NON-PERTURBATIVE $\mathcal{N}=1$ STRINGS

FROM

GEOMETRIC SINGULARITIES

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GEOMETRIC DUALITY:

* THEORY A ~ THEORY B

• PHYSICS		GEOMETRY W
• $\mathcal{M}_{PHYS}$	$\longleftrightarrow$	$\mathcal{M}_{CS}(W)$
• Symmetries, BPS spectrum, Correlation fcts ...	$\longleftrightarrow$	Geometric Data

* IF g_A ~ GEOMETRIC PARAMETER

↳ NON-PERTURBATIVE INFORMATION ON A

* W WILL BE SOME COMPLEX GEOMETRY

$$W: \; P_W(\underbrace{x_i}_{\text{COORDINATES}}, \underbrace{u_\alpha}_{\text{COMPLEX STRUCTURE}}) = 0$$

- PROTOTYPE:

* $\mathcal{N}=2$, $d=4$ SUSY SYM $\longleftrightarrow$ RIEMANN SF Σ — Seiberg, Witten

* $\mathcal{M}_{PHYS} \leftrightarrow \mathcal{M}_{CS}(\Sigma)$ Classical periods

$\Big\downarrow$ EXAMPLE OF A GENERAL RELATION

PHYSICS		GEOMETRY
GRAVITY FREE QFT's	$\longleftrightarrow$	CANONICAL SINGUL. OF CY SPACES

Witten, Vafa, Warner, Lerche, Katz, Klemm, P.M.

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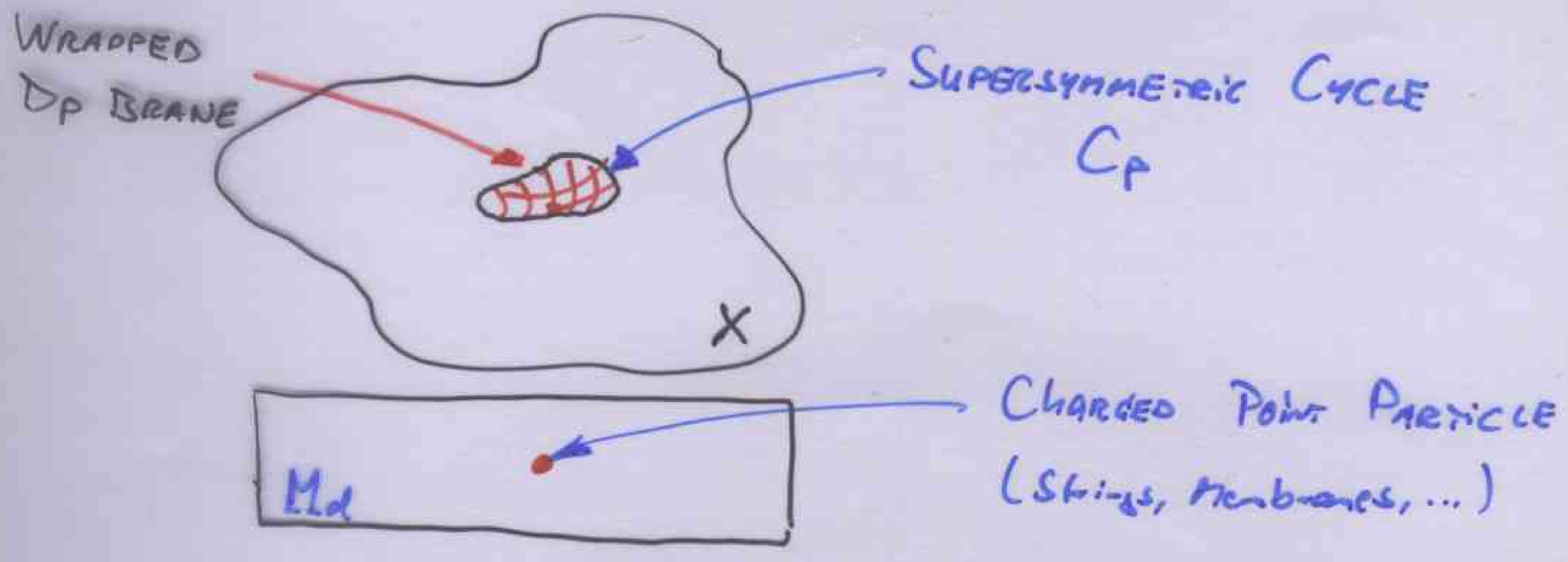
LINK PROVIDED BY D-BRANE GEOMETRIES OF TYPE II STRINGS



* "EXTRA" STATES IN THE FULL STRING THEORY

* INTERESTING QUESTION:

ISOLATED SYSTEM OF D-BRANE STATES

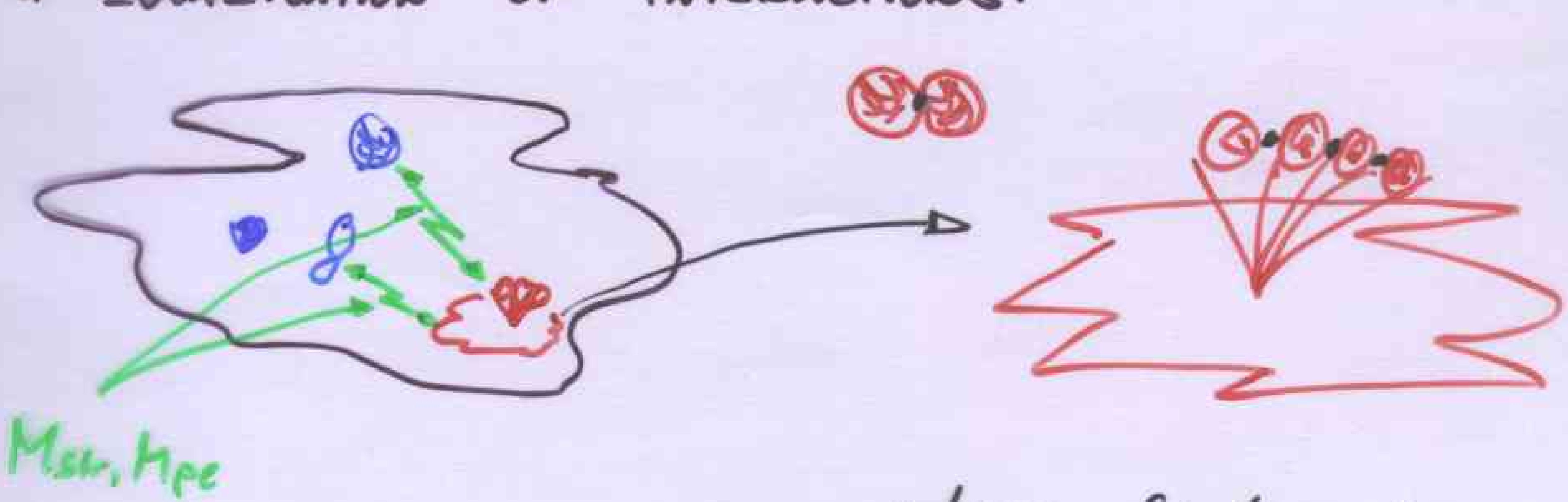
☒ TYPE II STRINGS ON LOCAL SINGULARITIES

* DECOUPLE MOST OF FUNDAMENTAL STRINGS / GRAVITY

- $M_{str}, M_{pl} \to \infty$
- $M_{DB} \sim \mathrm{Vol}\, C_p \cdot M_{str} = \text{fixed}$ (BPS)

↳ $\mathrm{Vol}\, C_p \to 0$ GEOMETRIC SINGULARITY

* LOCALIZATION OF INTERACTIONS:



- LOCAL SINGULARITY
- INTERSECTING CYCLES

* CORRESPONDENCE OF

GEOMETRY		PHYSICS
• HOMOLOGY H_p INTERSECTIONS	$\longleftrightarrow$	• BPS CHARGES INTERACTIONS
$\mathcal{M}_{geom} \Leftrightarrow$ VOLUMES C_p	$\longleftrightarrow$	BPS MASSES $\Leftrightarrow \mathcal{M}_{phys}$
(• $\mathcal{M}_{C_p}(X)$	$\longleftrightarrow$	LORENTZ REP)

⋮

☐ SUCCESSFULL APPLICATION:

$\mathcal{N}=2, d=4$ SYM $\longleftrightarrow$ CANONICAL 3-FOLD SING. OF CY X

- $g_i \Leftrightarrow$ GEOMETRIC PARAMETER
  ↳ NON-PERTURBATIVE SOLUTION

$$\mathcal{M}_{PHYS} \cong \mathcal{M}_{CS}(X)$$

$\int \Omega^{3,0}$ Classical 3-FOLD Periods

(generalizes $SU(2) \Leftrightarrow \Sigma$)

↳ NP FIELD THEORY PREDICTS 3-FOLDS!

- LARGE CLASS OF CFT'S HAS BEEN CONSTRUCTED AND SOLVED

- + S-DUALITY GROUPS,
  STABLE BPS SPECTRA
  ⋮

NOTE: COUPLING TO GRAVITY / STRINGS:
FIND GLOBAL EMBEDDINGS

QFT STRING THEORY

NP $\mathcal{N}=1$ STRINGS FROM GEOMETRIC SINGULARITIES

* ESTABLISH A DUALITY

$$\boxed{W_{n+1} \quad \longleftrightarrow \quad Z_n, V}$$

GEOMETRY $\qquad\qquad$ $\mathcal{N}=1$ QFT

* GEOMETRY W_{n+1}:
  - CLASS OF CANONICAL $n+1$ FOLD SINGULARITIES (NON-COMPACT)
  - IIA OR F-THEORY COMPACTIFICATION
  - FIBRATIONS OF ELLIPT. FIBERED ALE SINGULARITIES
* $\mathcal{N}=1$ QFT (Z_n, V):
  - COMPACTIFICATION $10 \to 10-2n$ DIMENSION ON A COMPACT CY Z_n
  - BACKGROUND GAUGE FIELDS V: HOLOMORPHIC STABLE VECTOR BUNDLES

* WE WILL:
  - ESTABLISH EQUIVALENCE USING TYPE IIA PHYSICS
  - NEED: Z_n IS ELLIPT. FIBERED
  - ANY STRUCTURE GROUP H OF V

* NOTE: • IF $H \subset G_{HET}$ (E_8^2, SO_{32})

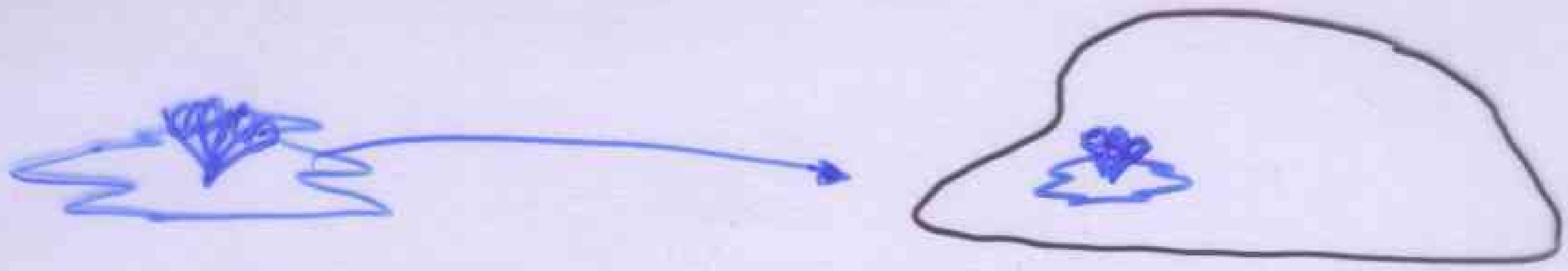
$\Rightarrow (Z_n, V)$ DEFINE A HETEROTIC COMPACTIF. IN POINT PARTICLE LIMIT.

$\longrightarrow$ IIA / HET $\quad$ F/HET DUALITY IN PP LIMIT

Dual Pairs of String Duality

* Couple to String Theory

↳ Find Global, Compact Embeddings



Local Singularity W_{n+1}

Compact CY $\hat{W}_{n+1}$

* Global Embeddings Exist Precisely for

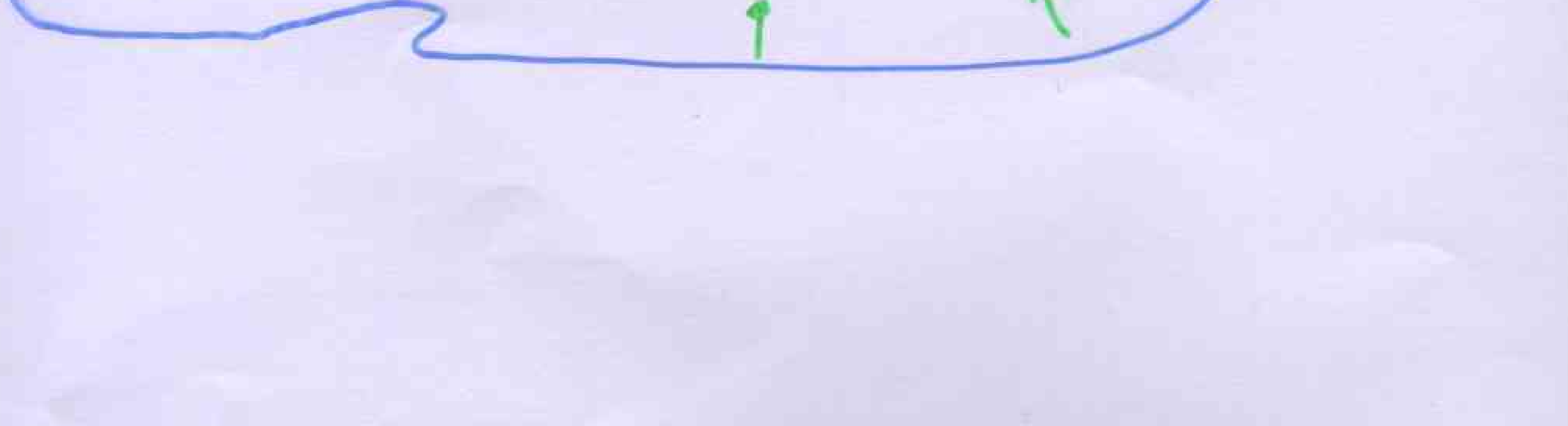
$$H \subset G_{HET}$$

* Extend

$$W_{n+1} \overset{\checkmark}{\leftrightarrow} (Z_n, V) \longrightarrow \hat{W}_{n+1} \leftrightarrow (Z_n, V)$$

FT

String Theory Duality
(F-Theory / Het)



THE SINGULARITIES

* ESTABLISH $W_{n+1} \leftrightarrow Z_{n,1} V$ IN TWO STEPS:

if $n=1$ $\rightarrow$ WILSON LINES ON $T^2 = Z_1$

$\updownarrow$

H SINGULARITIES OF E.F. ALE SPACES W_2

if $n>1$ HOLOMORPHIC FIBRATIONS

$\hookrightarrow$ $W_{n+1} \leftrightarrow (Z_{n,1} V)$

$\hookrightarrow$ Friedmann Morgan Witten

WILSON LINES ON T^2 AND MIRRORSYMMETRY OF K3

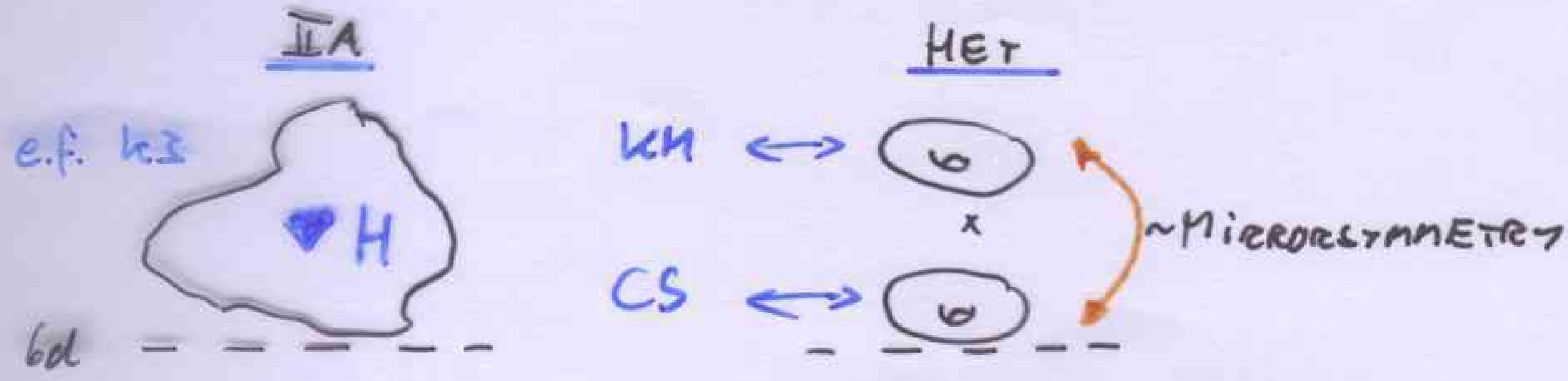
* CLAIM:

M		W		
KÄHLER BLOW UP OF H SINGULARITY E.F. ALE	~	COMPLEX DEF. H-SING. E.F. ALE	~	WILSON LINES STRUCTURE GROUP H ON T^2

* FOLLOWS FROM R-SYMMETRY OF $IIA/K3\times T^2$

* A DUAL HETEROTIC VIEW:

· DUALITY: $\frac{IIA}{K3} \sim \frac{HET}{T^4} \longrightarrow \frac{IIA}{e.f.\ K3} \sim \frac{HET}{T^2\times T^2}$



· 6d: H GAUGE SYMMETRY

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		$\mathcal{M}_{cs}(W)$	$\sim$	$\mathcal{M}_{phys}$

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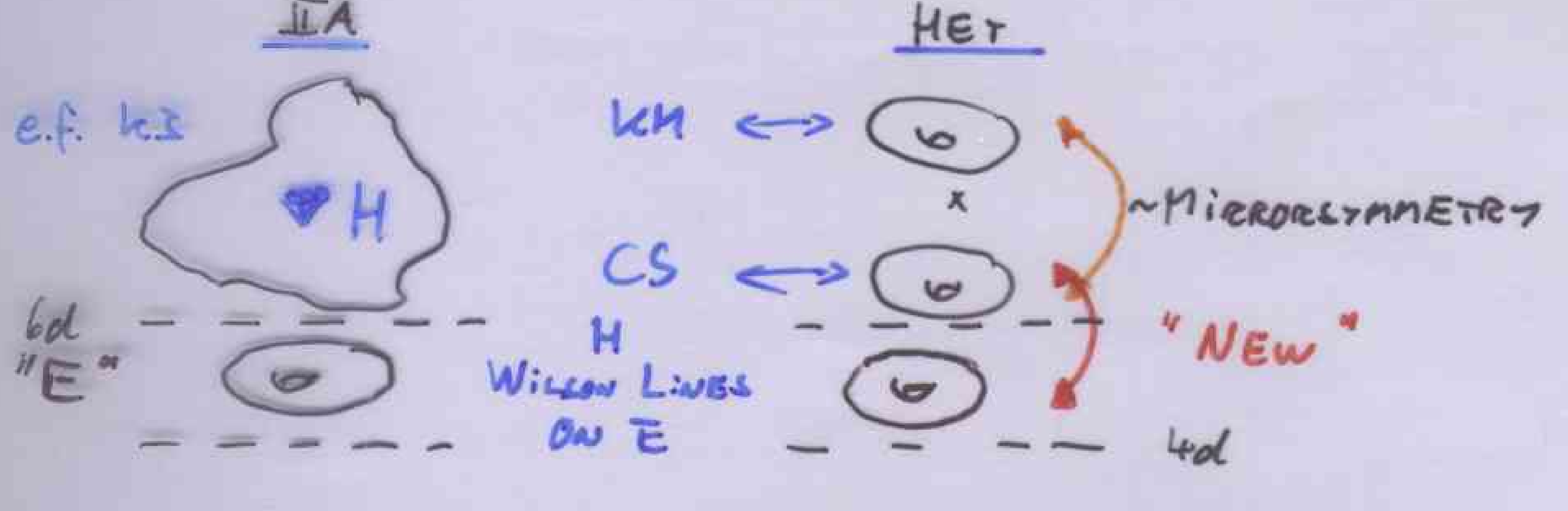


- 6d: H GAUGE SYMMETRY

* FIELD THEORY LIMIT

↳ K3 ⟶ ALE

The Complex Geometry W — From Toric Geometry + Local Mirrorsymmetry

- $p_W = p_0(y,x,z) + p_+(y,x,z;v) = 0$

  y,x,z : Homogeneous Coordinates on Ell. Fiber of ALE

  v : " on ALE

- $p_0 = p_W|_{v=0}$: Geometry

  $$p_0 = y^2 + x^3 + g_2 x + g_3 = 0 \qquad \text{Elliptic Curve}$$

- p_+ : Gauge Bundle

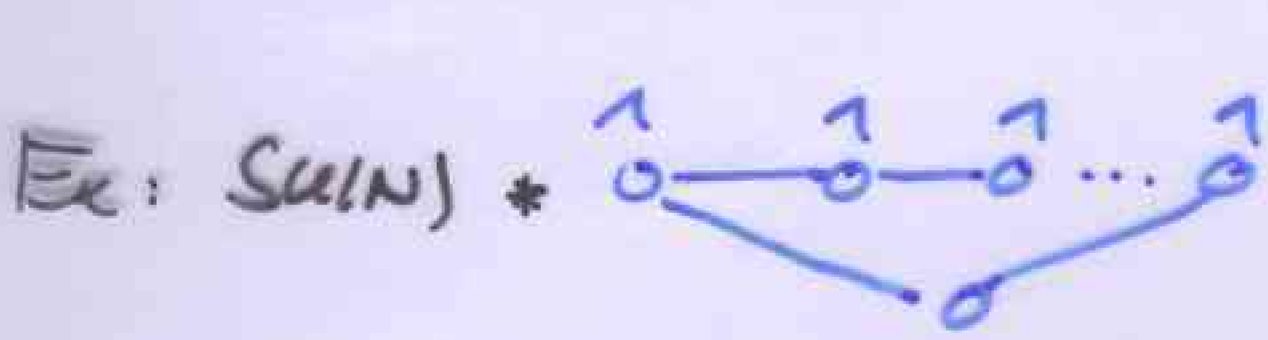
  $$p_+^{(H)} = \sum v^i p_i(y,x,z) \qquad H: \text{Structure Group}$$

Group Theoretical Structure: Affine Dynkin Diagrams

Each Node with Dynkin Index s_i Contributes

A Monomial to p_{s_i}

Ex: SU(N) *



N nodes $s_i = 1$

$$* \; p_+ = v^1 (z^N + z^{N-2} x + \ldots \quad)$$

↳ Degree N Equation on $E: p_0 = 0$

↳ N unordered pts = Values of W.L.

* Works also for

- Non-simply Laced H
- V without Vector Structure (CHL) (↳ with

$n>1$: HOL. STABLE VECTOR BUNDLES ON Z_n

* CONSIDER HOLOMORPHIC FIBRATIONS

$$\begin{array}{c} CY \\ W_{n+1} \end{array} \left\{ \begin{array}{c} W_2 \\ \downarrow \\ B_{n-1} \end{array} \right. \longleftrightarrow \begin{array}{c} (E,V) \\ \downarrow \\ B_{n-1} \end{array} \qquad Z_n: \begin{array}{c} E \\ \downarrow \\ B_{n-1} \end{array}$$

* TORIC GEOMETRY + MIRROR SYMMETRY

  ↳ · EXTRA SINGULARITIES OF FIBRATION

  · NON-SMOOTH FIBRATIONS

* $P_{W_{n+1}} = P_0 + P_+ = 0$ ↔ FMW

$P_0 \to Z_n$

$P_+ \to$ · FAMILY OF VB V ON Z_n

* $\mathcal{M}_{cs}(W_{n+1})$ HAS PRECISELY THE RIGHT STRUCTURE AS PREDICTED BY LOOIJENGA, FMW.

STABLE VECTOR BUNDLES IN THE HETEROTIC STRING

* HETEROTIC CY COMPACTIFICATION [(0,2) VACUA]

Candelas
Horowitz
Strominger
Witten

↳ CHOICE OF A BUNDLE V

$$\textcircled{\times}\quad g_{a\bar b} F^{a\bar b} = 0 \qquad V \text{ STABLE}$$

$$F_{ab} = F_{\bar a\bar b} = 0 \qquad V \text{ HOLOMORPHIC}$$

- $\textcircled{\times}$ IS VERY DIFFICULT TO SOLVE EXPLICITELY
- SPECIAL CONSTRUCTIONS:
  - $V = TZ$
  - ORBIFOLDS, LG MODELS, GEPNER

  → Friedman, Morgan, Witten

* USING THE GEOMETRIC CONSTRUCTION $W_{n+1} \Leftrightarrow (Z_n, V)$

WE CAN CONSTRUCT FAMILIES OF VB

- FOR ANY STRUCTURE GROUP V
- ANY E.F. CY Z_n
- TORIC GEOMETRY → SINGULARITIES
  - * NON-SMOOTH FIBS Z_n
  - * SHEAF GENERALIZATIONS
- g_{HET} → GEOMETRIC PARAMETER

  ↳ NON-PERTURBATIVE GENERALIZATIONS
  - 5 BRANES
  - NON-PERT GAUGE SYMMETRIES

WHY WAS THIS SO EASY ?

$$\begin{array}{ccc}
\text{* Consistent IIA / F-THEORY COMPACT.} & & \text{CONSISTENT HET COMP.} \\
\downarrow & & \uparrow \\
\text{CANONICAL CY SINGULARITIES } W_{n+1} & \xrightarrow{W_{n+1} \to (B_{n-1}, V)} & \text{STABILITY CONDITION ON } V
\end{array}$$

VERY GOOD CONTROL

VERY POOR CONTROL

* MATH. CONDITION "CANONICAL GORENSTEIN SINGULARITIES"

↳ CONDITION ON CHERN CLASS OF A LINE BUNDLE $\mathcal{N}$

$$\eta(\mathcal{N}) = c_1(\mathcal{N}) \geq V_G \cdot c_1(\mathcal{L})$$

INVARIANT OF SINGULARITY

B_{n-1}

$$Z_n : \begin{array}{c} T^2 \\ \downarrow \\ B_{n-1} \end{array}$$

↳ see also Rajesh

NON-PERTURBATIVE VACUA I : 5-BRANES

* ADDITION OF (HETEROTIC) 5-BRANES

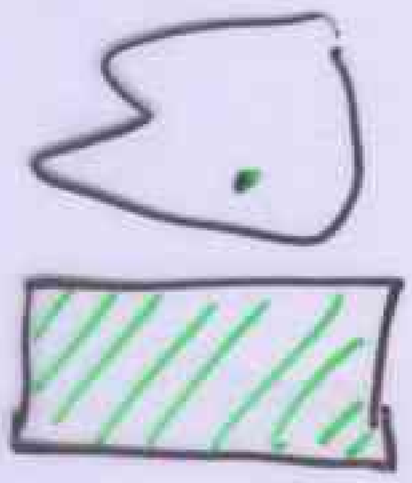
↳ MANY NEW VACUA

Duff
Minasian
Witten

$$c_2(TZ) = c_2(V) + \underbrace{[W]}_{\text{5 BRANES}}$$

* GEOMETRIC CONSTRUCTION INCLUDES 5-BRANES

6d: 5 BRANES FILL SPACETIME



⟷ Blow ups of B_2

Morrison
Vafa

4d: 5-BRANES WRAP CURVES

↳ CURVE DETERMINED BY A BLOW UP OF W_{n+1}

see also
Rajesh

* NOTE: WORLD-VOLUME FIELDS CONTRIBUTE TO SPACE TIME SPECTRUM

↳ · GAUGE SYMMETRIES
· EXTRA MATTER (CHARGED)

NON-PERT. VACUA II: GAUGE SYMMETRIES

* INTRODUCE EXTRA SINGULARITIES IN THE

FIBRATION $W_{n+1} = \begin{matrix} W_2 \\ \downarrow \\ B_{n-1} \end{matrix}$

↳ · EXTRA GAUGE SYMMETRIES IN IIA/F

· NON-PERT. IN HETEROTIC PICTURE

* $f: W_{n+1} \to (Z_n, V)$ → HETEROTIC CONFIGURATION

· CLAIM:

$$\boxed{\begin{array}{l}\text{CONSIDER HET. STRING ON CY } Z_n \\ \text{WITH } \hat{G} \text{ SINGULARITY.} \\ \text{IF } V|_{Sing} \text{ IS SUFFICIENTLY TRIVIAL} \\ \text{THERE IS A } \hat{G} \text{ GAUGE SYMMETRY}\end{array}}$$

* COMPARE WITH TYPE II STRINGS:

	SINGULARITY +	BG FIELDS →	GAUGE SYMM.
TYPE IIA	$\hat{G}$	$B = 0$ (Aspinwall)	$\hat{G}$ (D-BRANES)
HET. STRING	$\hat{G}$	$B = 0$, $V = 0$	$\hat{G}$ (?) ↳ Witten

NON-PERTURBATIVE DUALITY

HET. STRING		HET. STRING
ON	$\sim$	ON
Z_n WITH $\hat{G}$ SING		Z_n' WITH G' SING
V WITH SG. H		V WITH SG $\hat{H}$

H ($\hat{H}$) = COMMUTANT OF G ($\hat{G}$) IN $E_8 \times E_8$

- EXCHANGES GEOMETRIC $\leftrightarrow$ BUNDLE DATA

  (SIMILARITY TO A DUALITY IN L.σ. MODELS)

- EXTREME EXAMPLE:

$\frac{\text{HET}}{K3}$		$\frac{\text{HET}}{K3}$
$H = \emptyset$	$\sim$	$H = E_8 \times E_8$
$\hat{G} = \emptyset$		$\hat{G} = E_8 \times E_8$
$G_{pert} = E_8 \times E_8$		$G_{pert} = \emptyset$
np : $n_T = 24$		np : $n_T = 24$ & $E_8 \times E_8$

OUTLOOK

- A CLASS OF CY $n+1$ FOLD SINGULARITIES

$$\mathcal{M}_{CS}(W_{n+1}) \sim \mathcal{M}_{PHYS}(Z_n, V)$$

- MAP $W_{n+1} \to (Z_n, V)$ ALLOWS TO CONSTRUCT
  - VB ON Z_n, ANY H
  - GENERALIZATIONS:

    PERTURBATIVE: · SHEAF
    · NON-SMOOTH Z_n

    NON-PERT: · 5 BRANES
    · NP GAUGE SYM.

- NON-PERT HET. STRING PHENOMENA:
  - NP GAUGE-SYM. ON SINGULAR Z_n
  - NP DUALITIES

- NEXT STEP: CORRELATION FCTS FOR $N=1$ (0,2) VACUA

  ↳ MIRRORSYMMETRY ON W_{n+1}
  - (0,2) YUKAWA COUPLINGS
  - GAUGE COUPLINGS
  - SUPERPOTENTIALS F. SINGLETS (↳ WITTEN)

  NON-PERTURBATIVE INFORMATION

-