BRST QUANTIZATION in AdS_5 space

R. KALLOSH , STRINGS 99

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hep-th / 9905111
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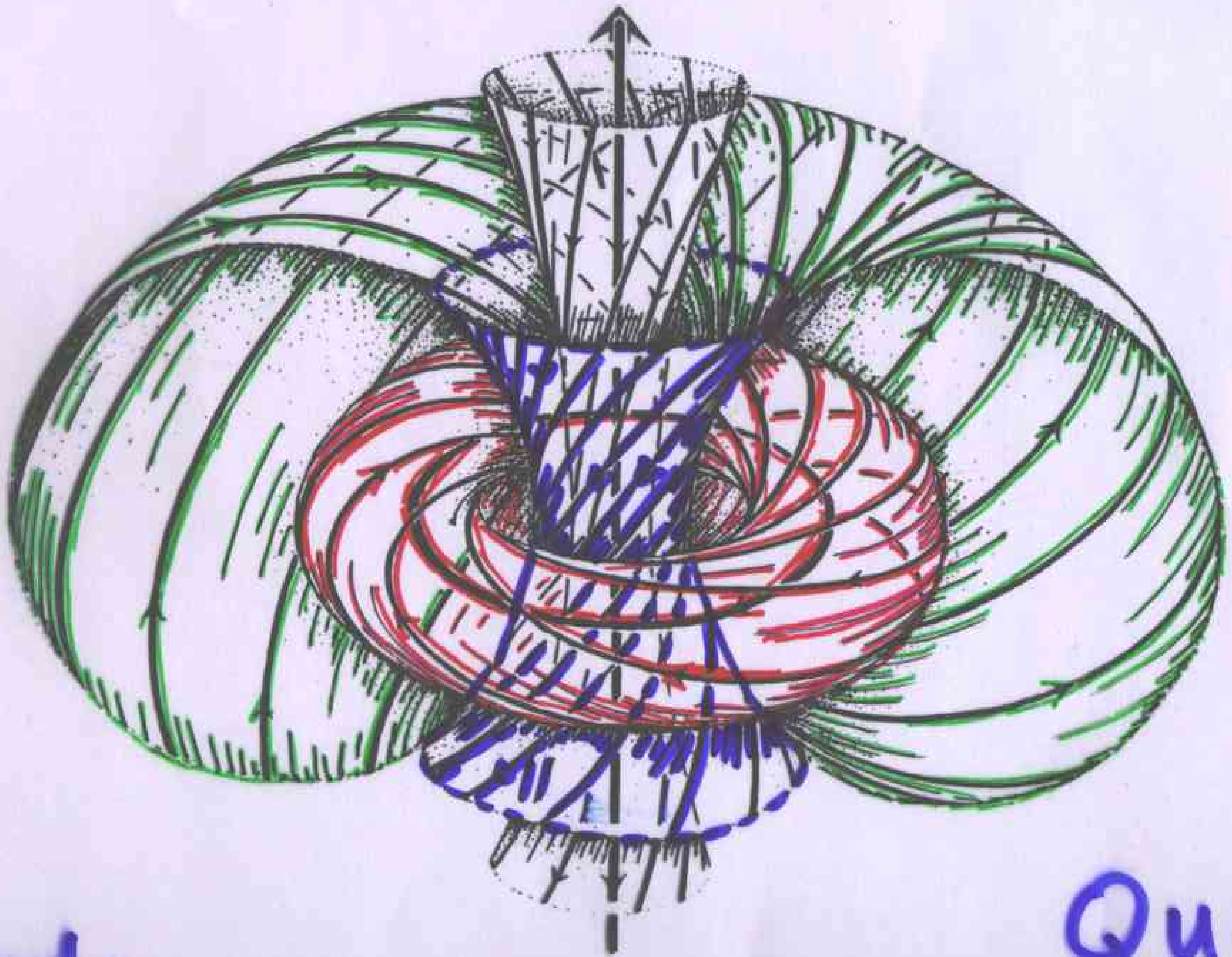
Twistors

Penrose
1967



Quarks

Important:

Supertwistors (Ferber, 1978) form a fundamental representation of **SU(2,2|4)**, the superalgebra of the isometries of the NEAR HORIZON $AdS_5 \times S^5$ SUPERSPACE

Outline

? 1. Status of GS Superstring Theory in $AdS_5 \times S^5$

Isometry: non-linearly realized $SU(2,2|4)$ superconformal symmetry

2. Supertwistors as Quarks (fund. repr.) of $SU(2,2|4)$

Toy model with linearly realized SUPERCONFORMAL SYM.

$\Downarrow$

Superoscillator construction of all UIR:

YM, sugra, KK sugra, massive string states

3. BRST quantization of massive particle in AdS_5

Diff. to quantize → Non-linear action in curved space (x^μ, ρ) with $SU(2.2)$ isometry

Classically equivalent

Easy to quantize → Quadratic action in TWISTOR VARIABLES

GS Superstring in $AdS_5 \times S^5$

Near Horizon Superspace

Metsaev, Tseytlin

$$\frac{SU(2,2|4)}{SO(1,4) \times SO(5)}$$ super-coset

R.K., A. Rajaraman, J. Rahmfeld

Closed form of the classical string action with

RR 5-form

Fermionic terms up to $\Theta^{32}(\partial\Theta)^2$ Strings 98

κ-symmetry gauge-fixing

R.K., J. Rahmfeld

R.K., A. Tseytlin

IIB D3 brane Killing Spinor

$$\boxed{\Theta^1 = \Gamma_{0123}\,\Theta^2}$$

Max dependence on fermions $\Theta^2(\partial\Theta)^2$

The gauge is admissible under certain conditions!

II B Green-Schwarz String action on $adS_5 \times S^5$, gauge-fixed κ-sym.

R.K., J. Rahmfeld

$$S = -\frac{1}{2}\int d^2\sigma\Big\{\Big[\sqrt{-g}\, g^{ij}\Big(y^2(\partial_i x^p - i\bar\theta\Gamma^p\partial_i\theta)(\partial_j x^p - i\bar\theta\Gamma^p\partial_j\theta) + \frac{1}{y^2}\partial_i y^t\partial_j y^t\Big]$$

$$+ 2i\,\varepsilon^{ij}\partial_i y^t\,\bar\theta\Gamma^t\partial_j\theta\Big\}$$

Supersolvable algebra

I. Pesando
all roads lead to Rome

Compare with the metric on $adS_5 \times S^5$

$$ds^2 = y^2 dx^p dx^p + \frac{1}{y^2}dy^t dy^t$$

Rajaraman, Rosali
attempt to quantize

P. Claus, R.K. (1998)

Isometries → symmetries of the action

$$\delta X(x,y,\theta)\ ,\ \delta y(x,y,\theta)\ ,\ \delta\Theta(x,y,\theta)$$

SU(2.2|4)

Penrose twistors „quarks" of the conformal group

1967

SU(2.2) is a covering group of the conformal group SO(4.2)

4-component commuting spinor

$$Z^{\Omega} = \begin{pmatrix} \lambda^{\alpha} \\ \mu_{\dot\alpha} \end{pmatrix} \qquad Z^{+\Omega} = \left(\bar\lambda_{\dot\alpha} \;\; \bar\mu^{\alpha} \right)$$

SU(2.2)

$$\begin{pmatrix} \delta\lambda \\ \delta\mu \end{pmatrix} = \begin{pmatrix} L - \frac{1}{2}D & -iK \\ -iP & -L + \frac{1}{2}D \end{pmatrix} \begin{pmatrix} \lambda \\ \mu \end{pmatrix}$$

Twistors are fundamental

X-SPACE is a derivable concept

$$\mu_{\dot\alpha} = -i \, \underline{X_{\dot\alpha\alpha}} \, \lambda^{\alpha}$$

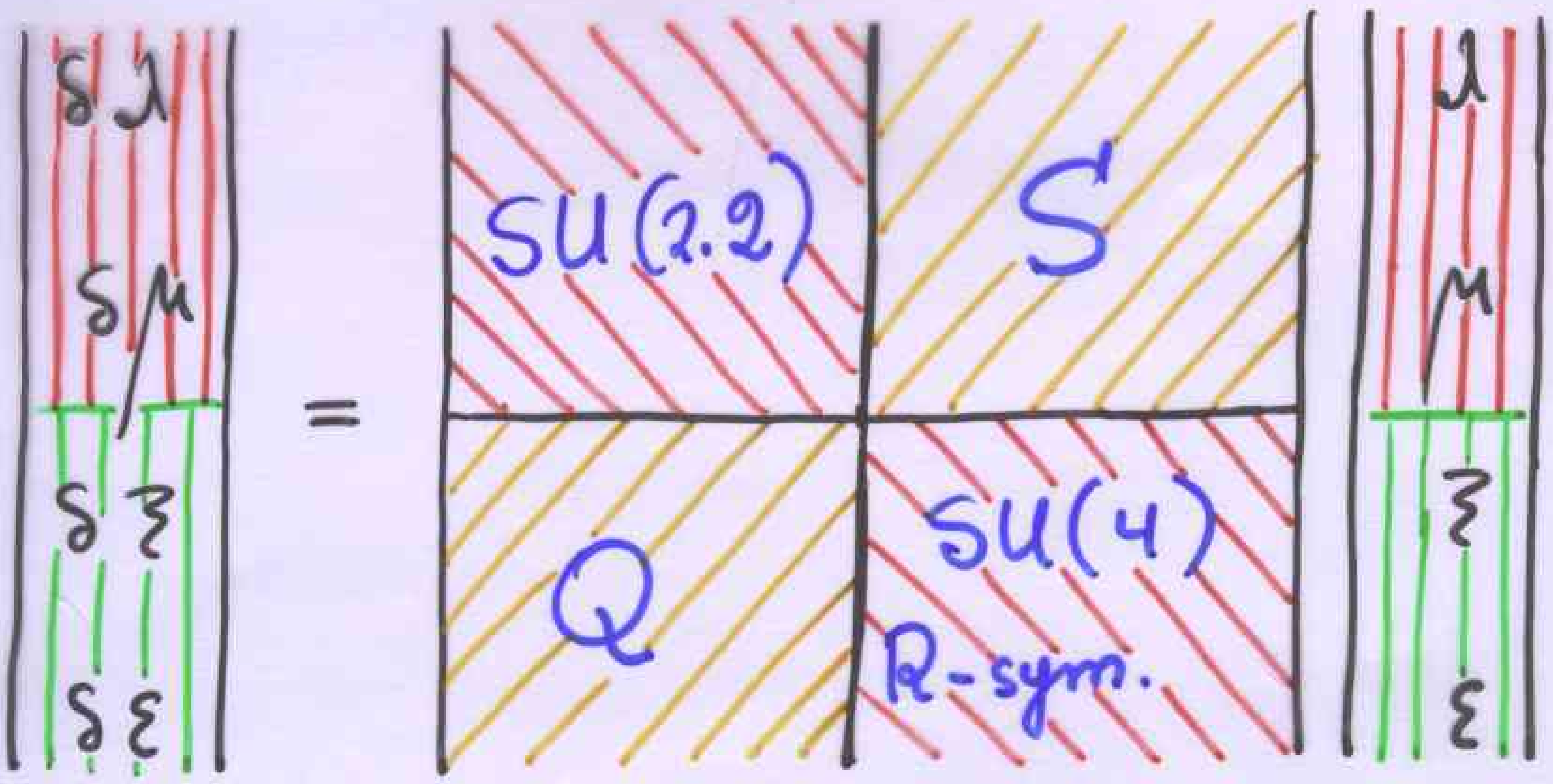
Ferber 1978

Supertwistors

„quarks" of the superconformal group

$SU(2,2|4)$

2+2



commuting spinors

anticom. spinors

Supertwistors are fundamental

(X, θ) - superspace is a derivable concept

$$\mu_{\dot\alpha} = -i\, \underline{X^{\text{chiral}}_{\dot\alpha\alpha}}\, \lambda^\alpha$$

$$\xi^i = \underline{\theta^i_\alpha}\, \lambda^\alpha$$

$\alpha, \dot\alpha, i, r =$
$= 1, 2$

$$\varepsilon^r = \underline{\theta^r_\alpha}\, \lambda^\alpha$$

2

Manifest $SU(2,2|4)$ Invariants

$$\mathcal{Z} \quad , \quad \bar{\mathcal{Z}} = \mathcal{Z}^{+} H$$

$$H = \left|\begin{array}{cc|cc} & 1 & & \\ 1 & & & \\ \hline & & -1 & \\ & & & -1 \end{array}\right|$$

$\bar{\mathcal{Z}}_1 \mathcal{Z}_2$ is invariant !

Simple rule for bilinear combination of quarks !

Supertwistor world-sheet action

Toy model

Implied by GS action on $AdS_5 \times S^5$, not derived!

String action: non-linear σ-model with $SU(2.2|4)$ sym
Twistor action: linearly realized $SU(2.2|4)$

$$S = -i\int d^2\sigma \left(\bar{Z}_L \partial_- Z_L + \bar{Z}_R \partial_+ Z_R + \cdots \right)$$

Quantization

$$\left[\bar{Z}_{x}(\sigma), Z_{x'}(\sigma') \right\}_{\tau=\tau'} = \delta_{xx'}(\sigma-\sigma')$$

Dynamical derivation of the **SUPEROSCILLATOR CONSTRUCTION** of UNITARY IRREDUCIBLE REP. of $SU(2.2|4)$

Gunaydin, Minic, Zagerman (1998) (Bars, Gunaydin (1982))

Ferrara et al
Superfields and UIR of $SU(2.2|N)$

$-\infty < n < +\infty$

$$\boxed{Z_{\substack{L\\R}}(\sigma_\pm) \sim e^{-in(\tau\pm\sigma)} \left(Z_n\right)_{\substack{L\\R}}}$$

Classification of UIR of $\overbrace{SU(2,2|4)}^{\text{central extension of}}$

$U_2(1)$ $\quad Z = \frac{1}{2}[\vec{a}^i\vec{a}_i + \vec{a}^\alpha\vec{a}_\alpha - \vec{b}^r\vec{b}_r - \vec{\beta}^x\vec{\beta}_x]$

(1) $Z=0$ multiplets

1) $N=4$ $d=4$ SYM

$P=1$
1 generation of
$(\xi,\eta) \to$ doubleton

2) "Massless" $N=8$ supergravity states in $d=5$ on adS_5

$P=2$
2 generations

From our action
ξ_L, η_L ; ξ_R, η_R

3) "Massive" KK supergravity states

$P>2$
ever increasing #

The "massless" graviton supermultiplet of $N=8$ adS_5 supergravity sits at the bottom of the infinite tower corresponding to $P=2$

From our action mode expansion

$\xi_{L,R}(\sigma), \eta_{L,R}(\sigma)$;

Massive String States in $AdS_5 \times S^5$

Long multiplets

$$P \geq 4$$

# of generations of superoscillators =
# of supertwistors

AdS energy is quantized

$$E = n + m + \frac{N_a}{2} + \frac{N_b}{2} + P$$

$U(1)$ Central charge is quantized

$$Z = n - m + \frac{N_a}{2} - \frac{N_b}{2}$$

Novel Multiplets

$$P \geq 2$$

$$Z = \pm j$$

Do not appear in tensor products of YM $N=4$ multiplets

Do not appear in KK IIB supergravity on adS_5

Conjecture
Gunaydin, Minic, Zagermann

novel multiplets may be states of

(p, q) superstrings on $adS_5 \times S^5$

Main result : our world-volume supertwistor action upon quantization contains all states of UIR of $SU(2.2|4)$

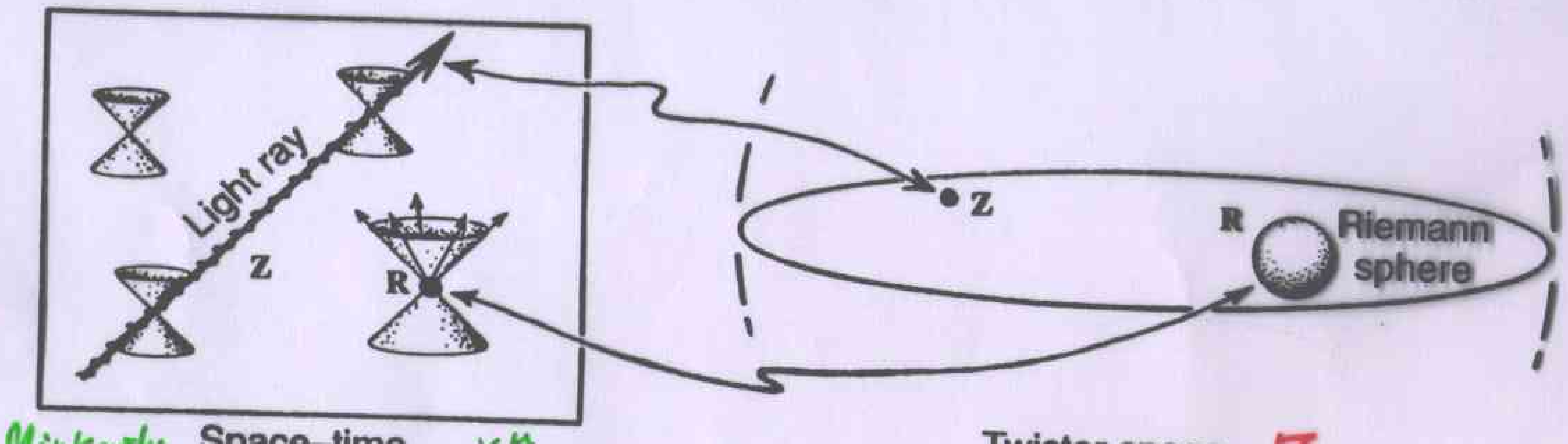
?

(p, q) string theory J. Schwarz (1995) E. Witten (1996)

Action (1997) M. Cederwall, P. Townsend

„The central programme of twistor theory" in
Chaos, Solitons and Fractals
Vol 10, p 581, 1999

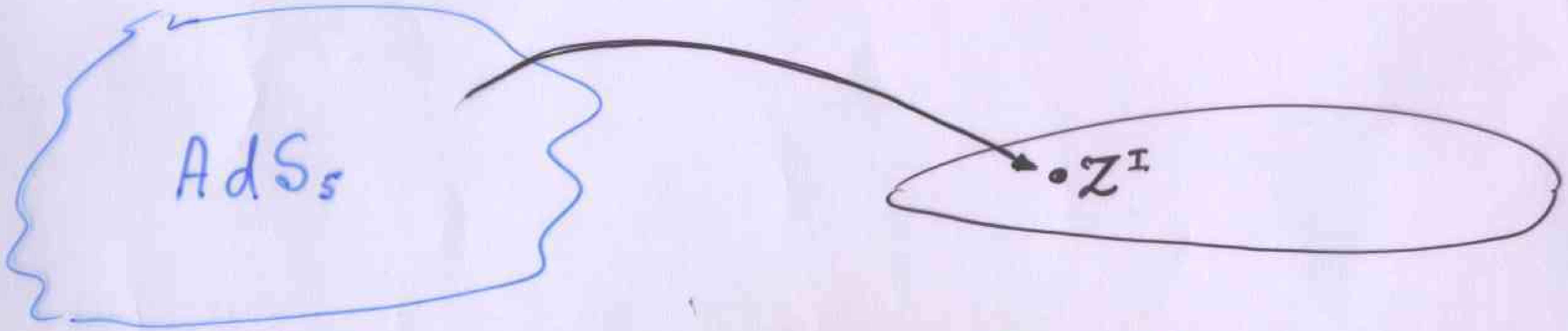
Penrose



$d=4$ Minkowski Space–time X^μ Twistor space $\mathcal{Z}$

Twistor theory provides an alternative physical picture to that of space-time, whereby entire light rays are represented as points.

P. Claus, J. Rahmfeld, Y. Zunger, hep-th/9906118



$d=5$ AdS space-time 2-twistor space $\mathcal{Z}^I$, $I=1,2$

X^μ, ρ

COLOR

2 quarks of SU(2.2)

$$\mathcal{Z}^I = \begin{pmatrix} \lambda_\alpha^I \\ \bar{\mu}^{\dot\alpha I} \end{pmatrix}$$

$$\boxed{\bar{\mu}^{\dot\alpha I} = -i\, x^{\dot\alpha\alpha} \lambda_\alpha^I + \frac{\varepsilon^{IJ}}{\rho} \bar{\lambda}^{\dot\alpha J}}$$

From fund. repr. $\quad \delta \mathcal{Z}^I = \left| su(2.2) \right| \mathcal{Z}^I$

the isometry of AdS_5 space

$$\begin{cases} \delta x^\mu = -\left(a^\mu + \lambda^{\mu\nu} x_\nu + \lambda_D x^\mu + x^2 \Lambda_K^\mu - 2x\cdot\Lambda\, x^\mu\right) - R^2 \dfrac{\Lambda_K^\mu}{\rho^2} \\ \\ \delta\rho = \left(\lambda_D - 2x\cdot\Lambda_K\right)\rho \end{cases}$$

example of non-linear realization of SU(2.2) algebra

BRST quantization of massive particle in $\underline{AdS_5}$

$$\left.\begin{aligned} S &= -m\int ds \\ ds^2 &= \rho^2 dx^2 + R^2\left(\frac{d\rho}{\rho}\right)^2 \\ & x^\mu(\tau),\ \rho(\tau) \end{aligned}\right\}$$

difficult to quantize

Classically equivalent theory which is easy to quantize

$$S_{tw} = -i\int d\tau\, \bar{Z}^I \partial_\tau Z^I - i\, u^a \varphi_a$$

4 1st class constraints

$$\varphi_0 = \bar{Z}^I Z^I - 2sR \qquad s = \pm m$$

$$\varphi_i = \bar{Z}^I \sigma_i^{IJ} Z^J \qquad SU(2)\times U(1)$$

$$\bar{Z}^I(\tau),\ Z^I(\tau)$$

The counting works!

$$5 - 1 = 4$$

$$8 - 4 = 4$$

Batalin - Fradkin - Fradkina - Vilkovisky

and/or BV

and/or BRST methods of quantization all work!

There exist a gauge where the action is quadratic!

$$S_{g.f.} = -i \int d\tau \Big[\bar{\mathcal{Z}}^I \partial_\tau \mathcal{Z}^I - i \left(\bar{\mathcal{Z}}^I \mathcal{Z}^I - 2sR \right) + i b_a \partial_\tau c^a \Big]$$

$$Q_{BRST} = c^0 \left(\bar{\mathcal{Z}}^I \mathcal{Z}^I - 2sR \right) + c^i \bar{\mathcal{Z}}^I \sigma_i^{IJ} \mathcal{Z}^J + i\, \varepsilon_{ijk} c^i c^j b_k$$

$$Q^2_{BRST} = 0$$

Normal ordering ambiguity is resolved by requiring CPT-symmetry to be valid at the quantum level.

Quadratic action ⟹ oscillator construction of UIR of $SU(2,2)$

Group Theory L^+, L^0, L^-

Physical states in Fock space

$$L^- |\Omega\rangle = 0$$

ewv

Multiplet

$$|\Omega\rangle, \quad L^+|\Omega\rangle, \quad L^+L^+|\Omega\rangle, \ldots$$

ewv

New additional requirement: physical states are quarks color blind!

$$Q_{BRST} |\Omega\rangle = 0$$

Not all UIR of $SU(2,2)$ are physical states!

Quantization of AdS energy and $s = \pm m$

conformal Hamiltonian eigenvalue

$$E = \frac{1}{2R}\left[N_a + N_b + 4\right]$$

$$Z = sR = \frac{1}{2}\left(N_a - N_b\right)$$

Resume

We learned how to go

From trajectories in curved AdS_5 space with $SU(2.2)$ isometry $\xrightarrow{\text{to}}$ Quantum Mechanics in Twistor space, we constructed

$$H = [Q_{BRST}, \Psi]$$

and physical states, multiplets of $SU(2.2)$

Gravity - - - → Quantum Gravity

Can this help to understand

D-branes, superstring theory in $AdS_5 \times S^5$

supermembrane theory in $AdS_4 \times S^7$

super-5-brane theory in $AdS_7 \times S^5$

?