from/to vacuum, define $K^{-1}(X)$ (hep-th/9812135)

Jump to the bound-state construction:

For simplicity, start with 2N 9-branes. The gauge group is $U(2N)$. The tachyon is expected to have a vev with two eigenvalues, $\pm T_0$. Thus, the gauge symmetry is broken to $U(N+k) \times U(N-k)$. It will be sufficient to consider cases with $k=0$, i.e., the same number of positive and negative eigenvalues.

The vacuum manifold of the tachyon is

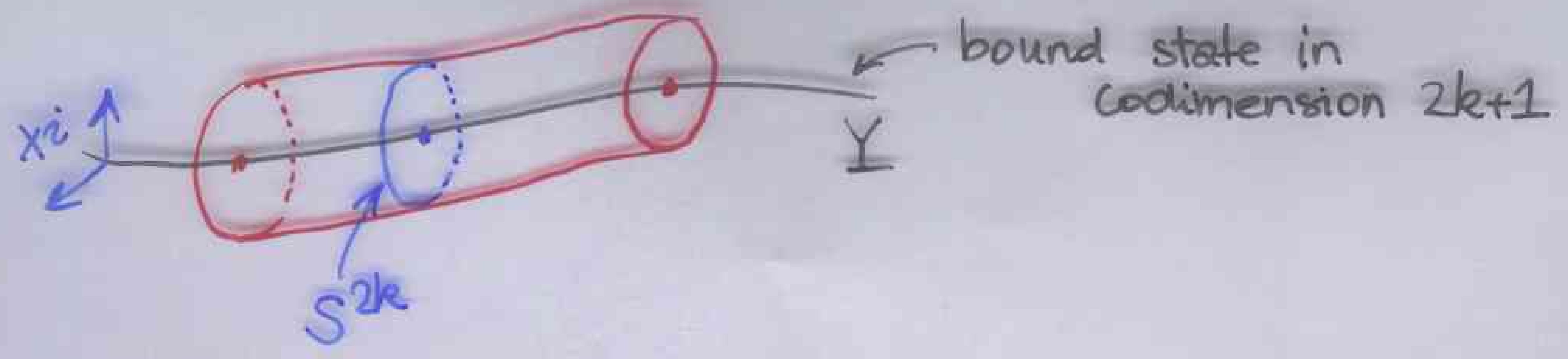
$$\mathcal{V}ac_1 = U(2N) / (U(N) \times U(N)) \, .$$

for "stable" values of N

The homotopy groups of $\mathcal{V}ac_1$ are: $\pi_{2k}(\mathcal{V}ac_1) = \mathbb{Z}$, $\pi_{2k+1}(\mathcal{V}ac_1) = 0$ (i.e., complementary to those of $\mathcal{V}ac_0$ of Type IIB 9-branes)

In fact, $\mathcal{V}ac_1$ is a finite-dimensional approximation to BU, a very important manifold in topology (and K-theory in particular), intimately connected with $U(\infty)$.

Given the structure of the homotopy groups, the tachyon field on the 9-branes of Type IIA can form stable vortices in <u>odd</u> codimensions = supersymmetric D-branes of Type IIA theory.



Consider x^i, $i=1,.. 2k+1$ coordinates in the transverse dimensions. The transverse rotation group has a spinor rep S. Extend S as a bundle over the neighborhood of Y in spacetime. Choose a configuration of $N = 2^k$ unstable 9-branes such that their Chan-Paton bundle is S.

The tachyon vortex representing the bound state is

$$T = \Gamma_i x^i.$$

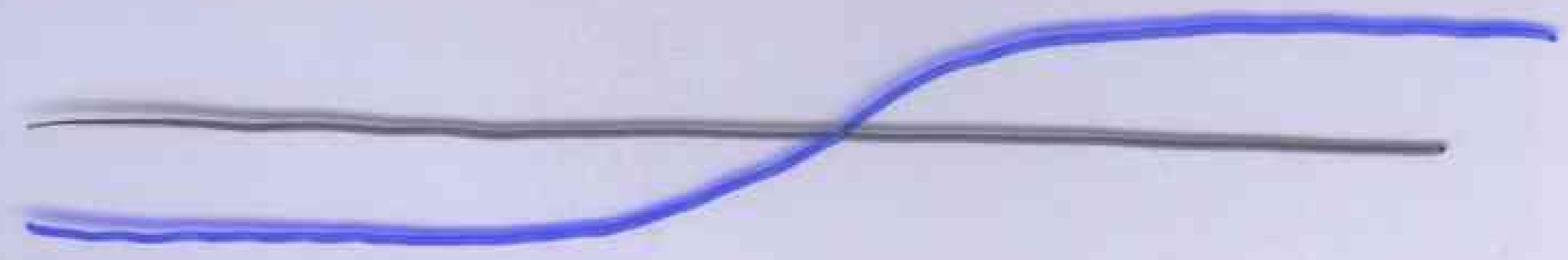
Even though this is formally the same formula as in Type IIB, the physics is different: The tachyon of Type IIB takes asymptotically values in $\mathcal{V}ac_0 \cong U(N)$, while in Type IIA, it takes values in $\mathcal{V}ac_1 = U(2N)/U(N)\times U(N)$.

It can be demonstrated (hep-th/9812135) that this construction of bound states is a direct generalization of the Atiyah-Bott-Shapiro construction from $K(X)$ to $K^{-1}(X)$.

EVERY TYPE IIA D-BRANE (OF CODIMENSION $2k+1$ IN $\mathbb{R}^{10}$) CAN BE OBTAINED AS A BOUND STATE OF 2^{k-1} UNSTABLE TYPE IIA D9-BRANES.

EXAMPLES:

Codimension one:



U(1) gauge theory on one unstable 9-brane;
one real scalar tachyon; symmetry unbroken;
the (anti) kink can be identified as (anti) D8-brane

worldvolume coupling: $\sim \int dT \wedge e^F$

For N D8-branes, we need N D9-branes.

Codimension three:

U(2) gauge theory w/ adjoint tachyon;
cf. Georgi-Glashow model;
the topological defect of codimension three describes the 't Hooft-Polyakov magnetic monopole, can be identified w/ D6-brane of Type IIA.
Multiple D6-brane config's = multi-monopole config's in U(2).

Codimension nine:

We need 16 D9-branes now, can describe any # of D0-branes as topo. defects in this U(16) theory.

MATRIX THEORY FROM TACHYON CONDENSATION ON SPACETIME-FILLING D-BRANES

Matrix theory describes the sector of M-theory carrying a fixed lightcone momentum, in terms of quantum mechanics of N D0-branes of Type IIA string theory.

BFSS, Susskind, Sen, Seiberg, ... Polchinski

Start w/ Type IIA theory, in the sector w/ N D0-branes, go to a limit in which both the closed-string gravity sector & the higher string modes decouple (= the Sen-Seiberg scaling limit).

By the scaling argument, this is equivalent to the sector of full M-theory carrying the corresponding lightlike momentum, but technically described by the decoupled D0-brane Q.M.

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NOW: Set up the same configuration of D0-branes using the spacetime-filling branes & tachyon condensation; follow the Sen-Seiberg scaling limit.

- individual D0-brane is a bound state of (topo. defect on) $n = 2^4 = 16$ spacetime-filling branes, $U(16)$ gauge theory (plus higher string modes, plus closed strings)
- any number N of D0-branes still needs only 16 spctm-filling branes
- the Sen-Seiberg limit: weakly coupled, non-relativistic $U(16)$ theory (possibly defined by string field theory $\int (A*QA + A*A*A)$)

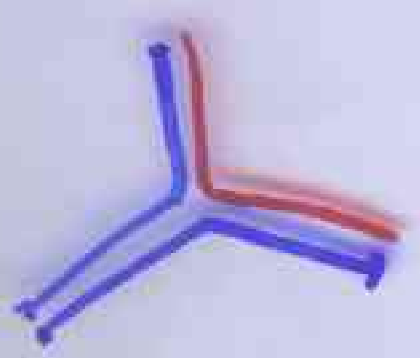
MATRIX THEORY VERSUS OPEN STRING FIELD THY.

Thus:

In the limit where the gravitational & closed string sector decouples from the D0-brane system, it is natural to describe the Matrix theory system as a multi-vortex system in (a certain limit of) covariant open string field theory of sixteen spacetime-filling Type IIA D9-branes.

$$\int \left(A \star QA + \tfrac{2}{3} A \star A \star A\right)$$

BRST charge



also: see A. Sen, C.V. Johnson

Possible advantages:

- closer to background independence? (cubic action: $\int A^3$)
- naively just open strings, but gravity & closed strings are implicitly contained
- simple formulation, transparent geometry (Chern-Simons) (wealth of results from late '80)
- possible generalizations to other N brane systems, such as those in Maldacena's conjecture

Also: a very interesting limit of open string field theory!

Similarities with holographic field theory

Holographic field theory:

- M-theory as a gauge field thy,
  large spacetime requires
  a large number **N** of partons

- Gauge group to reproduce
  11d sugra at low energies:

  $$OSp(1|32) \times OSp(1|32)$$

- At low energies, gauge symmetries
  are traded for diffeomorphisms

- The gauge group is super-Lorentz
  in (10,2) dimensions,
  the Lagrangian is related to
  the (supersymmetrized) Euler density:

  $$\mathcal{L} \sim \int_{\mathcal{M}_{11}} \omega_{11} \sim \int_{\substack{\mathcal{M}_{12},\\ \partial\mathcal{M}_{12}=\mathcal{M}_{11}}} d\omega_{11} \sim \int_{\mathcal{M}_{12}} (\text{Euler})$$

- The most straightforward representation
  of gauge group is in terms of
  64 x 64 Γ-matrices in (10,2) dim's:

  $$\left(\begin{array}{c|c|c} 0 & Q & Q' \\ \hline Q & \Gamma & \\ \hline Q' & & \Gamma' \end{array}\right) \text{ acting on } \left(\begin{array}{c} S_+ \\ \hline S_- \end{array}\right)$$

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- M(atrix) theory as gauge thy, in sector with N magnetic vortices
- Gauge group following from K-theory: $U(16)$

  $$U(16) \hat{\times} U(16)$$

  maximal compact subgroup! (= same spectrum of defects)

- The mag. vortex background identifies gauge transfmtn's and spacetime rotations
- In K-theory, the most important object – which realizes the embedding of lower branes to higher ones – is the Euler character of the normal bundle:

  

  $$K(Y) \xrightarrow[\text{euler}]{} K(X)$$

- In K-theory, the spinor bundles S_+, S_- play a crucial role in the ABS bound-state construction of branes