ABSORPTION BY 3-BRANES and THE AdS/CFT CORRESPONDENCE

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I. Normalized 2-point functions of $\mathcal{N}=4$ SYM from 3-brane absorption cross-sections. New results on higher partial waves of the dilaton.

II. AdS approach to operators with dimensions $\frac{d}{2} - 1 < \Delta < \frac{d}{2}$

Correlation functions from Legendre transform.

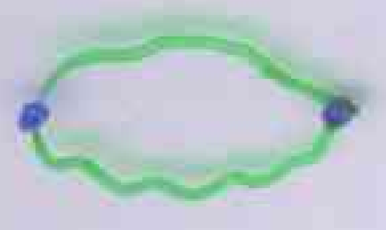
III. AdS/CFT duality in the context of type 0 string theory without spacetime SUSY.

IV. Arguments for tachyon stabilization for small AdS_5 radius.

$$\sigma = \frac{2\kappa^2}{2i\omega}\, \text{Disc}\, \Pi(p) \Big|^{-p^2=\omega^2+i\epsilon}_{-p^2=\omega^2-i\epsilon}$$

$$\Pi(p) = \int d^4x\, e^{ip\cdot x}\langle O(x)\, O(0)\rangle$$

For the dilaton, $O = \frac{T(3)}{4}\, \mathrm{tr} F^2$;

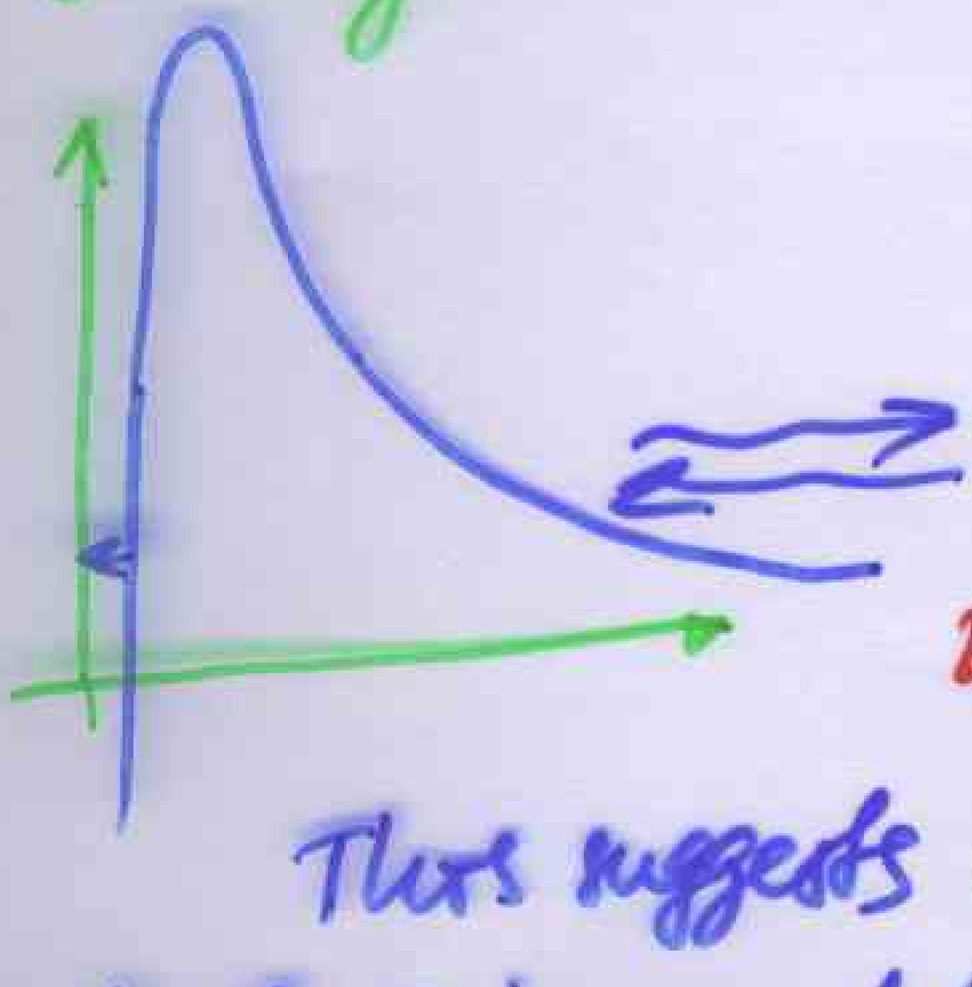
 gives $\langle O(x)O(0)\rangle = \frac{3N^2}{\pi^4|x|^8}$ which may be converted into the cross-section

$$\sigma = \frac{\kappa^2\omega^3N^2}{32\pi}.$$

On the SUGRA side (strong coupling) absorption is determined from $\Box\phi = 0 \Rightarrow \phi = \rho^{-5/2}\chi$

$$\left[\frac{d^2}{d\rho^2} - \frac{15}{4\rho^2} + 1 + \frac{(\omega R)^4}{\rho^4}\right]\chi(\rho) = 0$$ (for s-waves)

Solving the tunneling problem for $\omega R \ll 1$,



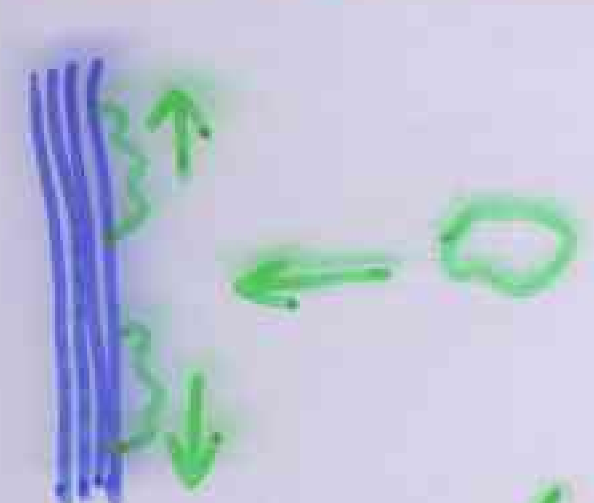
$$\sigma = \frac{\pi^4}{8}\omega^3 R^8;$$

Using $R^4 = \frac{N\kappa}{2\pi^{5/2}}$, this agrees with the 1-loop result!

This suggests a non-renormalization theorem: 1-loop is exact (related by SUSY to $\langle T(x)T(0)\rangle = \frac{c}{|x|^8}$).

There are TWO substantially equivalent but somewhat different in detail approaches to two-point functions at strong coupling.

The older approaches relies on absorption by the 3-brane geometry.



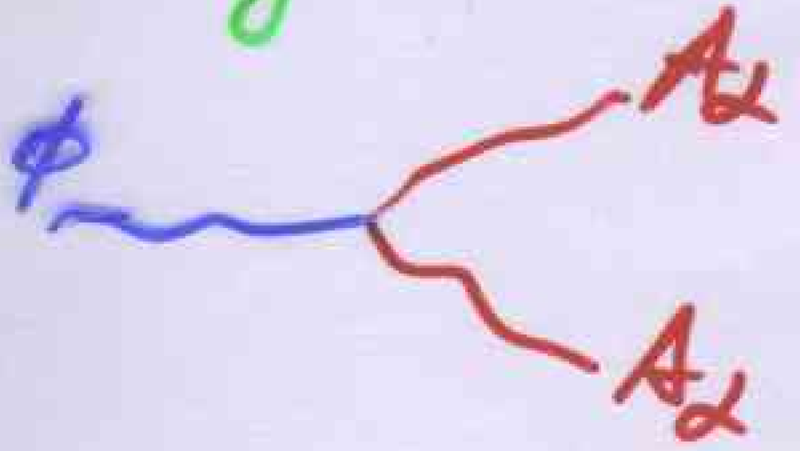
Absorption by the stack of D3-branes at weak coupling converts a closed string to a number of open strings.

$$S = S_0 + T_{(3)} \int d^4x \left[\frac{1}{4} \phi \, \mathrm{tr} F_{\alpha\beta}^2 - \frac{1}{4} C F_{\alpha\beta} \tilde{F}^{\alpha\beta} + \frac{1}{2} h^{\alpha\beta} T_{\alpha\beta} + \dots \right]$$

The DBI action establishes a correspondence between bulk fields and gauge invariant operators in the $\mathcal{N}=4$ SYM theory.

$$S_{bulk} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{g} \left(R - (\partial_\mu \phi)^2 + \dots \right)$$

For leading order dilaton absorption,

$\mathcal{M} =$ 

$$\sigma = \int [\text{phase space}] \, |\mathcal{M}|^2.$$

Recently, similar exact agreement was found for ALL partial waves of the dilaton (IK, W. Taylor, M. Van Raamsdonk)

On SUGRA side, the equation is

$$\left[\rho^{-5}\frac{d}{d\rho}\rho^{5}\frac{d}{d\rho} - \frac{\ell(\ell+4)}{\rho^2} + 1 + \frac{(\omega R)^4}{\rho^4}\right]\phi = 0;$$

$$\sigma^{(\ell)} = \frac{\pi^4}{24}\,\frac{(\ell+3)(\ell+1)}{[(\ell+1)!]^4}\left(\frac{\omega R}{2}\right)^{4\ell}\omega^3 R^8 .$$

Agrees exactly with leading order $\langle \mathcal{O}^{(\ell)}(x)\mathcal{O}^{(\ell)}(0)\rangle$

$$\mathcal{O}^{(\ell)} = \frac{T_{(3)}}{4\ell!}\,\mathrm{STr}\left[F_{\alpha\beta}F^{\alpha\beta}X^{(i_1}\cdots X^{i_\ell)}\right] +$$

+ fermion^2 term + fermion^4 term.

contributes for $\ell > 0$ contributes for $\ell > 1$

$$\mathcal{O}^{(\ell)} = Q_\alpha Q_\beta Q_\gamma Q_\delta\ \mathrm{Tr}\left[X^{(i_1}\cdots X^{i_\ell)}\right]$$

supercharges

This suggests non-renormalization theorems for ALL operators $\mathcal{O}^{(\ell)}$.

Summary: absorption cross-sections yield NORMALIZED 2-point functions at strong coupling.

If the AdS limit is taken first (Maldacena) then correlation functions may be calculated via the GKPW method

$$e^{-I(\phi_0)} = \left\langle e^{\int d^dx\, \phi_0 \mathcal{O}(x)} \right\rangle_{\text{gauge theory}}$$

$I(\phi_0)$ = minimum of Euclidean action subject to $\phi(x, z_{\text{cut-off}}) = \phi_0(x)$

$$ds^2 = \frac{1}{z^2}\left[dz^2 + (d\vec{x})^2\right]$$

$z_{\text{cut-off}}$ is the UV cut-off from the gauge theory point of view.

If try to remove cut-off first ($z_{\text{cut-off}} \to 0$) then the action needs careful regularization (IK, E. Witten).

Find the 2-point function

$$\langle \mathcal{O}(x)\mathcal{O}(0)\rangle = \frac{2}{\pi^{d/2}}\left(\Delta - \frac{d}{2}\right)^2 \frac{\Gamma(\Delta)}{\Gamma(\Delta - \frac{d}{2} + 1)} |x|^{-2\Delta},$$

where Δ is determined from

$\Delta(\Delta - d) = m^2$ (mass-squared in AdS_{d+1}).

$$\Delta_\pm = \frac{d}{2} \pm \sqrt{\left(\frac{d}{2}\right)^2 + m^2};$$

In the Breitenlohner–Freedman range

$$-\frac{d^2}{4} < m^2 < -\frac{d^2}{4}+1$$

Both Δ_+ and Δ_- are allowed!

The absolute lower bound on Δ is NOT $\frac{d}{2}$ but $\boxed{\frac{d}{2}-1}$, which is the UNITARITY BOUND in CFT_d.

The boundary behavior of a field in AdS_{d+1} is

$$\phi(z,x) \longrightarrow z^{d-\Delta}\left[\phi_0(x)+O(z^2)\right]+z^{\Delta}\left[A(x)+O(z^2)\right]$$

ϕ_0 is the "source"; $A\sim$ the "field" (BKLT)

$$A(x)=\frac{1}{2\Delta-d}\langle O(x)\rangle.$$

To interchange Δ and $d-\Delta$, $\phi_0 \leftrightarrow (2\Delta-d)A$ which is the Legendre transform of classical action.

We get the 2-point function which is positive for $\Delta > \frac{d}{2}-1 \Rightarrow$ the theory in AdS_{d+1} is indeed unitary.

<u>SUMMARY</u>: In the B-F range <u>TWO</u> different CFT's corresponding to same classical AdS action.

One theory where there are operators of dimension

$$\frac{d}{2}-1 < \Delta < \frac{d}{2}$$

is the CFT on N D3-branes placed on the CONIFOLD: $\sum_{a=1}^{4} z_a^2 = 0$

This CFT is dual to type IIB strings on $AdS_5 \times T^{1,1}$.

$T^{1,1} = (SU(2)\times SU(2))/U(1)$ is the Einstein space which is the BASE of the conifold: $ds^2_{\text{conifold}} = dr^2 + r^2 ds^2_{T^{1,1}}$

The IR limit of the field theory on D3-branes is $\mathcal{N}=1$ supersymmetric $SU(N)\times SU(N)$ gauge theory coupled to chiral superfields in bifundamental reps.

$$A_i \text{ in } (N,\bar{N}) \qquad (i,j=1,2)$$
$$B_j \text{ in } (\bar{N},N)$$

with exactly marginal superpotential (R-charge $=2$)

$$W = \epsilon^{ij}\epsilon^{kl}\,\mathrm{Tr}(A_i B_k A_j B_l)$$

$U(1)_R$ anomaly cancellation requires that the A's and the B's have R-charge $\frac{1}{2}$ and, therefore, dimension $\frac{3}{4}$.

The simplest class of chiral operators in the conifold field theory are

$$\mathrm{tr}\left(A^{i_1} B_{j_1} A^{i_2} B_{j_2} \ldots A^{i_k} B_{j_k}\right)$$

where the i's and j's are separately symmetric. The R-charge is k and $SU(2)\times SU(2)$ quantum numbers are $\left(\frac{k}{2}, \frac{k}{2}\right)$.

Since $\Delta = \frac{3k}{2}$, for $k=1$ we have $\Delta < \frac{d}{2}$.

Corresponding scalar in AdS_5 has $m^2 = -\frac{15}{4}$ (conformally coupled)

$$-4 < m^2 < -3$$

SUSY requires that we pick $\Delta_- = 2 - \sqrt{4+m^2} = \frac{3}{2}$ for this operator.

Had we picked Δ_+ the theory on AdS_5 would not be supersymmetric.

D-branes and Gauge Theories in Type 0 Strings

(I.K., A. Tseytlin)

Type 0 theory is the NSR string with a non-chiral GSO projection: $(-1)^{F_L+F_R}=1$.

No space-time fermions but twice as many bosons as in corresponding type II theory.

$(NS+,NS+);\ (NS-,NS-);\ (R+,R+);\ (R-,R-) \Leftarrow$ 0B

$(NS+,NS+);\ (NS-,NS-);\ (R+,R-);\ (R-,R+) \Leftarrow$ 0A

In addition to R-R boson $C_{(m)}$ present in type II theory we also find $\bar{C}_{(m)}$;

we can form
$$C^{\pm}_{(m)} = \frac{1}{\sqrt{2}}\left[C_{(m)} \pm \bar{C}_{(m)}\right]$$

There are TWICE AS MANY types of D-branes as in type II: Dp+ branes couple to $C^{+}_{(p+1)}$;
Dp− " " $C^{-}_{(p+1)}$.

Type 0B theory is non-chiral: the 4-form $C_{(4)}$ is unrestricted (while in type IIB $F_{(5)} = * F_{(5)}$).

D3+ branes couple to electric components of $F_{(5)}$

D3− " " magnetic "

The type 0 D-brane effective actions are (KT, Garousi)

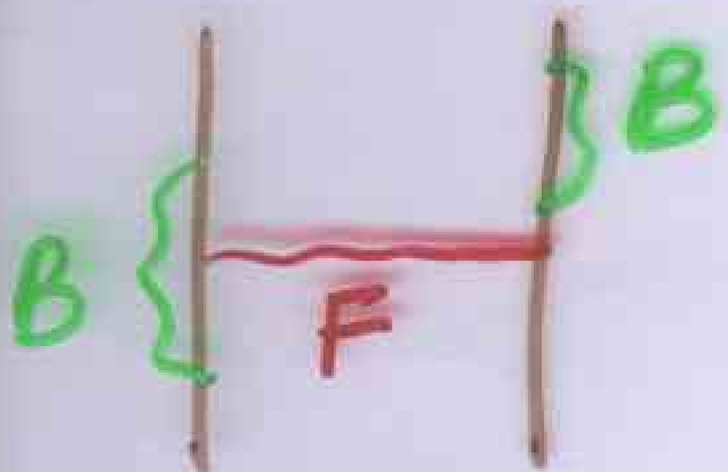
$$\int d^{p+1}\sigma\, e^{-\Phi} k_\pm(T)\sqrt{-\det\left(\hat{g}_{\alpha\beta}+2\pi\alpha' F_{\alpha\beta}\right)}\,,$$

$$k_\pm(T) = 1 \pm \frac{1}{4}T + O(T^2).$$

Putting together a Dp+ and a Dp− we cancel the tachyon tadpole.

On a single brane no spacetime fermions; no tachyons due to open string GSO projection $(-1)^{F_{open}}=1$.

Open strings connecting Dp+ and Dp− are spacetime fermions (BG)



On N parallel Dp+ and N Dp− branes find $U(N)\times U(N)$ gauge theory: dimensional reduction of 9+1 d theory coupled to a M–W fermion in $(N,\bar{N})$ and a " " $(\bar{N},N)$.

For 3-branes we find in the IR limit:

3+1 d $SU(N)\times SU(N)$ gauge theory with 6 adjoint scalars X^i of $SU(N)_1$; 6 adj. scalars Y^i of $SU(N)_2$; 4 Weyl fermions in $(N,\bar{N})$ and 4 in $(\bar{N},N)$.

This non-SUSY theory is <u>conformal</u> in the planar (large N) limit.

* 1-loop and 2-loop β-functions cancel for large N.
* The near-horizon region of the type 0 3-brane solution is $AdS_5 \times S^5$ with $T=0$ and $\Phi = \text{const}$ (since only $F_{(5)} = *F_{(5)}$ components are excited by selfdual 3-branes the solution is same as in IIB).
* This type 0 example of AdS/CFT duality is a $(-1)^{F_S}$ orbifold on the $\mathcal{N}=4$ duality found in IIB.

Indeed type 0 string is a $(-1)^{F_S}$ orbifold of type II: (NS−, NS−) and (R−, R∓) sectors appear as the TWISTED sectors of the orbifold.

* The gauge theory on N selfdual D3-branes is a $\mathbb{Z}_2$ orbifold of $U(2N)$ $\mathcal{N}=4$ SYM:

the $\mathbb{Z}_2$ is generated by $(-1)^{F_S}\gamma$;

γ is conjugation by $\begin{pmatrix} I & \\ & -I \end{pmatrix}$;

$(-1)^{F_S}$ acts as -1 from the center of $SU(4)_R$ (Nekrasov-Shatashvili)

$\Rightarrow$ This is one of the "orbifold" CFT's.

Planar untwisted sector correlators are same as in the "parent" $\mathcal{N}=4$ theory (Bershadsky, Kakushadze, Vafa).

Twisted sector crucially depends on the type 0B nature of the theory.

As $\lambda = g_{YM}^2 N \to \infty$, $AdS_5 \times S^5$ becomes locally flat $\Rightarrow$ has tachyon instability: $m_T^2 = -\frac{2}{\alpha'}$;

In the GKPW map the operator corresponding to T is

$$\mathcal{O}_T = \left[\frac{1}{4}F_{\alpha\beta}^2 - \frac{1}{4}G_{\alpha\beta}^2 + \frac{1}{2}(\partial_\alpha X^i)^2 - \frac{1}{2}(\partial_\alpha Y^i)^2 + \ldots\right]$$

For large λ, $\Delta = 2 \pm \sqrt{4+(mR)^2} = 2 \pm \sqrt{4 - 2\sqrt{\lambda}}$ is COMPLEX (a sign of CFT instability), but is real for $\lambda < \lambda_c$.

Another stabilizing effect is $m_T^2 \to m_T^2 + \frac{\text{const}}{R^2}$ due to the $F_{(5)}^2 T^2$ term in the spacetime action.

This effect increases λ_c.

The fact that theory is stable for sufficiently small $\lambda > 0$ is supported by perturbative gauge theory:

$$\Delta(\lambda) = 4 + \frac{\lambda^2}{8\pi^4} + \mathcal{O}(\lambda^3)$$

Dimension of $\mathcal{O}_T$ is real $\Rightarrow$ no instability!

AdS/CFT duality suggests that at $\lambda = \lambda_c > 0$ the sum over planar graphs defining $\Delta(\lambda)$ becomes singular (λ_c is the finite radius of convergence).

Singularities of planar graphs are typical in large N theories: here they correspond to transition from stability to instability in string theory on $AdS_5 \times X_5$ as the AdS radius is increased.