BRANE DIAMONDS AND DUALITIES IN $\mathcal{N}=(2,2)$ GAUGE THEORIES IN D=2

M. AGANAGIC, AK

BASED ON A CONSTRUCTION PRESENTED IN

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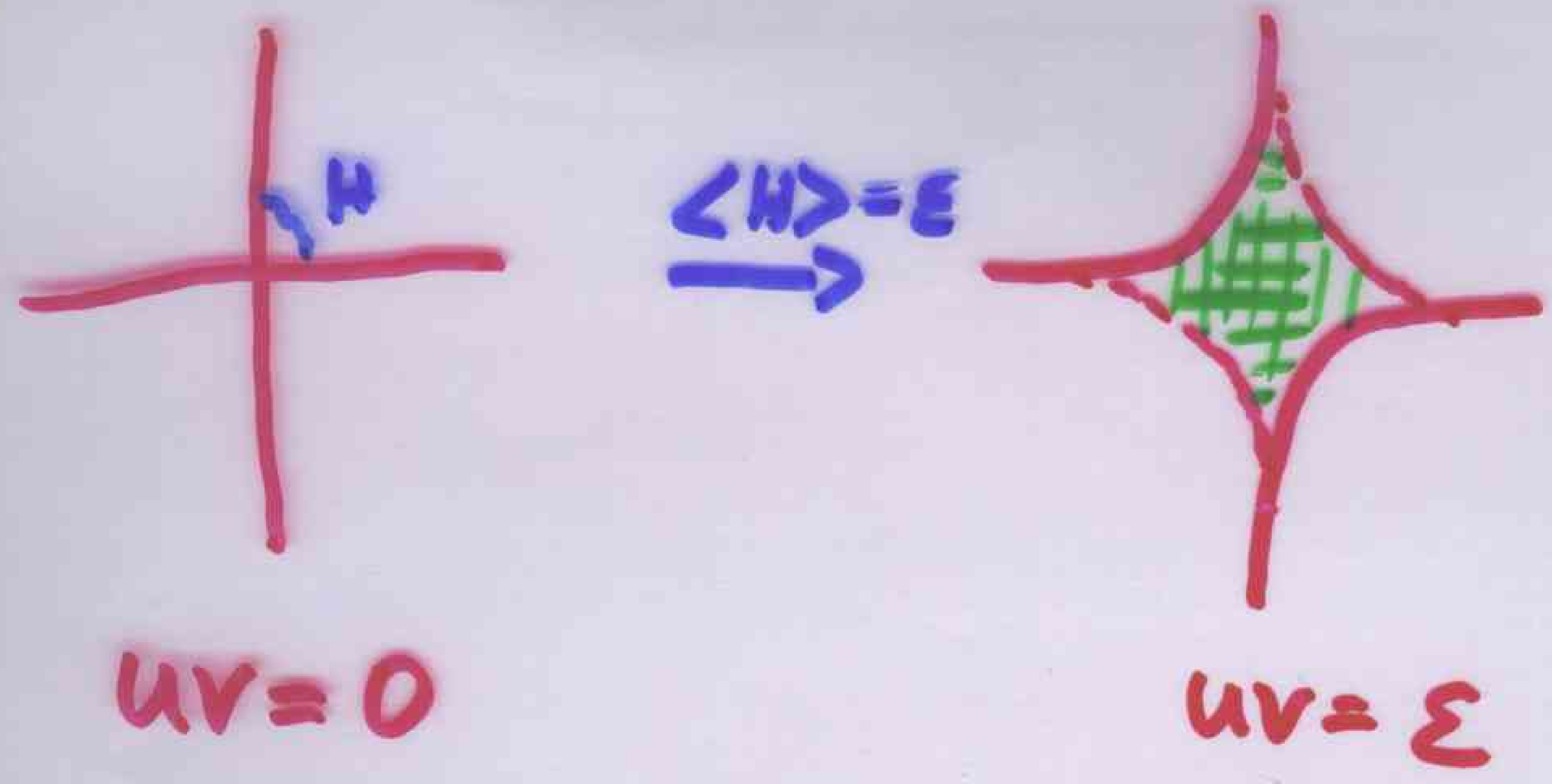
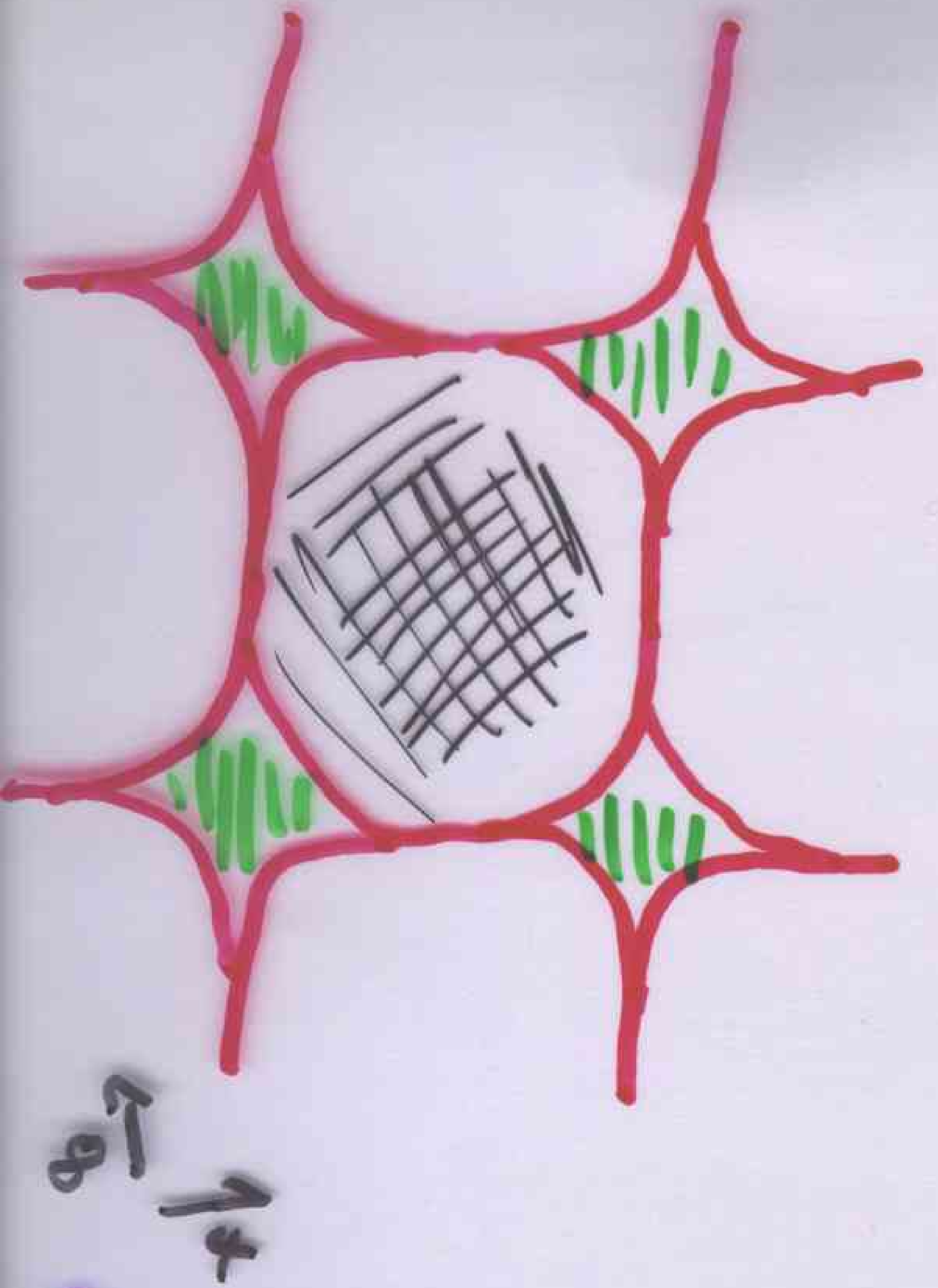
hep-th/9903093



UV=ε has NON-CONTRACTIBLE CIRCLE => STUDY D-BRANE ENDING ON SMOOTH NS-BRANE CURVE

NS : 012345
NS' : $\underbrace{0123}$ 89
4D INTERSECTION

BRANE DIAMOND



BOX (HANANY ZAFFARONI)
& DIAMONDS

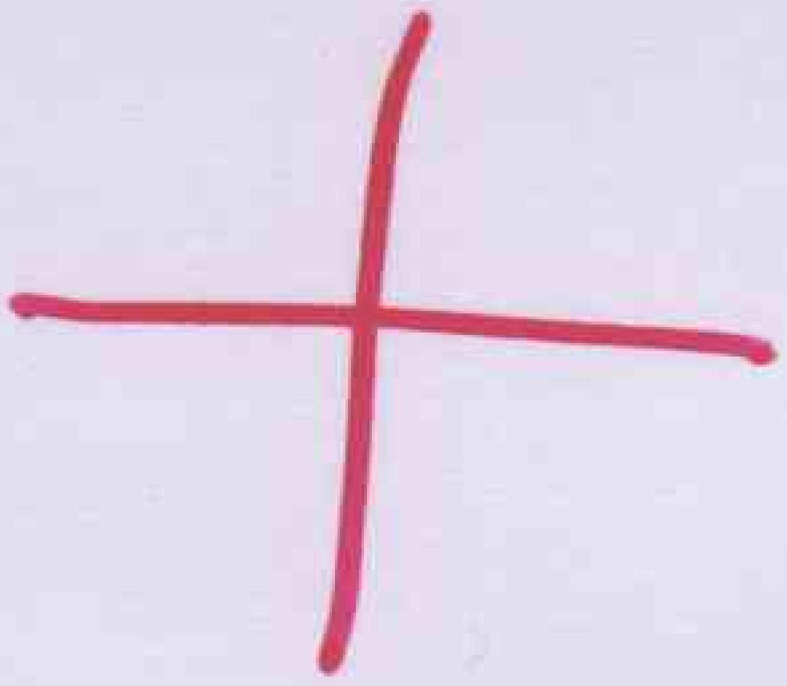
WHAT ARE THE RULES OF THE GAME?

$\rightsquigarrow$ T-DUALIZE TO GEOMETRY

BASIC RELATION:

NS: 012345

NS': 0123 89

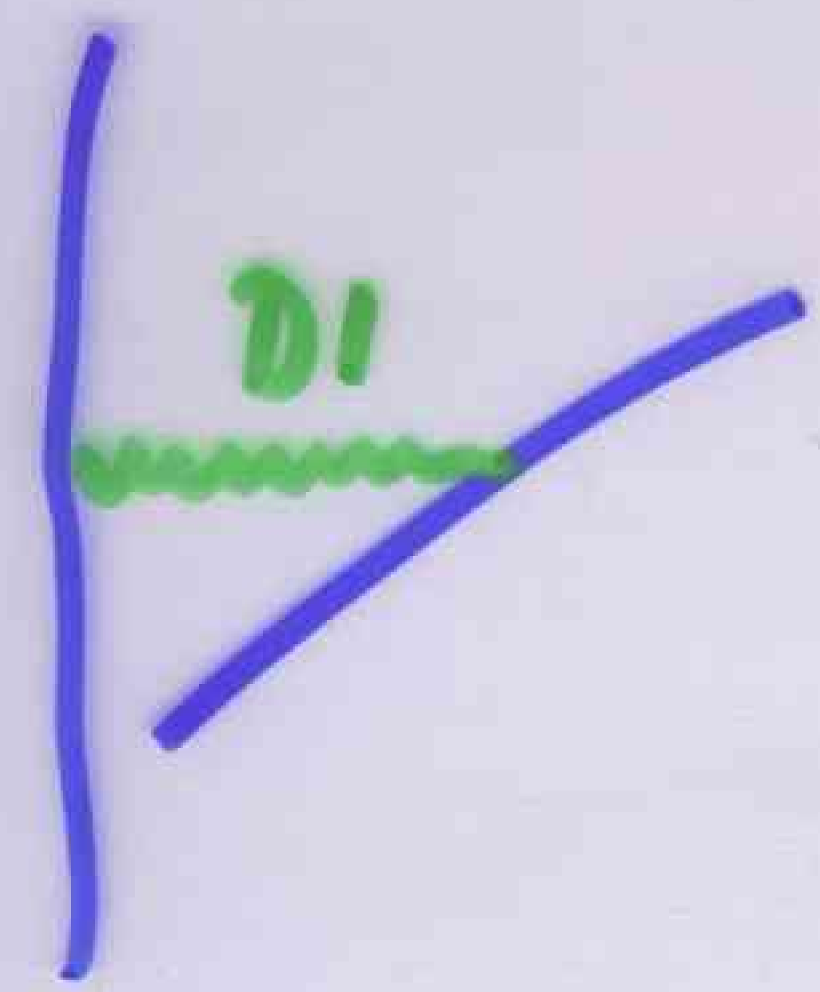


$\downarrow$ T-DUAL T_{48}

CONIFOLD

(BERSHADSKY, VAFA, SADOV; DASGUPTA; MUKHI)

ESPECIALLY:

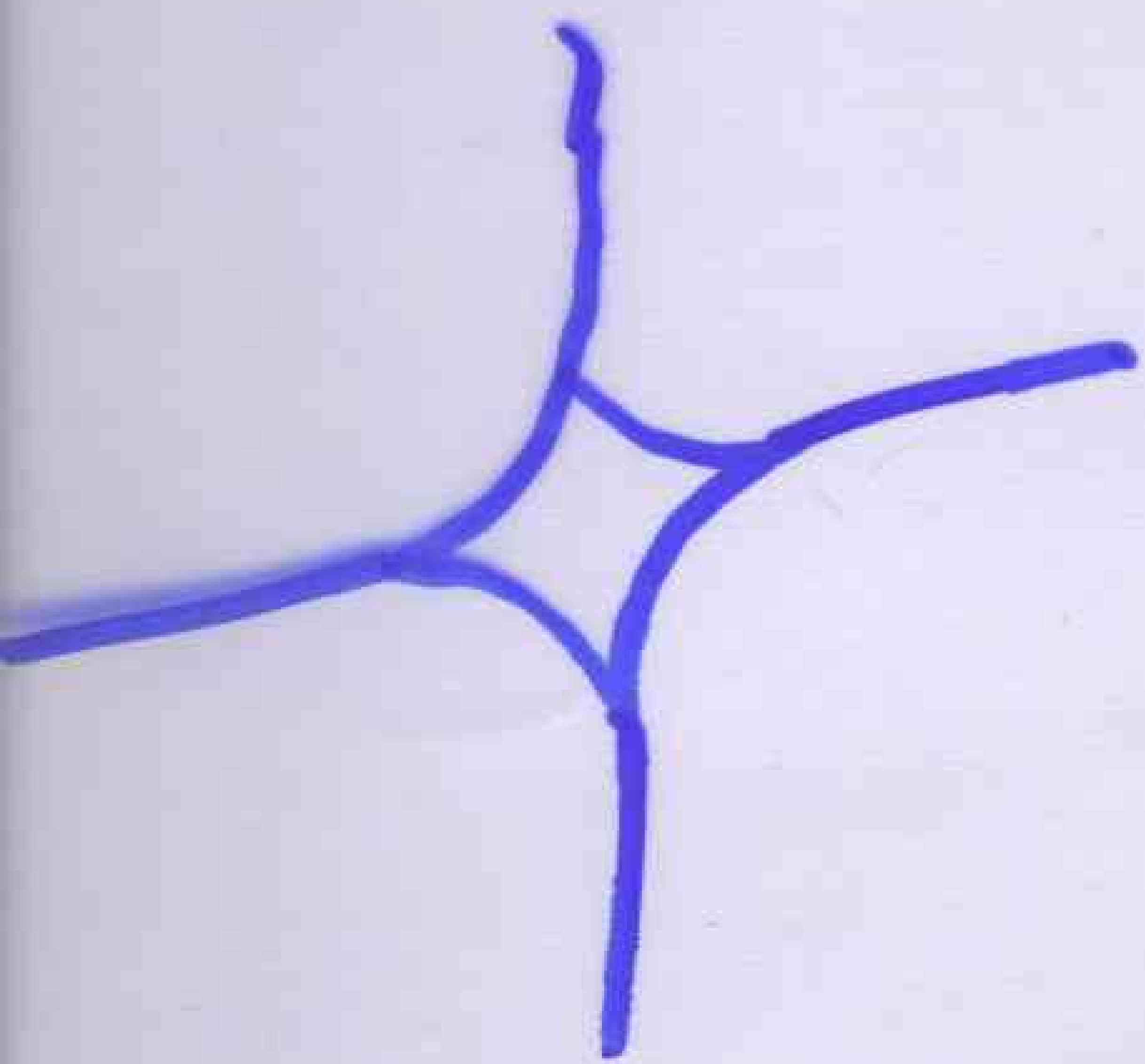


DEFORMATION:

$$uv - xy = \varepsilon$$

STROMINGER'S HYPER: S^3

IIB



SMALL RESOLUTION ("BLOWUP")

S^2

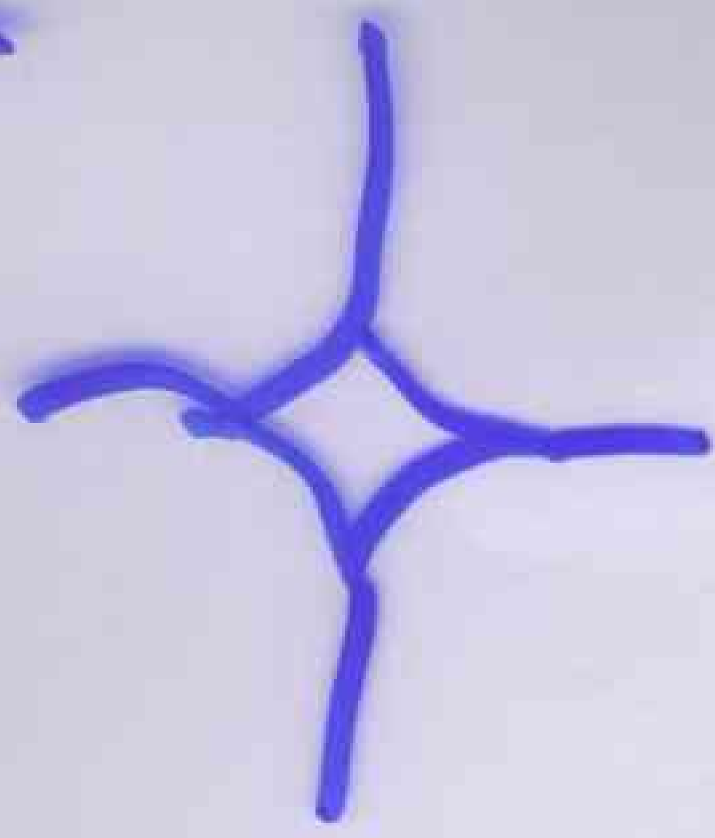
NOTE:

- SIMILARLY ONE COULD DO T_6 TO MAP BRANE SETUPS TO GEOMETRY (URANGA, DASGUPTA & MUKHI)
  → D4 ON INTERVAL

- DOING $T_{48} \times T_6$ WE CAN CONSTRUCT MIRROR SYMMETRY AS T^3- DUALITY (SYZ) STEP BY STEP (AKLM, SEE ALSO: LEUNG-VAFA)

- PROCEDURE STILL WORKS WHEN THE SPECIAL LAGRANGIAN T^3-FIBER DEGENERATES

SO:



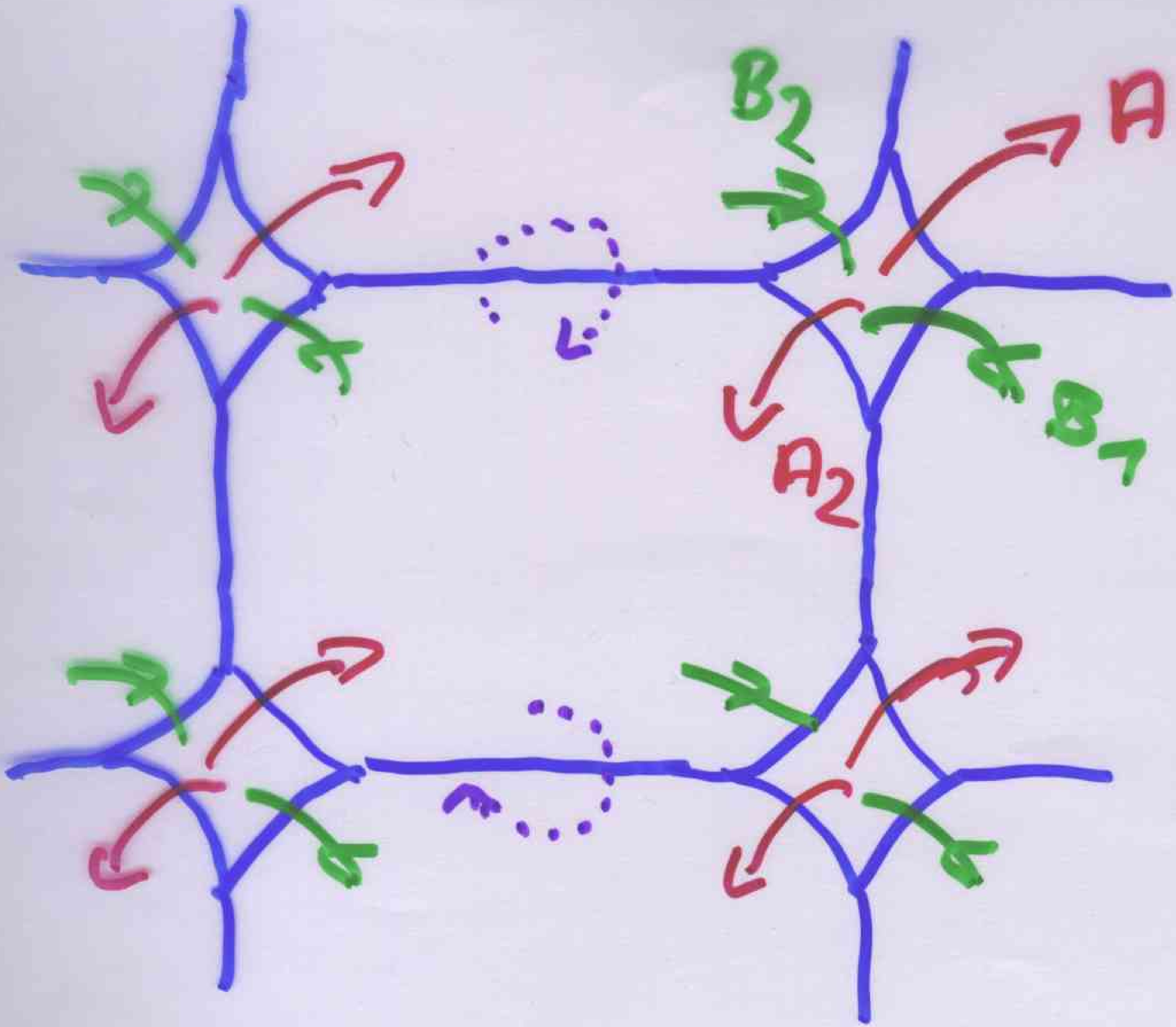
$\longleftrightarrow$ D3 ON BLOWUP OF CONIFOLD

KLEBANOV, WITTEN; MORRISON, PLESSER:

	$SU(N)$	$\times$	$SU(N)$
A_1	$\square$		$\bar{\square}$
A_2	$\square$		$\bar{\square}$
B_1	$\bar{\square}$		$\square$
B_2	$\bar{\square}$		$\square$

$$W = A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1$$

$\Longrightarrow$ DIAMOND RULES



$$W = \sum_{c} B_1 A_1 B_2 A_2 \; - \; \sum_{c} B_2 A_2 B_1 A_1$$

ESPECIALLY:

NOT $\mathcal{N}=4$

BUT:

	$SU(N)_A$	$\times\ SU(N)_B$
A_1	$\square$	$\bar{\square}$
A_2	$\square$	$\bar{\square}$
B_1	$\bar{\square}$	$\square$
B_2	$\bar{\square}$	$\square$

$g^2_{YM} \to \infty$

AFTER ALL: NS NS' INTERSECTION HAS INTERACTING 4D THEORY. CAN'T BE DUAL TO FLAT SPACE

(IT'S DUAL TO THE CONIFOLD)

APPLICATION:

MIRROR-SYMMETRY

IN

$\mathcal{N} = (2,2)$ $\quad d = 2$

SUSY GAUGE

THEORIES!

AFTER ALL THE (RE) DISCOVERY OF $\mathcal{N}=4$; $d=3$ MIRROR SYMMETRY (INTRILIGATOR, SEIBERG) BY HANANY AND WITTEN WAS ONE OF THE MAJOR SUCCESSES OF THE BRANE APPROACH ....

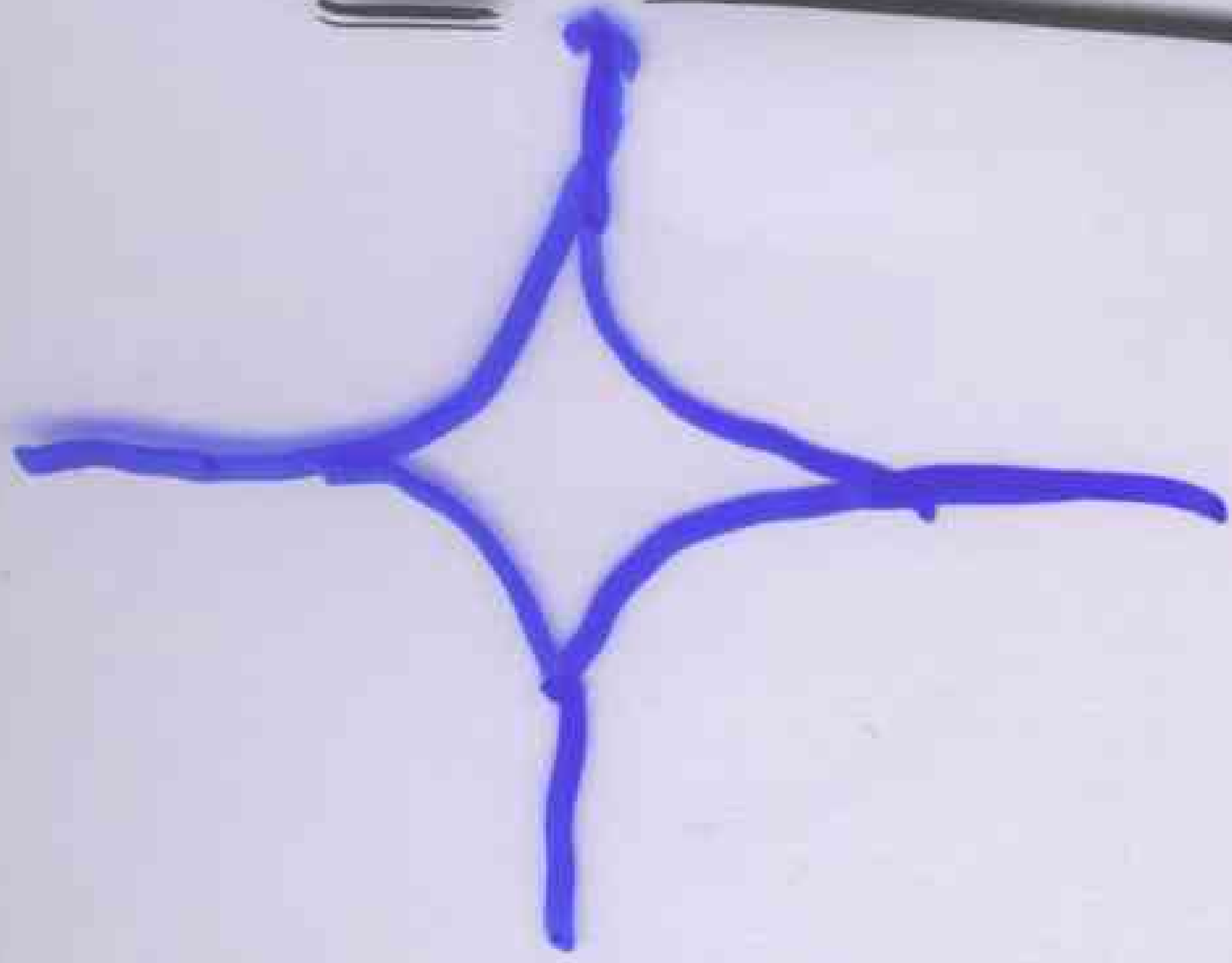
$SU(2)_{345} \leftrightarrow SU(2)_{789}$

FI $\leftrightarrow$ MASSES

HIGGS $\leftrightarrow$ COULOMB

SO TO REDO THIS SUCCESS STORY: PUT A D3 BRANE IN THE DIAMOND.....

" ELECTRIC THEORY "



ELLIPTIC MODEL

U(1) + 2 FLAVORS

LINEAR SIGMA MODEL (WITTEN)

HIGGS BRANCH: BLOWUP OF CONIFOLD

+ DECOUPLED U(1) C.O.M.

NONABELIAN VERSION:

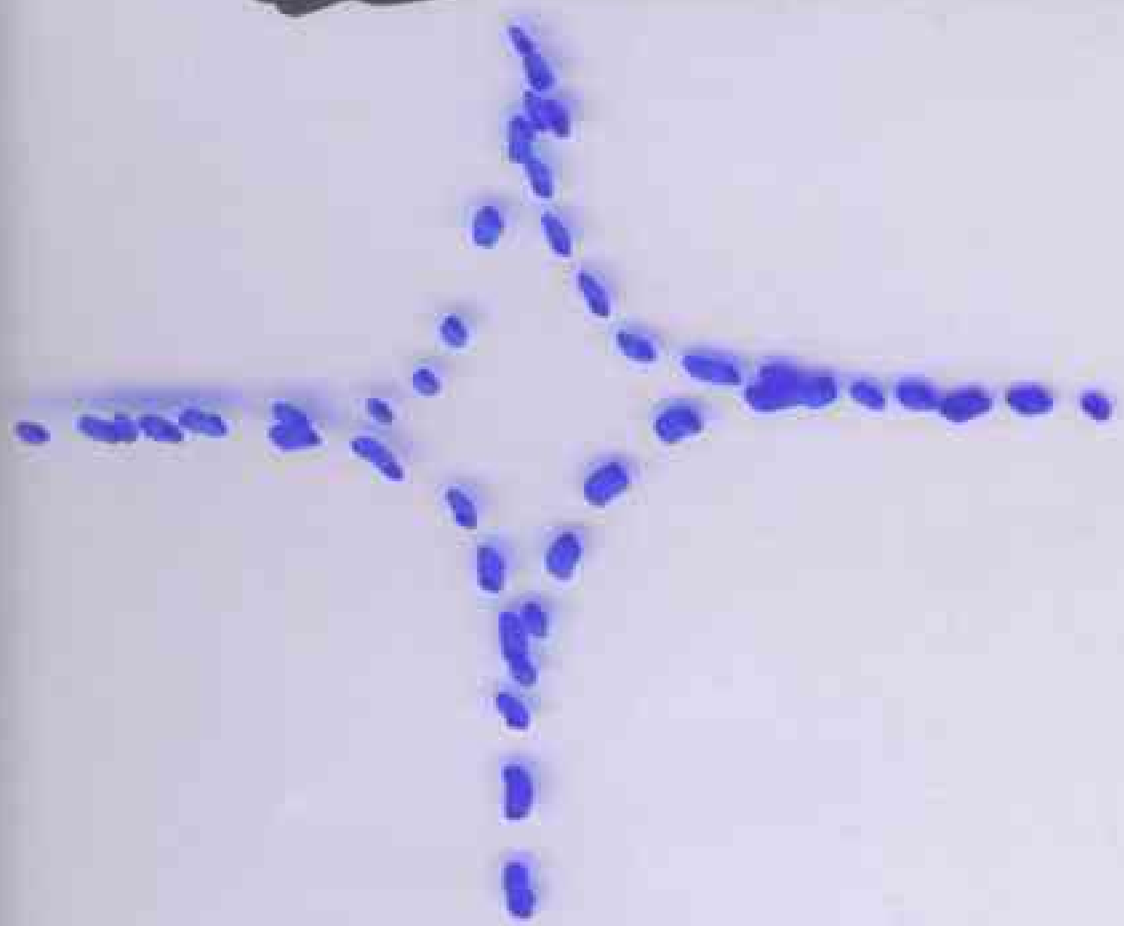
SU(N) × SU(N) × U(1)

+ QUARTIC SUPO

+ DECOUPLED U(1)

N D1-BRANES ON BU OF CONI

"MAGNETIC THEORY"



D5 – DIAMOND

$\downarrow T_{48}$

$D1_{01} - D5_{01\,4567} - D5'_{01\,45\,89}$

= 16 SUSY THEORY

+ 2 EXTRA FLAVORS

+ SUPO

$$U(1) + \underset{0}{Y} + \underset{0}{Z} + \underset{+1}{Q} + \underset{-1}{\tilde{Q}} + \underset{+1}{T} + \underset{-1}{\tilde{T}}$$

$$W = YQ\tilde{Q} + ZT\tilde{T} + hQ\tilde{T} + \tilde{h}\tilde{Q}T$$

$h, \tilde{h}$: D5 – D5' HYPERMULTIPLETS

(KACHRU, OZ, YIN) = DIAMOND

+ $\underset{X}{\text{FREE}}$ CHIRAL (C.O.M.)

→ FREE VM

~ FREE CHIRAL

→ $U(1)$ + 2 FLAVORS

~ $U(1)$ + 2 FLAVORS +
+ SINGLETS + SUPO

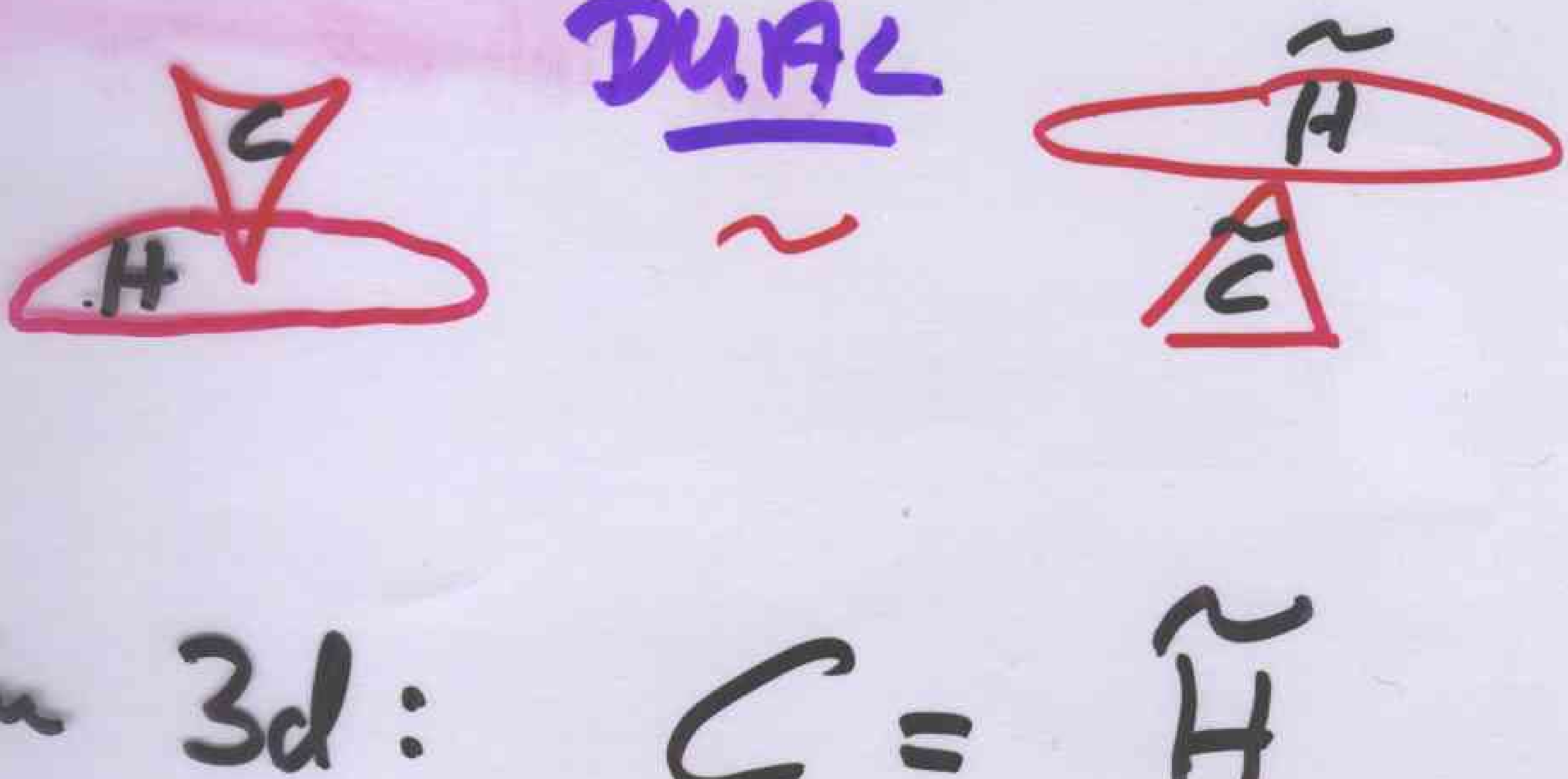
→ $SU(N) \times SU(N) \times U(1)$

~ $U(N)$ + 3 ADJOINTS
+ SUPO + 2 FLAVORS

MANY MANY MORE
MIRRORS CAN BE
EASILY DERIVED THIS
WAY ....

WHAT DO WE MEAN BY "MIRROR - DUALITY" IN TWO DIMENSIONS ?

IN 2D THERE ARE TWO DIFFERENT MIRROR GAUGE DUALITIES POSSIBLE



in 3d:

$$C = \tilde{H}$$

$$H = \tilde{C}$$

IN 2D THE CONCEPT OF MODULI SPACE IS ILL DEFINED

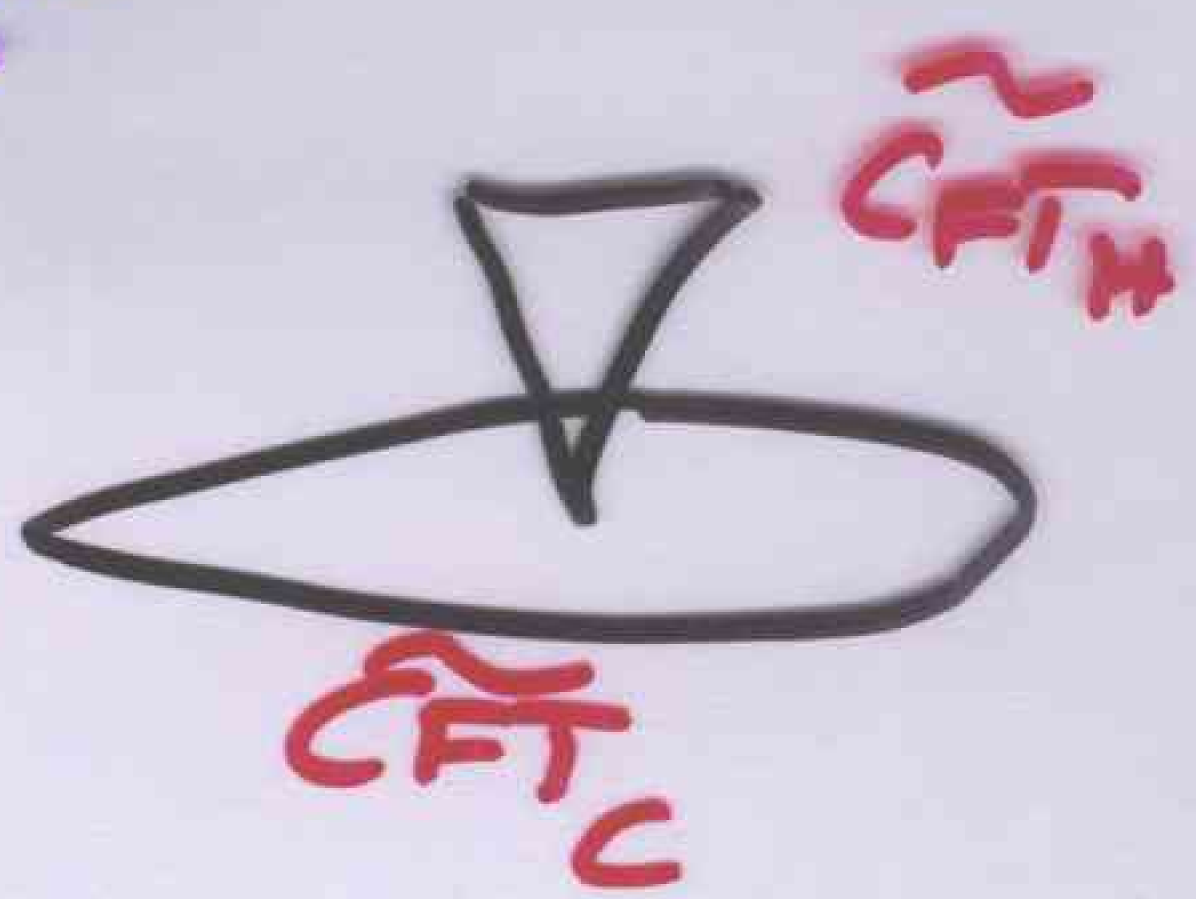
IF WE FLOW TO AN INTERACTING IR THEORY (AS WE DO), IT IS THE NL σM WITH THE "MODULI" SPACE AS ITS TARGET SPACE

IN THE EXAMPLES WE STUDY THERE ARE ACTUALLY 2 DIFFERENT CFTs FOR THE 2 (QUANTUM-MECHANICALLY) DISJOINT BRANCHES

(WITTEN)

DUAL

$\longleftrightarrow$



IN 2D:

$$CFT_C = \widetilde{CFT}_H$$

$$CFT_H = \widetilde{CFT}_C$$

DOES NOT REQUIRE

$$C = \tilde{H}$$

$$H = \tilde{C}$$

"BORING MIRROR"

- → INHERITED FROM $\mathcal{N}=4$; $d=3$
- → $H = \tilde{C}$
  $C = \tilde{H}$

$\Rightarrow$ $CFT_H = NL\sigma M$ on H

$$\parallel$$

$\widetilde{CFT}_C = NL\sigma M$ on $\tilde{C} = H$

"EXCITING MIRROR"

$$CFT_H = \text{NL}\sigma\text{M on } H$$

$$\|$$

$$\widetilde{CFT}_C, \quad \text{NL}\sigma\text{M on } \tilde{C}$$

$\tilde{C}$ BEING THE GEOMETRIC MIRROR OF H, THAT IS A DIFFERENT SPACE LEADING TO THE SAME CFT !

BLOWUP OF CONI $\overset{S^2}{}$ ⟷ DEF. OF CONI $\overset{S^3}{}$

HOW TO DISTINGUISH ?
WHICH ONE IN (WHICH)
BRANE SETUP ?

- R-SYMMETRIES:

$U(1) \times \widetilde{U(1)}$

$U(1)_{23}$ $\downarrow$ 4d-R-SYMM.

$U(1)_{L,R} = U(1) \pm \widetilde{U(1)}$

DIFFERENT
ON COULOMB
AND HIGGS
BRANCH

BORING: $\left[U(1)\times\widetilde{U(1)}\right]_H \longleftrightarrow \left[U(1)\times\widetilde{U(1)}\right]_C$

EXCITING: $\left[U(1)\times\widetilde{U(1)}\right]_H$

$\updownarrow$

$\left[\widetilde{U(1)}\times U(1)\right]_C$

FLIP SIGN
OF
$U(1)_R$!

- PARAMETER MAP:

START WITH CONI AS HIGGS BRANCH, BLOWUP MODES ARE FI-TERMS

FI $\xrightarrow{\text{BORING}}$ TWISTED MASSES ("BLOWUP-MODES") = BACKGROUND VM

$\xrightarrow{\text{EXCITING}}$ SUPO- MASS = BACKGROUND CM

- AND OF COURSE: MODULI SPACE

BLOWUP OF CONI $\xrightarrow{\text{BORING}}$ BLOWUP

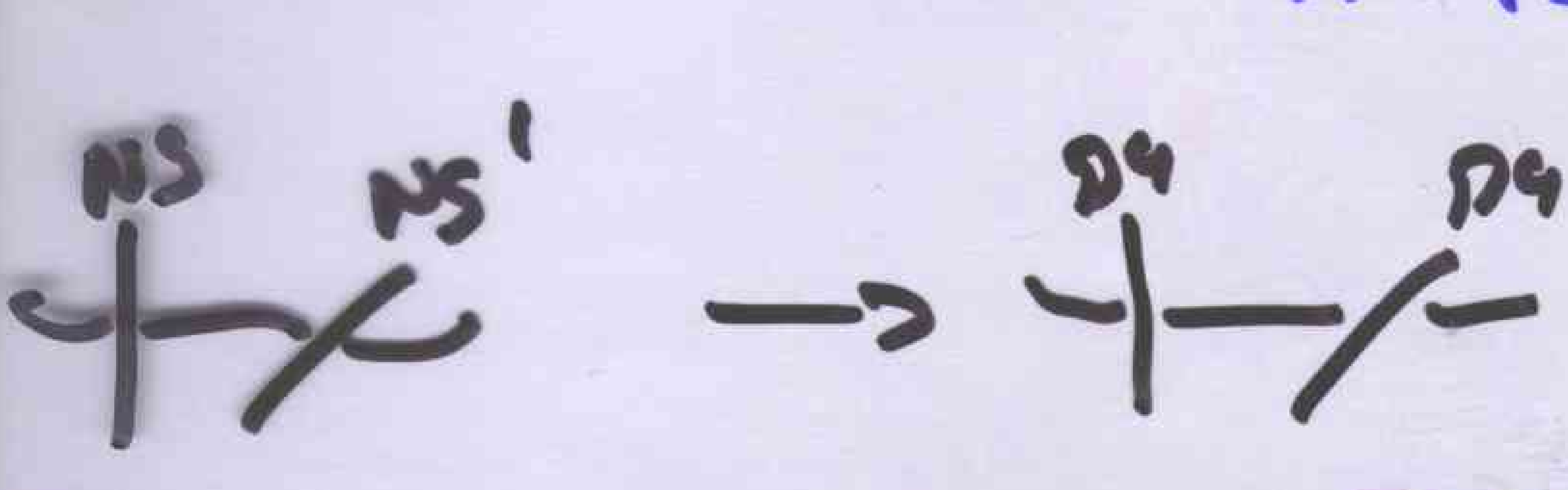
$\xrightarrow{\text{EXCITING}}$ DEFORMATION

BORING MIRROR SYMMETRY CAN BE SEEN IN "INTERVAL" THEORIES VIA 2-10 FLIP

(IIA-VERSION OF S-DUALITY)

BRODIE $\mathcal{N}=(4,4)$

HANANY-HORI $\mathcal{N}=(2,2)$



BLOWUP OF CONI

BLOWUP OF CONI

BUT IT IS THE

"EXCITING" VERSION

WE OBTAIN FROM

THE DIAMONDS!

→ SEE FOR EXAMPLE

THE $U(1)$ - R - CHARGES

HOW DOES THIS WORK IN OUR EXAMPLE?

$U(1)$:

σ VM

$$\left.\begin{array}{c} Z \\ Y \end{array}\right\} \text{neutral chiral}$$

$$\left.\begin{array}{cc} Q & \tilde{Q} \\ T & \tilde{T} \end{array}\right\} \text{CM}$$

$$\begin{array}{cc} +1 & -1 \end{array}$$

$$W = Z Q \tilde{Q} + Y T \tilde{T} + h Q \tilde{T} + \tilde{h} \tilde{Q} T$$

WANT TO SHOW:

QUANTUM - COULOMB BRANCH
IS DEFORMATION OF
CONIFOLD

CB - BRANCH: NEUTRAL SCALARS

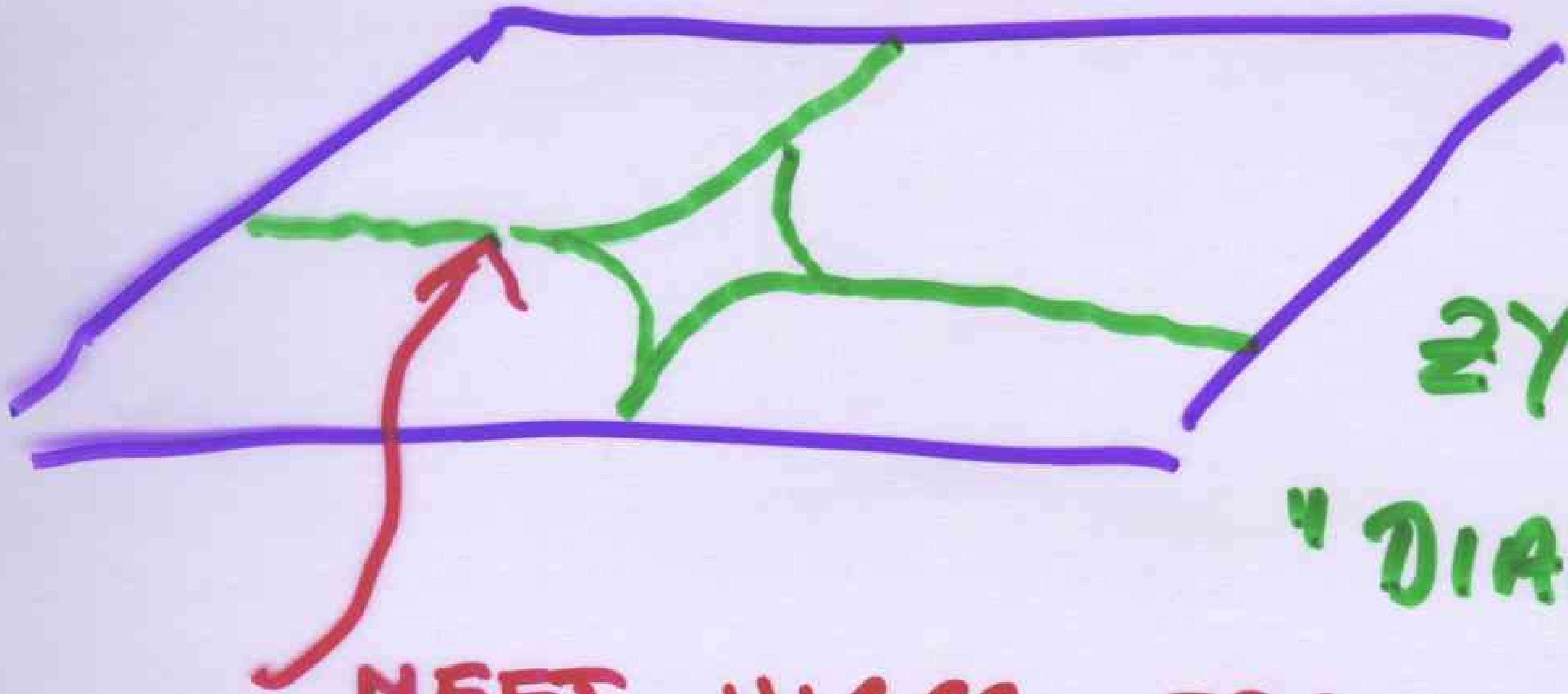
$$\underset{(VM)}{\sigma} + \underbrace{Z + Y}_{\text{SINGLET CM}}$$

AT $ZY = \varepsilon$ WE TOUCH THE HIGGS BRANCH SINCE A FLAVOR COMES DOWN TO ZERO MASS

$$\varepsilon = h^{Y}_{h}$$

$\Rightarrow$ CLASSICAL CB-BRANCH

Z-Y-PLANE



$ZY = \varepsilon$

"DIAMOND"

MEET HIGGS BRANCH

HOW DOES THE FULL QUANTUM-STORY LOOK LIKE?

WE CAN SEE THE DEFORMED CONIFOLD EXPLICITELY IN $D=3$

D6 - D6' - D4 DIAMOND

D2 PROBE ON D6 - D6'

INTRODUCE EFFECTIVE VARIABLES ON COULOMB BRANCH V_+, V_- AND WRITE DOWN

(AHARONY, HANANY, INTRILIGATOR, SEIBERG, STRASSLER)

UNIQUE SUPERPOTENTIAL CONSISTENT WITH SYMMETRIES AND HOLOMORPHY

$$V_+ V_- + Z Y = \underbrace{h\tilde{h}}_{=\varepsilon}$$

DOES THIS TEACH US SOMETHING ABOUT 2D ?

(FOLLOWING: SEIBERG & DIACONESCU)

FOR: $g^2_{YM}\big|_{3D} > \frac{1}{R}$:

3D GAUGE-THEORY

$\downarrow$ 3D GAUGE DYNAMICS

$V_+V_- + YZ = \Sigma$

$\downarrow$ COMPACTIFY

NLσM ON: $V_+V_- + YZ = \Sigma$

THIS IS CONFORMAL AND HENCE DOES NOT EVOLVE ANY FURTHER!

UV

$g^2_{YM}\big|_{3D}$

$1/R$

IR

⇒ IN APPROPRIATE VARIABLES WE WILL SEE THE DEF. OF THE CONIFOLD AS THE TARGET SPACE OF THE 2D - THEORY AS WELL

"STRINGY VERSION"

D2 ON D6 + D6'

$\downarrow$ GO TO IR

M2 ON CALABI YAU

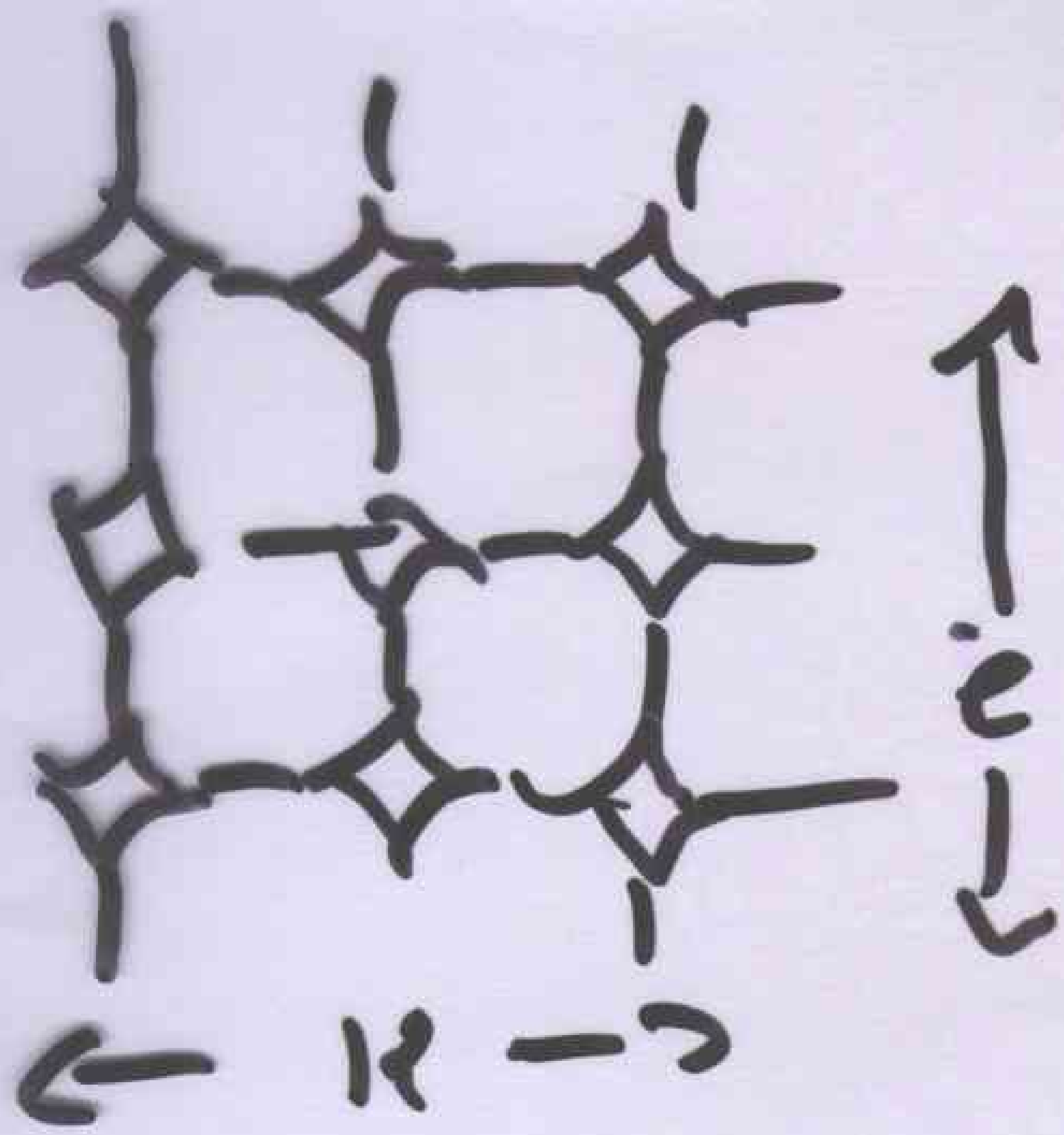
V_+, V_- DESCRIBE DEG. OF S^1 FIBERS

$\downarrow$ COMPACTIFY

F1 ON CALABI YAU

WV THEORY IS THE CALABI YAU SPACE

GENERAL:



$U(N)^{2K\ell}$

+ BIFU

+ QUARTIC

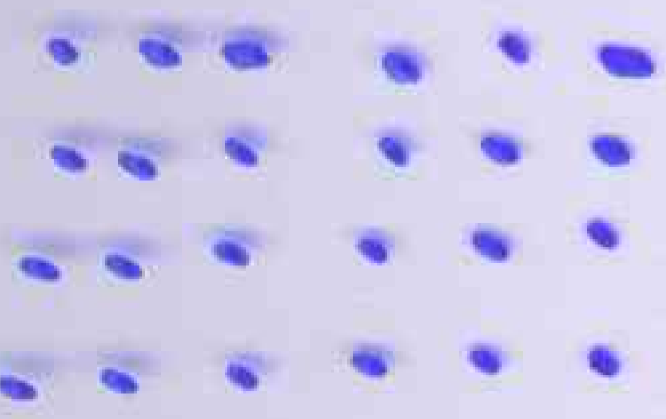
$\updownarrow$

$U(N)$ + 3 adj.

+ $(K+\ell)$ FLAVORS

BLOWUP OF $C_{K\ell}$

DEFORMATION OF $G_{K\ell}$



$\longleftrightarrow V_+ V_- =$

$= \sum_{ij} m_{ij} z^i y^j$

AS LINEAR σ MODEL

WE HENCE SEE:

- GEOMETRIC MIRROR SYMMETRY AS A GAUGE-THEORY DUALITY
  (AS SUGGESTED BY MORRISON AND PLESSER)
- A DESCRIPTION OF D-BRANES WITH THE DEFORMATION OF THE CONIFOLD AS THEIR MODULI SPACE
  ("NONABELIAN VERSION")