ISSUES

IN

AdS

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- DIJKGRAAF, J.M., MOORE
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$\left.\right\}$ TO APPEAR

PLAN

- AdS_2
- BRANE CREATION & AdS DECAY
- AdS_3 THERMAL PHASE TRANSITION
- D1/D5 GREYBODY FACTORS
- GRAVITY DUALS OF NON-COMMUTATIVE GAUGE THEORIES

AdS_2

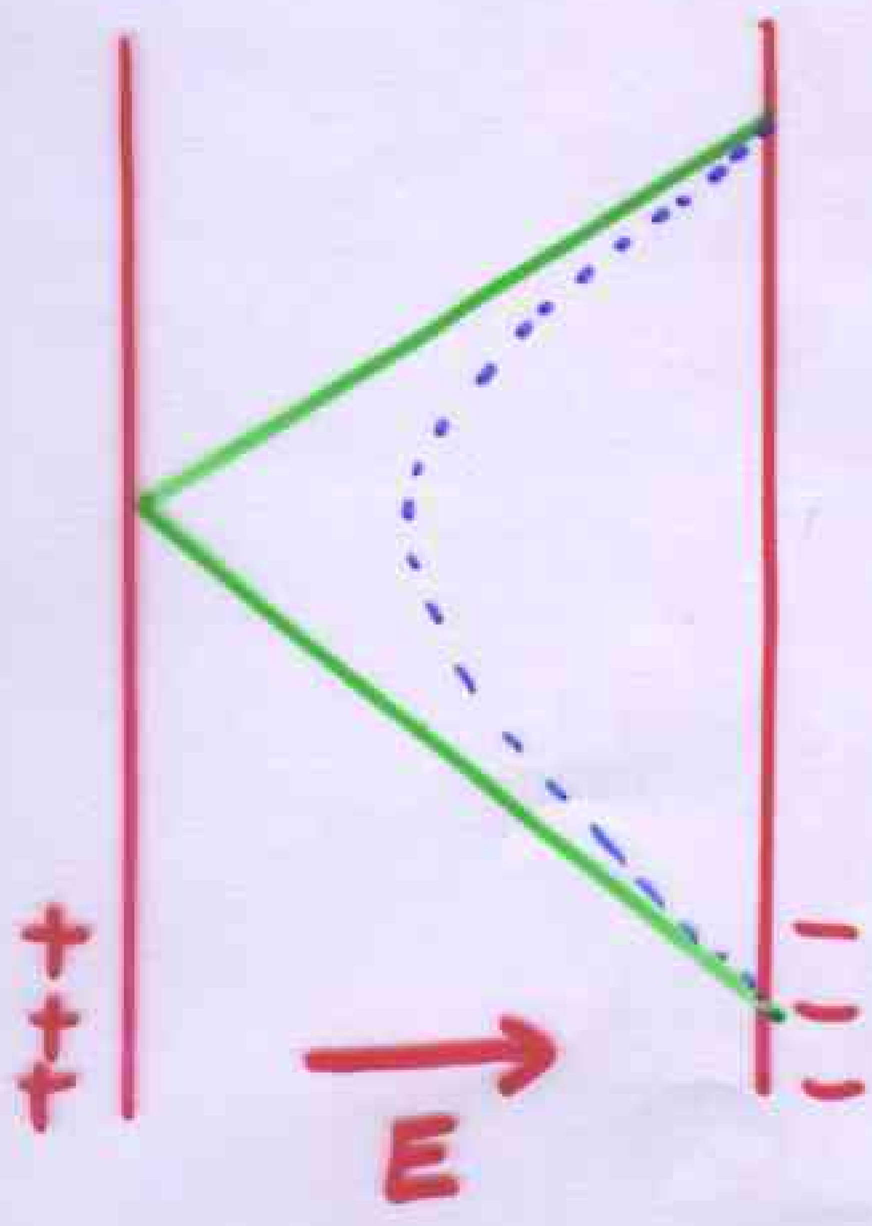
. NEAR HORIZON GEOMETRY OF BLACK HOLES

WITH $A_H \neq 0$ – $AdS_2 \times S_2$, $AdS_2 \times S_3$

. STANDARD LOW ENERGY LIMIT

$l_s \to 0$ $\qquad \frac{r}{l_s^2}$ = FIXED $\qquad N$ = FIXED

$$ds^2 = l_s^2 N^2 \left[\frac{-dt^2 + dz^2}{z^2} \right] = l_s^2 N^2 \left[\frac{-d\tau^2 + d\sigma^2}{\sin^2\sigma} \right]$$



2 BOUNDARIES

• SOME Q. M. ON THE BOUNDARY?

. FINITE ENERGY CONFIGURATIONS ARE <u>NOT</u> ALLOWED IF WE INSIST ON AdS BOUNDARY CONDITIONS

. NEAR EXTREMAL BLACK HOLE

$E \sim T^2 \, l_p$

T FIXED (ASYMPTOTIC GEOMETRY FIXED)

$\Rightarrow E \to 0$ WHEN $l_p \to 0$

(OTHER AdS/CFT's $\to E \sim T^{p+1} V_p$)

• $S = \int d^2\sigma \sqrt{g}\, e^{-2\phi} R$ + "MATTER"

$\downarrow$ CONSTRAINT

$e^{-\phi} \sim$ RADIUS OF S^2

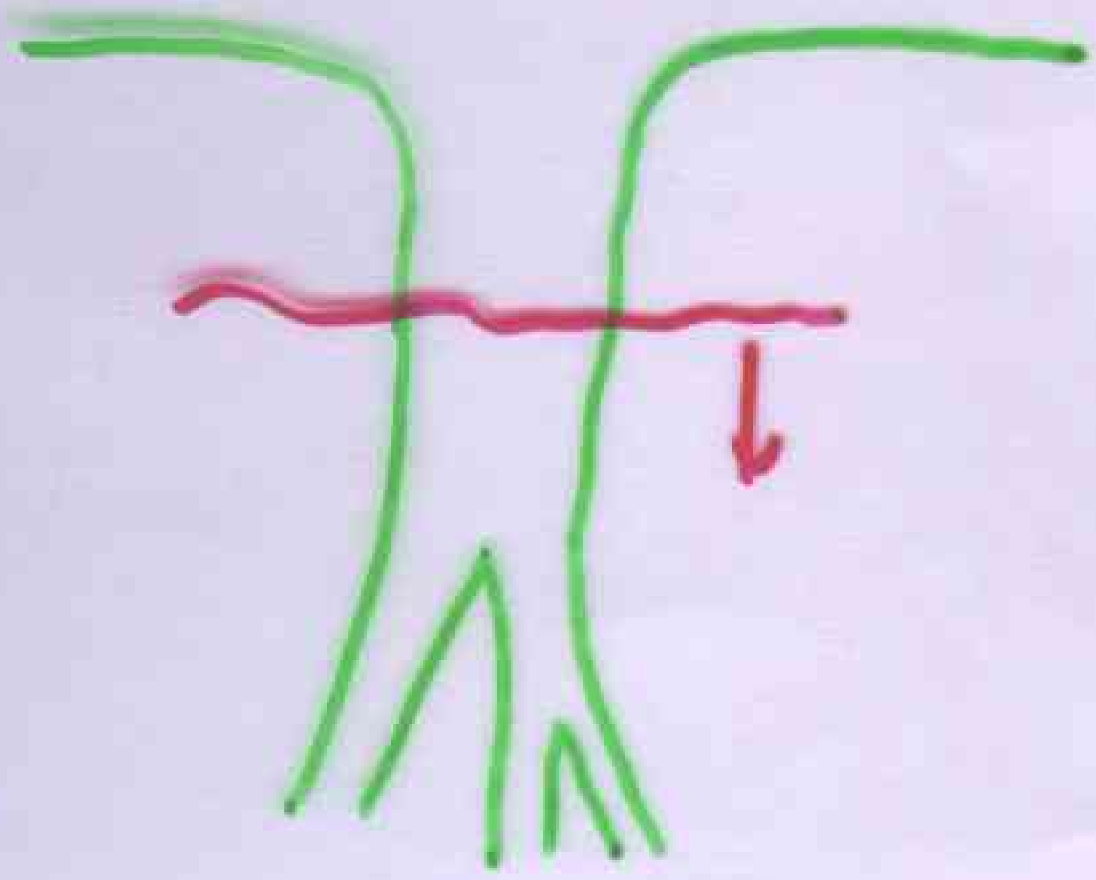
$\nabla_+ \nabla_+ e^{-2\phi} = T_{++}$

$\downarrow$ NEAR BOUNDARY OF AdS

$E \sim \sigma \partial_\sigma e^{-2\phi} \longrightarrow e^{-2\phi} \sim -E \log(\sigma) \xrightarrow[\sigma \to 0]{} \infty$

. ONLY GROUND STATES ARE ALLOWED

. MULTI-BLACK HOLE CONFIGURATIONS



$$ds^2 = -V^{-2}dt^2 + V^2 d\vec{x}^2$$

$$V = \cancel{1} + \sum_i \frac{Q_i l_p}{|\vec{x}-\vec{x}_i|}$$

. TWO BLACK HOLES

$\vec{U} = \vec{U}_1 - \vec{U}_2$ = RELATIVE COORDINATE

$$S = \frac{1}{2}\left(Q_1^3 Q_2 + Q_1 Q_2^3 \right) \int dt \, \frac{(\partial_t \vec{U})^2}{|\vec{U}|^3}$$

$$r \sim \frac{1}{\sqrt{|U|}}$$

$$S \sim \int 4\dot{r}^2 + r^2 \dot{\Omega}_3^2$$

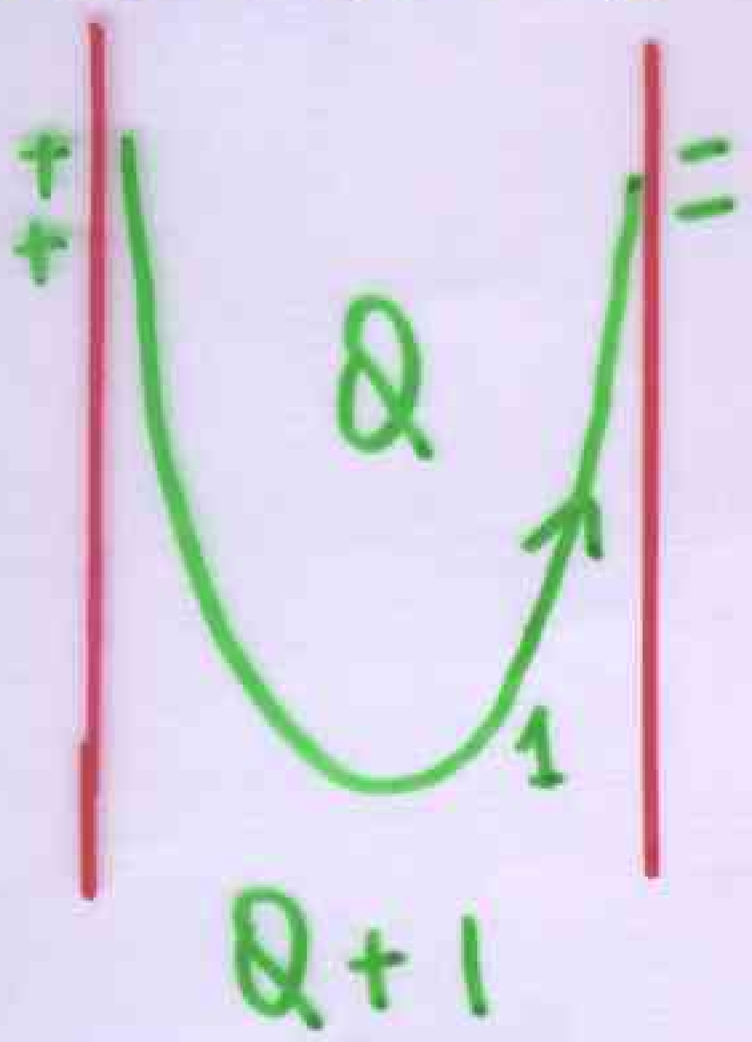
. FINITE VOLUME AT $\vec{U} \sim \infty$, $r \sim 0$

. INFINITE VOLUME AT $U \sim 0$, $r \sim \infty$

. BH ENTROPY FROM B.H. MODULI SPACE?

. GROUND STATES IN THE STRIP COORDINATES

INSTANTON - TUNNELING SOLUTION



. PAIR CREATION

$$E(r) \sim 2\left[\sqrt{(1+r^2)} - r\right] \xrightarrow[r \to \infty]{} 0$$

. TAKES ∞ TIME

. FINITE ACTION

BRILL INSTANTON

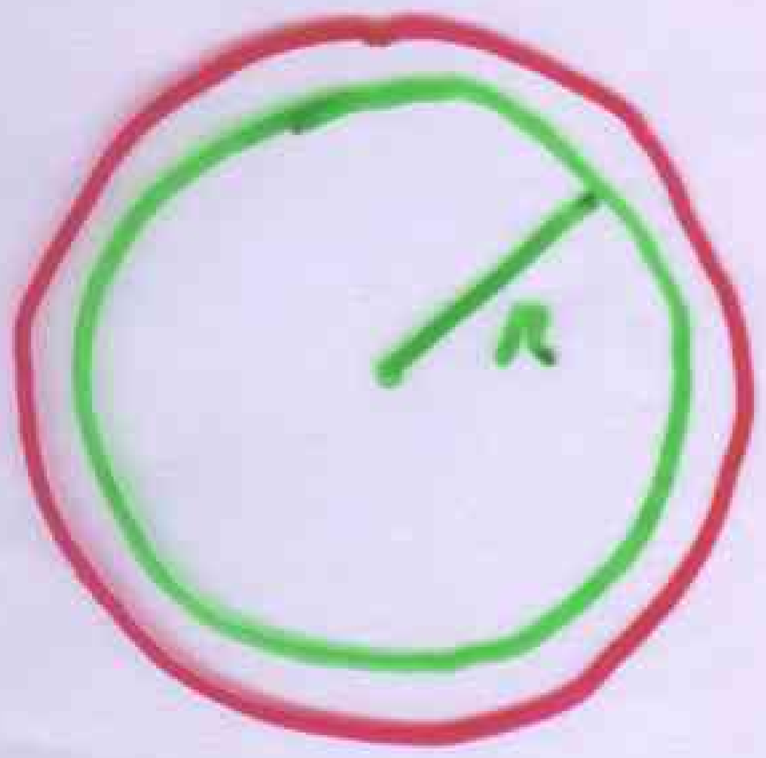
$$\text{ACTION} = \pi\, Q_1 Q_2 = -\tfrac{1}{2}\Delta S_{BH}$$

$\downarrow$

DIFFERENCE IN ENTROPIES

BRANE CREATION

. CONSTANT F_{p+2} FIELD STRENGTH IN AdS_{p+2} MIGHT LEAD TO BRANE CREATION → DECAY?



$$S = T_p \int d\tau \left\{ r^p \sqrt{1 + \dot{r}^2} - q\, r^{p+1} \right\}$$

$$q = \frac{\text{CHARGE}}{\text{CHARGE BALANCING GRAV. FIELD}}$$

SUSY $\Rightarrow q \leq 1$,

NON- SUSY CAN HAVE $q > 1$

(ELECTRON $q = 10^{21}$)

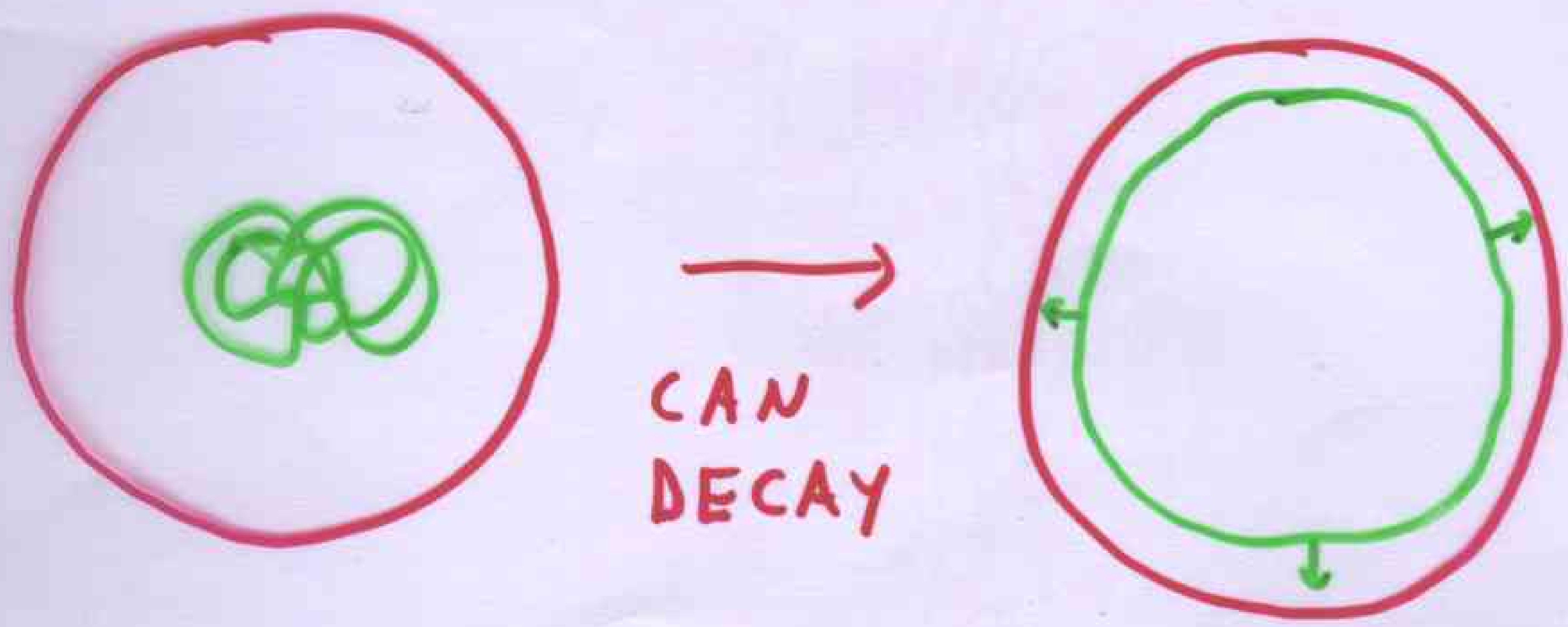
• $q > 1 \rightarrow$ UNSTABLE

• $q = 1$

• $p = 1$ $\quad E \rightarrow$ CONST $\quad$ WHEN $r \rightarrow \infty$

• $p > 1$ $\quad E \rightarrow \infty$ $\quad$ WHEN $r \rightarrow \infty$

IN AdS_3 WE CAN HAVE LONG STRINGS NEAR THE BOUNDARY



DECAY OF A NON SUSY AdS_3

- IIB ON K3

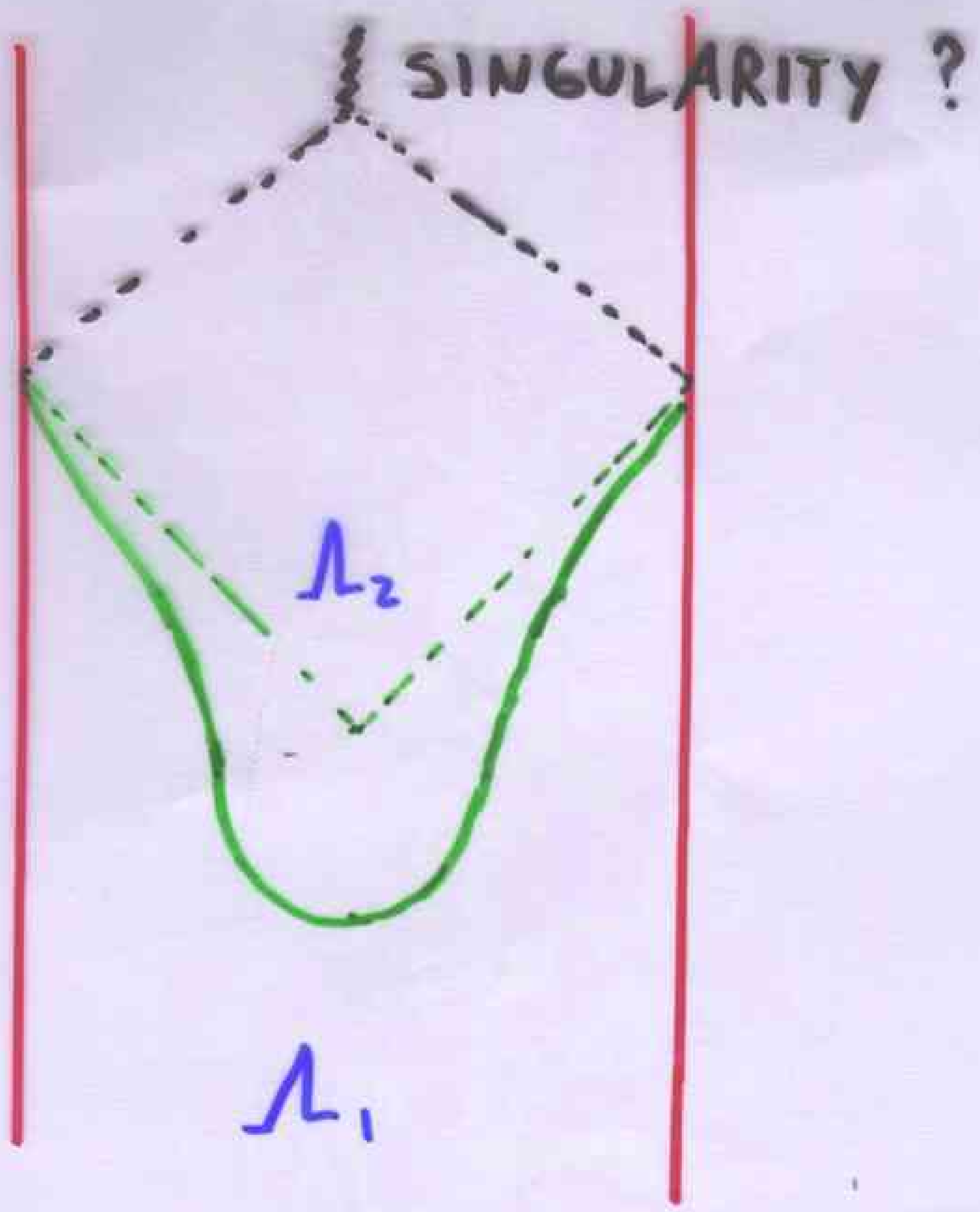
  Q_5 D5 ON K3 $\quad Q_5 > 0$

  Q_1 D1

SUSY: 1). $Q_1 \geq 0$ $\quad T_{(Q_5, Q_1)} = |V_4 Q_5 + Q_1|$

2). $Q_5 = 1$, $Q_1 = -1$

- TAKE $Q_1 = -|Q_1|$
  - CLASSICAL SOLUTION → SAME $AdS_3 \times S_3 \times K3$
  - $T_{(1,-1)} = V_4 - 1 < V_4 + 1 \Rightarrow q > 1$
- DECAYS
  - (PERTURBATIVELY IT WAS OK)



COLEMAN
DE LUCIA

$\Lambda_2 < \Lambda_1$

THERMAL PHASE TRASITION

IN AdS_3

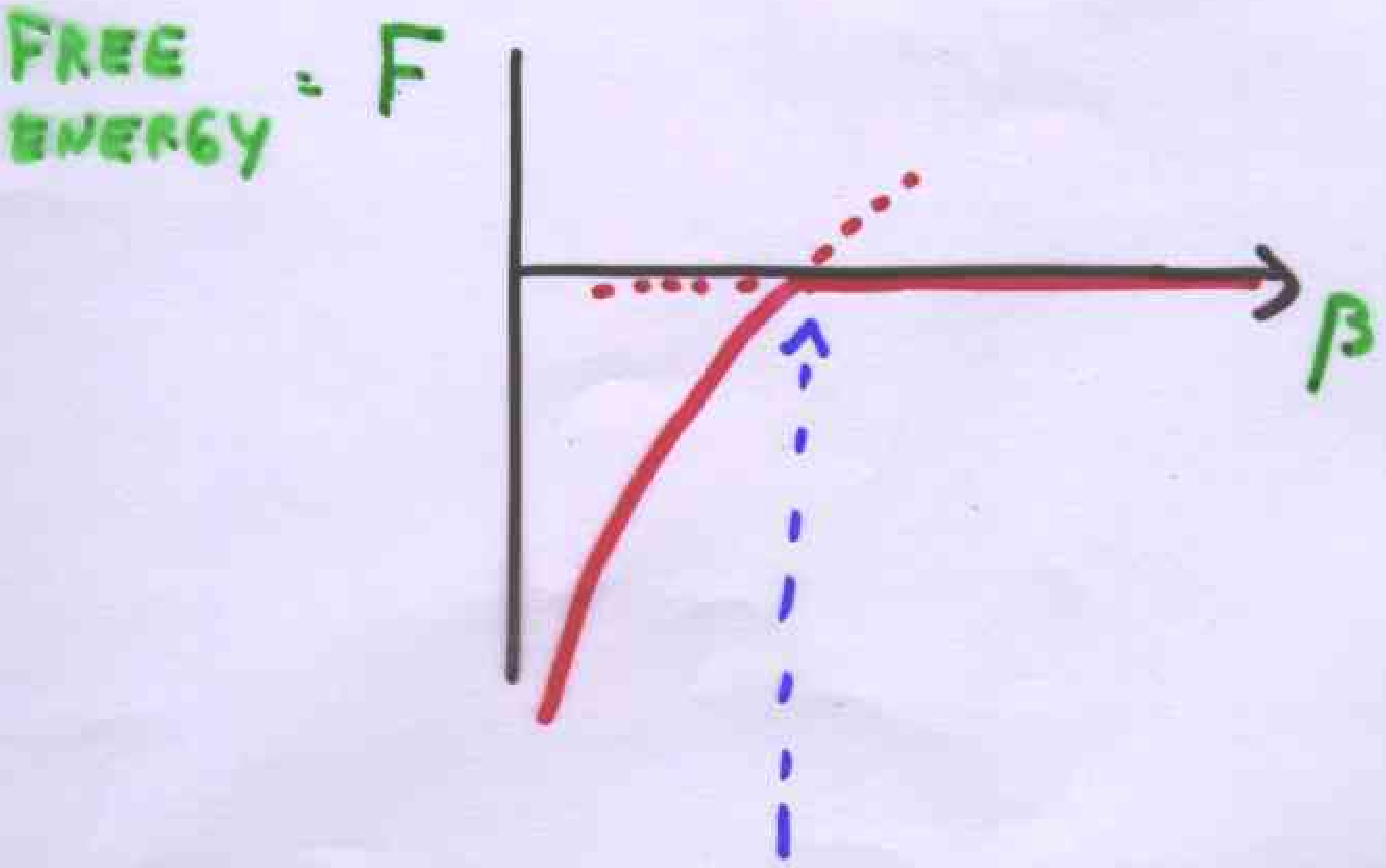
J.M.

- FINITE TEMPERATURE

- BOUNDARY IS A T^2 OF SIZES $(2\pi) \times \beta$

- 2 POSSIBILITIES
  - ADS_3 WITH TIME IDENTIFIED (S^1 CONTRACTIBLE)
  - BTZ (S^1_β CONTRACTIBLE)

  (NO GEOMETRY WHERE BOTH CYCLES ARE CONTRACTIBLE)

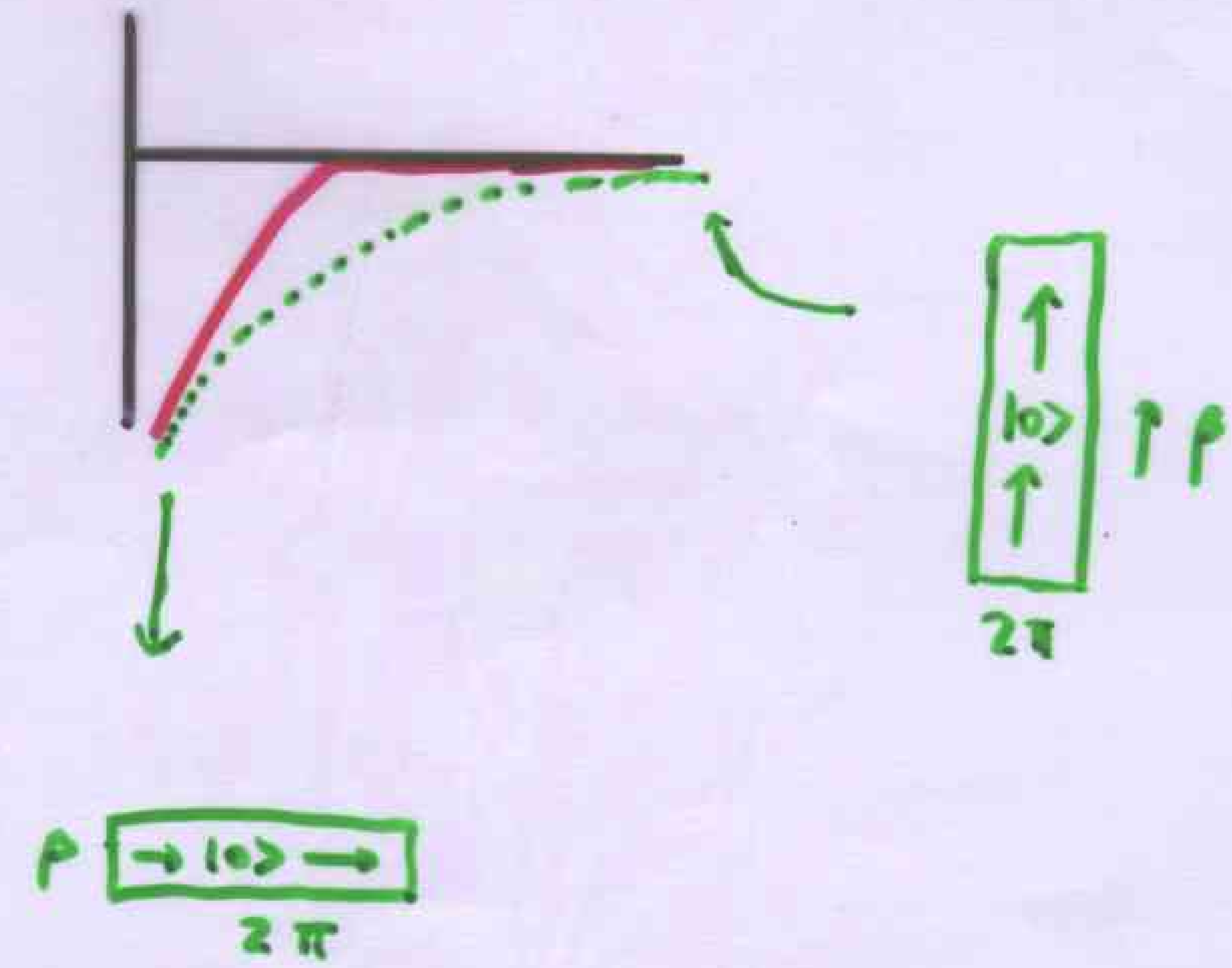


- CHOOSE LOWEST FREE ENERGY SOLUTION

HAWKING-PAGE PHASE TRANSITION

. WHY IS THIS PHASE TRANSITION HAPPENING?

IN A GENERIC CFT



. IN STRING THEORY

$$AdS_3 \times S_3 \times T^4 \longrightarrow$$ 2-d SIGMA MODEL WHOSE TARGET SPACE IS A DEFORMATION OF $Sym(T^4)^k$

$$\downarrow$$

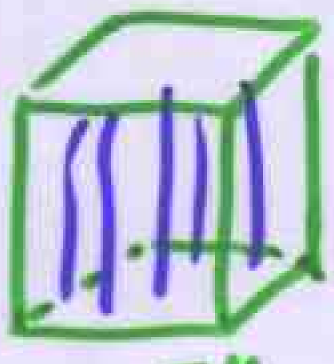
$$\frac{R}{l_p} \sim k^{1/4}$$

. IF WE TAKE EXACTLY THE SYMMETRIC PRODUCT → SUGRA APPROXIMATION IS NOT VALID

WE CAN SEE THE PHASE TRANSITION ALREADY AT THE SYMMETRIC PRODUCT POINT.

$\mathrm{Sym}(T^4)^k \longrightarrow$ GAS OF k STRINGS WINDING ON A CIRCLE AND MOVING AROUND A T^4

S^1



T^4

. MINIMUM ENERGY STATE $\rightarrow$ ALL SINGLY WOUND

. RAISE T $\rightarrow$ $\}\uparrow$ OSCILLATION ON THESE STRINGS $\rightarrow$ GAP ~ 1.

. MULTIPLY WIND w TIMES $\rightarrow$ LOWERS THE GAP

J.M. SUSSKIND

$$F = \underbrace{\frac{w}{2} + 2\pi^2 w T^2}_{E} - \underbrace{4\pi^2 w T^2}_{-TS}$$

$$F = \frac{w}{2}\left(1 - \frac{4\pi^2}{\beta^2}\right) < 0 \quad \text{FOR } \beta < 2\pi$$

. WE CAN ALSO SEE THIS PHASE TRANSITION IN THE ELLIPTIC GENUS (VALID EVEN AFTER DEFORMATION)

GREYBODY FACTORS FOR D1-D5

. IIB ON T^4

D1-D5 $\Rightarrow$ STRING IN 6-D

Q_1 D1

Q_5 D5

- ABSORPTION CROSS-SECTIONS FOR

  $m=0$ SCALARS _ S-WAVES

- GRAVITY

$$\sigma(\omega) = \pi^3 Q_1 Q_5\, \omega \left(1+\rho_L\left(\tfrac{\omega}{2}\right)\right)\left(1+\rho_R\left(\tfrac{\omega}{2}\right)\right)$$

- "EFFECTIVE STRING"

T^4

$Q_1 Q_5$

$$S = T\int\sqrt{G}$$

$$S \simeq T\int (\partial X)^2 + \underbrace{\partial X^I \partial X^J h_{IJ}}_{\text{COUPLING}}$$

- SAME ABSORPTION CROSS-SECTION

DAS - MATHUR

DHAR MANDAL WADIA

. WHY?

• $S_{INT} = \int d^2x \; \mathcal{O}(\vec{x}) \, h(\vec{x},0)$

• $A \sim \langle f | S_{INT} | i \rangle$

$\sigma \sim |A|^2$

$\sigma \sim \frac{1}{N_i} \sum_i \langle i | \int d^2x \; \mathcal{O}^{+}(\vec{x}) \mathcal{O}(0) e^{i\omega \cdot x} | i \rangle$

$\sigma \sim \langle \int \mathcal{O}^{+} \mathcal{O} \rangle_{\beta}$ → RIGHT DEPENDENCE ON ω.

• COEFFICIENT:

$$\langle \mathcal{O}^{+}(\vec{x}) \, \mathcal{O}(0) \rangle = \frac{c}{|\vec{x}|^4}$$

• c IS WELL DEFINED SINCE THE NORMALIZATION OF $\mathcal{O}$ IS FIXED BY SAYING IT IS THE OPERATOR THAT COUPLES TO h.

• ϕ^a = SET OF MASSLESS FIELDS

→ $SO(4,5)/SO(4)\times SO(5)$, $S = \int g_{ab}(\phi) \, \partial\phi^a \partial\phi^b$

$\cdot\ S_{int} = \int \mathcal{O}_a(\phi, \vec{x})\, \delta\phi^a$

$$\langle \mathcal{O}_a(\vec{x})\, \mathcal{O}_b(0) \rangle = \frac{G_{ab}(\phi)}{|x|^4}$$

G_{ab} → ZAMOLODCHIKOV METRIC

ϕ^a : MODULI OF THE CFT

· FIELD THEORY

. DEFORMATION OF $\mathrm{Sym}\,(T^4)^{Q_1 Q_5}$

. (4,4) SUSY $\Rightarrow$

MODULI SPACE IS CECOTTI

$$SO(4,5)\big/_{SO(4)\times SO(5)}$$

. SO $G_{ab}(\phi) = A\, g_{ab}(\phi)$

SUGRA

. WE DETERMINE A BY GOING TO THE ORBIFOLD POINT → DAS & MATHUR CALCULATION

THE LARGE N LIMIT OF NON-COMMUTATIVE GAUGE THEORIES

J.M., J. RUSSO - TO APPEAR

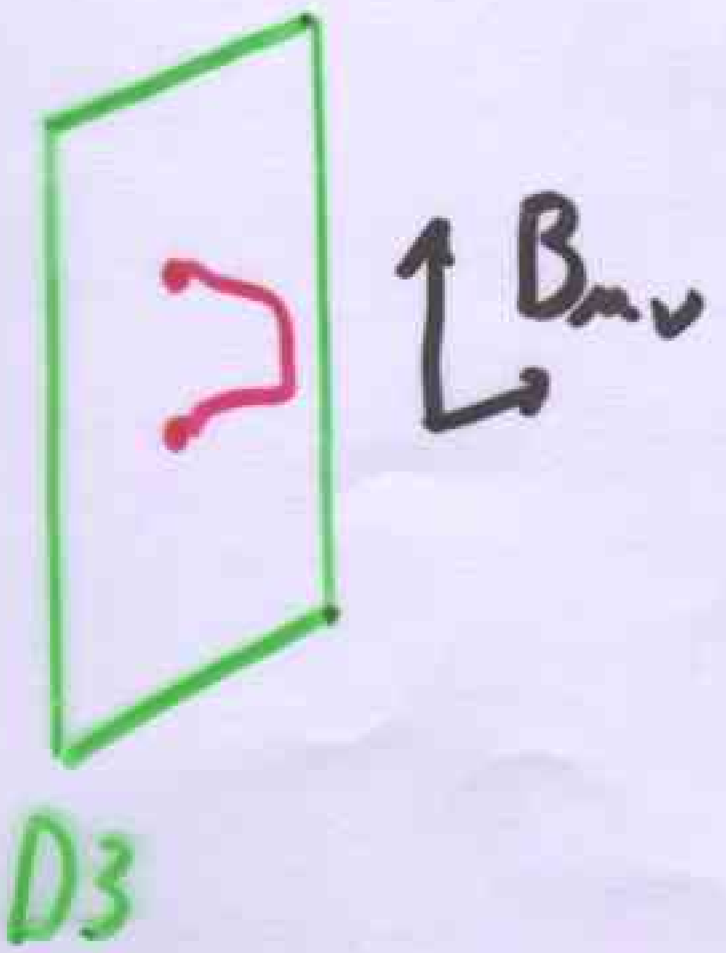
THE FIELD THEORY/GRAVITY CORRESPONDENCE ENABLES US TO STUDY NON-COMMUTATIVE GAUGE THEORIES AT STRONG COUPLING

- $[x^\mu, x^\nu] = i\,\theta^{\mu\nu}$ → SPACE-TIME COORDINATES DON'T COMMUTE

- $\mathcal{N}=4$ SYM ON NON-COMMUTATIVE $\mathbb{R}^4$

$\downarrow$

WORLVOLUME THEORY ON D3 BRANES IN A $B_{\mu\nu}$ - FIELD BACKGROUND



D3

- $\alpha' \to 0$

$B_{\mu\nu}$ = CONST

. FULL GRAVITY SOLUTION

. DECOUPLING LIMIT

→ SAME AS SEIBERG & WITTEN

$\alpha' \to 0$

$g_{ii} \sim \alpha'^2$ IN DIRECTIONS WHICH HAVE A B

$g_{ii} \sim 1$ IN DIRECTIONS $\perp$ TO B

$g_s \sim \alpha'^{\frac{r}{2}}$ r: RANK OF B.

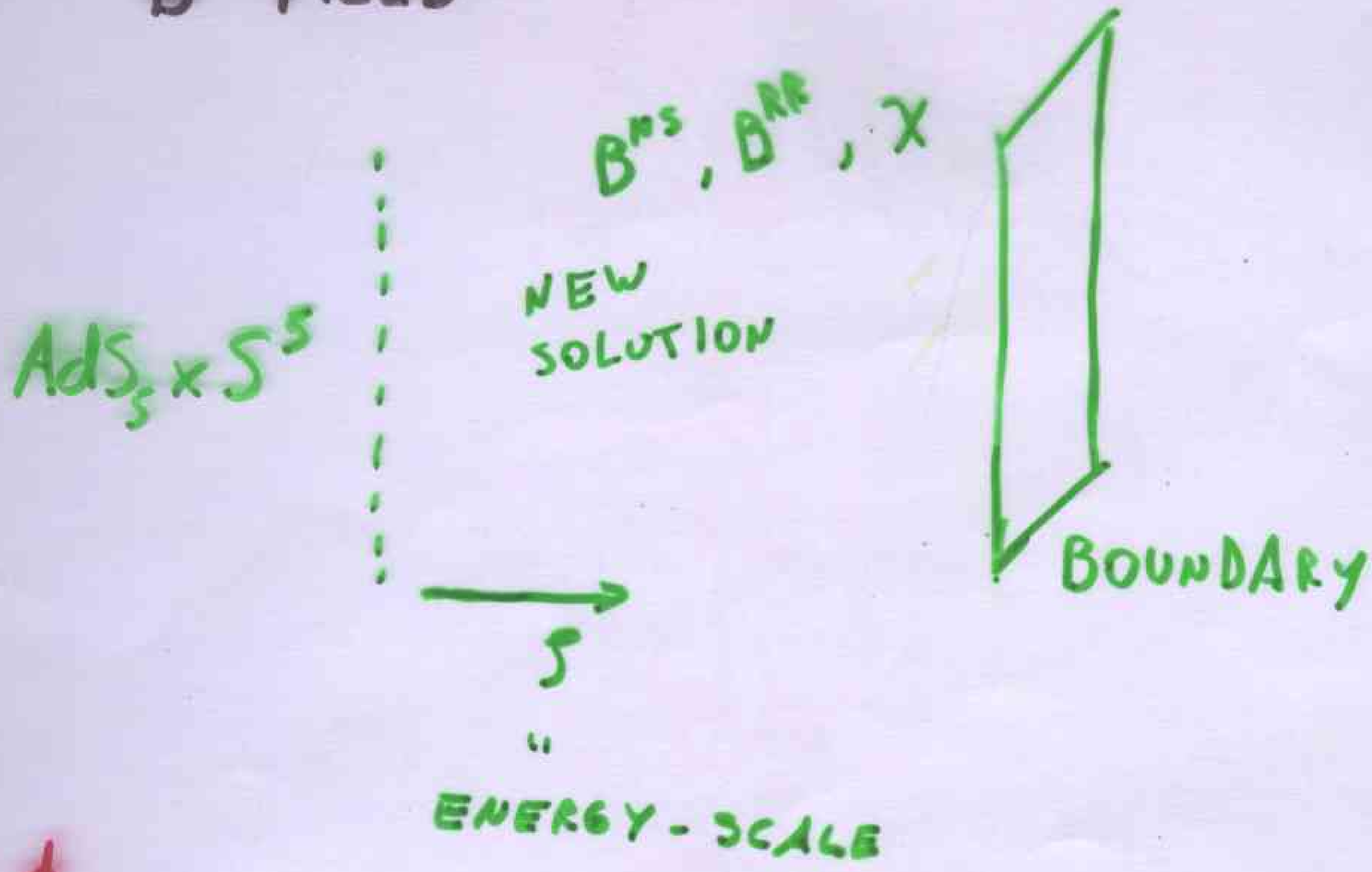
. NEAR HORIZON SOLUTION $B_{12} \neq 0$, $B_{34} \neq 0$

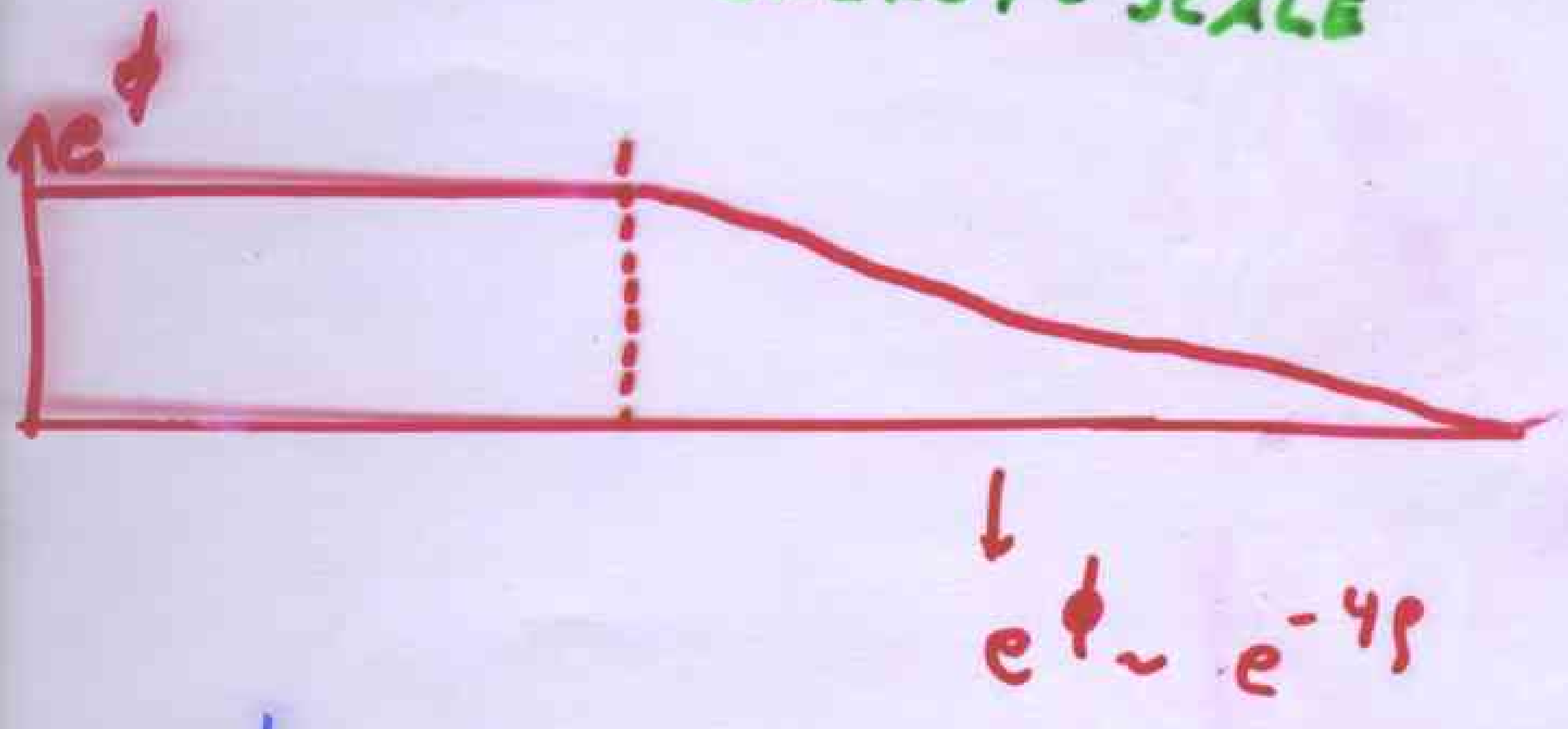
$$ds^2_{STR} = r^2 \left[h (dx_1^2 + dx_2^2) + h' (dx_3^2 + dx_4^2) \right] + \frac{dr^2}{r^2} + d\Omega_5^2$$

$$h = \frac{1}{1 + b^2 r^4}$$

$$e^{2\phi} \sim g^2 h h'$$

- NEAR HORIZON GEOMETRY OF D3 BRANES IN THE PRESENCE OF A B-FIELD





→ COUPLING GETS WEAK AT HIGH ENERGIES

AT LOW ENERGIES WE HAVE THE USUAL $AdS_5 \times S_5$ SOLUTION

- COMPACTIFY ON V_4 → SINGULAR BEFORE REACHING THE BOUNDARY
  ↓
  ANOTHER DESCRIPTION

- COMPUTE THE NUMBER OF DEGREES OF FREEDOM WITH A CUTOFF :

  - AREA (r) ~ INCREASES AT THE SAME RATE AS IN THE COMMUTATIVE CASE

  - FINITE TEMPERATURE BEHAVIOUR → SAME AS IN THE COMMUTATIVE CASE

CONCLUSIONS

- $E = 0$ IN AdS_2
- MULTIPLE AdS_2's ?
- F_D IN AdS_D CAN LEAD TO BRANE CREATION (NON - SUSY → VACCUM DECAY)
- AdS_3 PHASE TRANSITION → MULTIPLY WOUND STRINGS.
- D1-D5 GREYBODY FACTORS $m = 0$ S-WAVES → "NON- RENORMALIZATION" THEOREM
- GRAVITY → STUDY OF NON-COMMUTATIVE GAUGE THEORIES