NON-PERTURBATIVE VACUA IN HETEROTIC M-THEORY

(THREE FAMILY THEORIES WITH

$G = SU(3)_C \times SU(2)_L \times U(1)_Y$)

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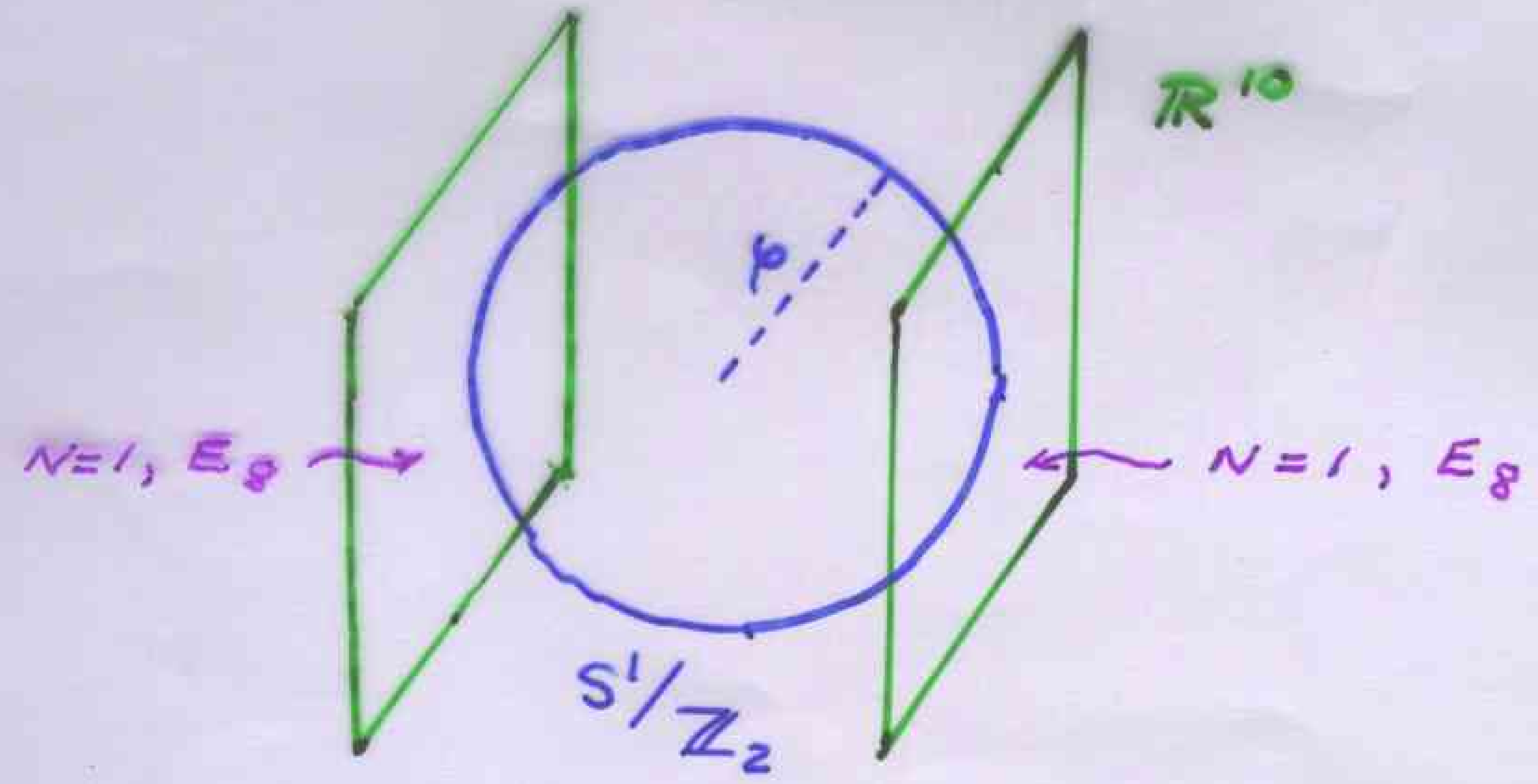
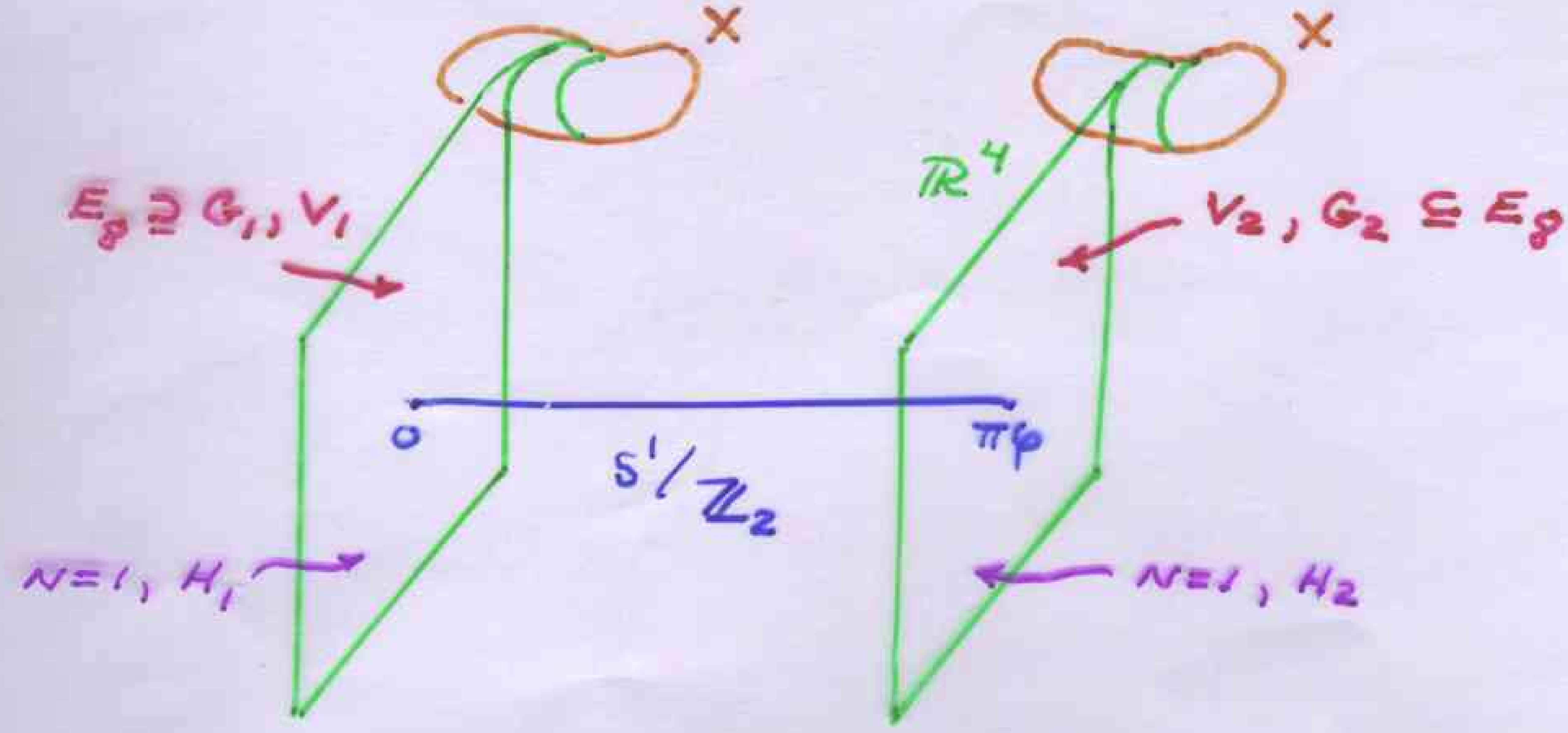
$M = \mathbb{R}^{10} \times S^1/\mathbb{Z}_2$: HORAVA-WITTEN



COMPACTIFY ON A SMOOTH CY 3-FOLD X

$M = \mathbb{R}^4 \times S^1/\mathbb{Z}_2 \times X$:



V_i ARE VECTOR BUNDLES OVER X WITH STRUCTURE GROUPS $G_i \subseteq E_8$

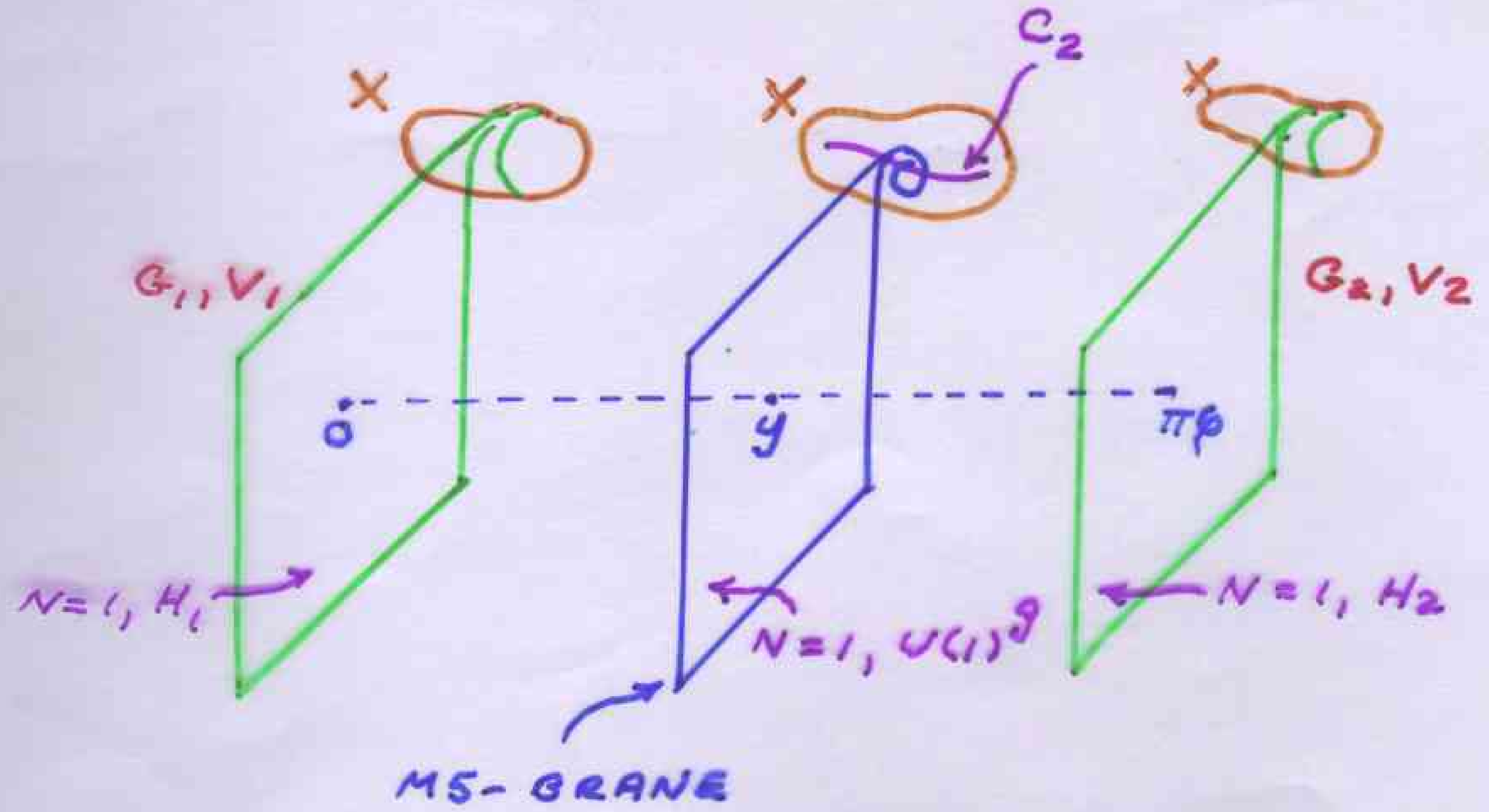
$H_i \subseteq E_8$ SATISFY

$$[H_i, G_i] = 0$$

AND ARE THE UNBROKEN GAUGE GROUPS. FOR EXAMPLE

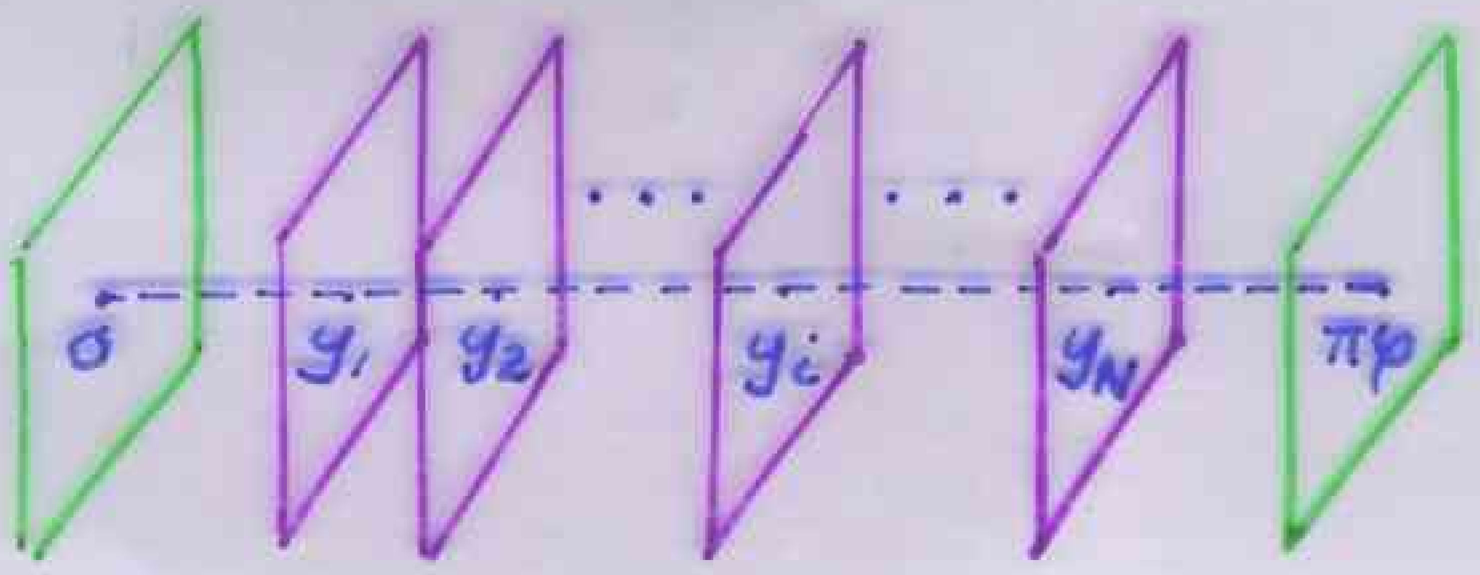
	G_i	H_i
$E_8 \supset$	$SU(3) \times$	E_6
	$SU(4) \times$	$SO(10)$
	$SU(5) \times$	$SU(5)$
	$\vdots$	

NON-PERTURBATIVE VACUA:



C_2 IS A HOLOMORPHIC CURVE OF GENUS g IN X

IN GENERAL



ANOMALY CANCELLATION $\Rightarrow$

$$dG = J^{(0)}\delta(y) + J^{(\pi\rho)}\delta(y-\pi\rho) + \sum_{i=1}^{N} J^{(i)}\delta(y-y_i)$$

WHERE

$$J^{(0)} \propto F^{(1)}\wedge F^{(1)} - \frac{1}{2} R\wedge R$$

$$J^{(\pi\rho)} \propto F^{(2)}\wedge F^{(2)} - \frac{1}{2} R\wedge R$$

$$J^{(i)} \propto \delta(C_2^{(i)})$$

INTEGRATE OVER A 5-CYCLE INCLUDING $S^1/\mathbb{Z}_2$ AND AN ARBITRARY C_4 $\Rightarrow$

$$\boxed{c_2(V_1) + c_2(V_2) + W = c_2(TX)}$$

WHERE

$$W = \sum_{i=1}^{N} [\delta(C_2^{(i)})]$$

ALSO

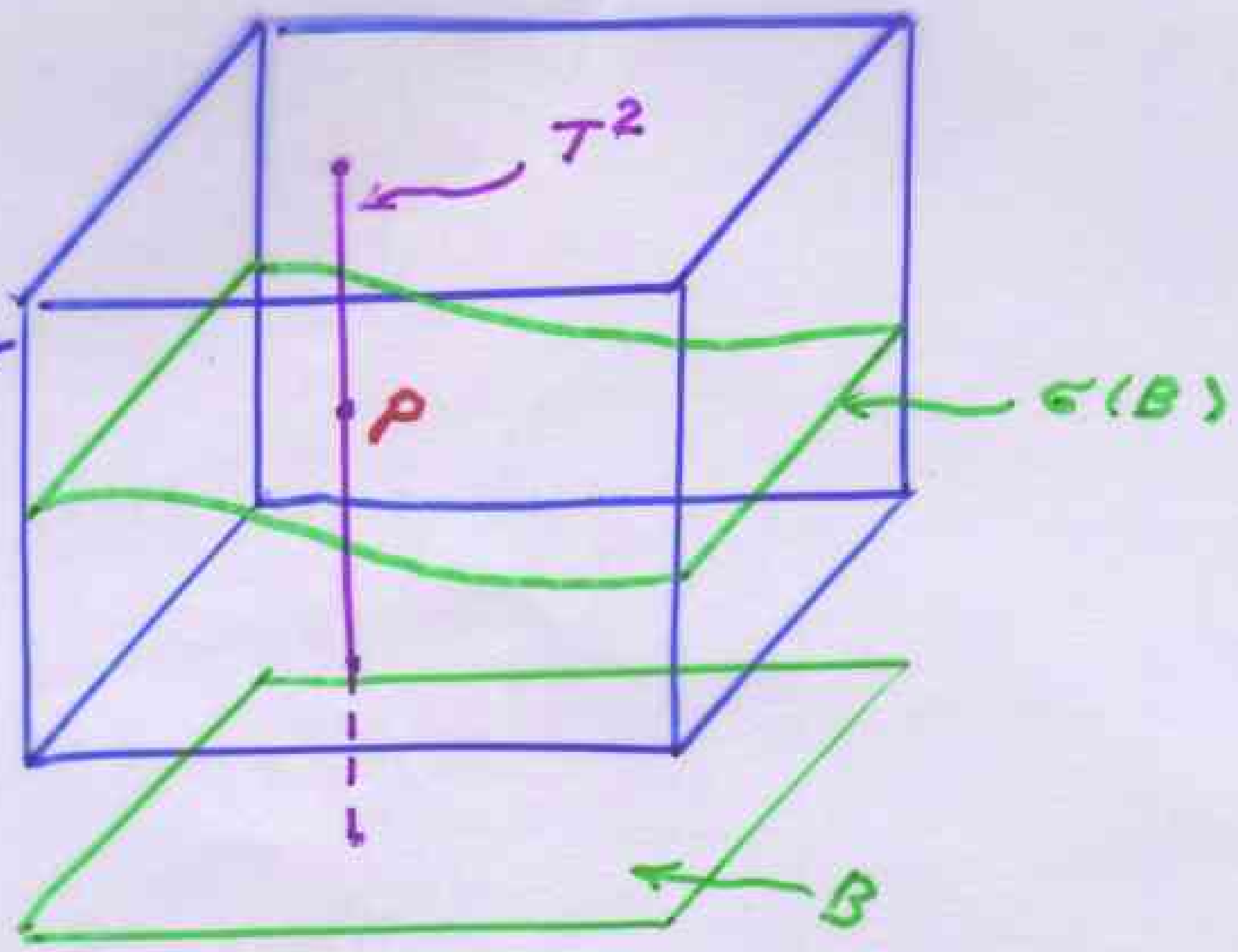
$$\boxed{N_{gen} = \frac{1}{2} c_3(V_1)}$$

WANT TO <u>EXPLICITLY</u> SOLVE THESE EQUATIONS

FOR <u>REALISTIC</u> PARTICLE PHYSICS VACUA. DISTLER, GREENE, KACHRU, F,M,W

TAKE <u>CY</u> X TO BE <u>TORUS-FIBERED</u> WITH A <u>GLOBAL SECTION</u>

$\Longleftrightarrow$ X IS AN <u>ELLIPTIC FIBRATION</u>



X CY $\Rightarrow c_1(TX) = 0 \Rightarrow$

$$B = \begin{cases} dP_i & i = 1, \ldots, 9 \\ F_r & r = 0, 1, \ldots \text{ + BLOW-UPS} \\ \mathcal{E} & \end{cases}$$

WE <u>KNOW</u> THE EFFECTIVE HOMOLOGY CLASSES AND CHERN CLASSES ON THE BASES. ON SUCH A CY X

$$c_2(TX) = c_2(B) + 11\, c_1(B)^2 + 12\, \sigma c_1(B)$$ F,M,W

RANK n HOLOMORPHIC VECTOR BUNDLES OVER X:

PRESERVING $N=1$ SUPERSYMMETRY $\Rightarrow$

$$V \text{ OVER } X \iff \mathcal{C}, \mathcal{N}$$

WHERE

$\mathcal{C}$ - SPECTRAL (n-FOLD) COVER (OF B)

$\mathcal{N}$ - LINE BUNDLE ON $\mathcal{C}$ ($\Leftarrow V|_{T^2} = \mathcal{N}_1 \oplus \ldots \oplus \mathcal{N}_n$)

THE CLASS OF $\mathcal{C} \in H_4(X, \mathbb{Z})$ IS OF THE FORM

$$\mathcal{C} = n\sigma + \pi^* \eta$$

WHERE $\eta \in H_2(B, \mathbb{Z})$. $SU(n) \Rightarrow c_1(V) = 0 \Rightarrow$

$$c_1(\mathcal{N}) = n\left(\frac{1}{2} + \lambda\right)\sigma + \left(\frac{1}{2} - \lambda\right)\pi_{\mathcal{C}}^* \eta + \left(\frac{1}{2} + n\lambda\right)\pi_{\mathcal{C}}^* c_1(B) \quad \text{F,M,W}$$

WHERE

$$n \text{ ODD} \Rightarrow \lambda - \frac{1}{2} \in \mathbb{Z}$$

$$n \text{ EVEN} \Rightarrow \begin{cases} \lambda \in \mathbb{Z} \\ \eta = c_1(B) \text{ MOD } 2 \end{cases}$$

THEREFORE

$$V \iff \lambda, \eta$$

FOR $c_1(V) = 0 \implies$

$$c_2(V) = \sigma\eta - \frac{1}{24}c_1(B)^2(n^3-n) + \frac{1}{2}\left(\lambda^2-\frac{1}{4}\right)n\eta\big(\eta - nc_1(B)\big)$$ F, M, W

AND

$$c_3(V) = 2\lambda\sigma\eta\big(\eta - nc_1(B)\big)$$ CURIO

RETURN TO THE ANOMALY CONDITION

$$c_2(V_1) + \cancel{c_2(V_2)} + W = c_2(TX)$$

$$\implies W = c_2(TX) - c_2(V_1)$$

WRITE

$$W = W_B + a_F F$$

INSERTING ABOVE $\implies$

$$W_B = \sigma\big(12c_1(B) - \eta\big)$$

$$a_F = c_2(B) + \left(11 + \frac{1}{24}(n^3-n)\right)c_1(B)^2 - \frac{n}{2}\left(\lambda^2 - \frac{1}{4}\right)\eta\big(\eta - nc_1(B)\big)$$

FURTHERMORE

$$N_{gen} = \frac{1}{2}c_3(V_1)$$

BECOMES

$$N_{gen} = \lambda\left(W_B^2 - (24-n)W_B c_1(B) + 12(12-n)c_1(B)^2\right)$$

PARTICLE PHYSICS <u>RULES</u>:

1. n ODD $\Rightarrow$ $\lambda - \frac{1}{2} \in \mathbb{Z}$

$$n \text{ EVEN} \Rightarrow \begin{cases} \lambda \in \mathbb{Z} \\ W_B = c_1(B) \text{ MOD } 2 \end{cases}$$

2. W MUST BE AN <u>EFFECTIVE</u> CLASS

* W EFFECTIVE $\Longleftrightarrow$ W_B EFFECTIVE IN B, $a_F \geq 0$

3. $N_{gen} = 3$

EXAMPLE:

CHOOSE STRUCTURE GROUP

$$G = SU(5) \qquad (\Rightarrow n = 5)$$

$\Rightarrow$ GUT GROUP IS

$$H = SU(5)$$

n ODD $\Rightarrow$ $\lambda - \frac{1}{2} \in \mathbb{Z}$. CHOOSE

$$\lambda = 3/2$$

CHOOSE BASE

$$B = dP_8$$

AN EFFECTIVE BASIS FOR $H_2(dP_8, \mathbb{Z})$ IS

$$\ell, \; E_i \quad i = 1, \dots, 8$$

AND

$$c_1(B) = 3\ell - \sum_{i=1}^{8} E_i$$

$$c_2(B) = 11$$

TAKE

$$W_B = 2E_1 + E_2 + E_3$$

E_i EFFECTIVE $\Rightarrow$ W_B IS EFFECTIVE. INSERTING ABOVE $\Rightarrow$

$$a_F = 17$$

$a_F > 0 \Rightarrow$

W IS AN EFFECTIVE CLASS ✓

ALSO, INSERTING ABOVE $\Rightarrow$

$$N_{gen} = 3 \; ✓$$

<u>CONCLUSION:</u> NON-PERTURBATIVE VACUUM WITH 3-FAMILIES GUT GROUP $H = SU(5)$ AND FIVE-BRANE CLASS

$W = 2E_1 + E_2 + E_3 + 17F$.

PROBLEM! NO 24 HIGGS TO BREAK $SU(5)$ TO THE STANDARD GAUGE GROUP $SU(3) \times SU(2) \times U(1)$!

SOLUTION: WILSON LINES

HOWEVER! FOR ELLIPTICALLY FIBERED CY'S X

$$\pi_1(X) = \mathbb{1}$$

(EXCEPTION: $B = \mathcal{E}$ BUT $\Rightarrow N_{gen} \neq 3$)

$\Rightarrow$ NO WILSON LINES!

SOLUTION:

CONSTRUCT TORUS-FIBERED CY MANIFOLDS Z

WITHOUT GLOBAL SECTIONS

SO THAT

$$\pi_1(Z) \neq \mathbb{1}$$

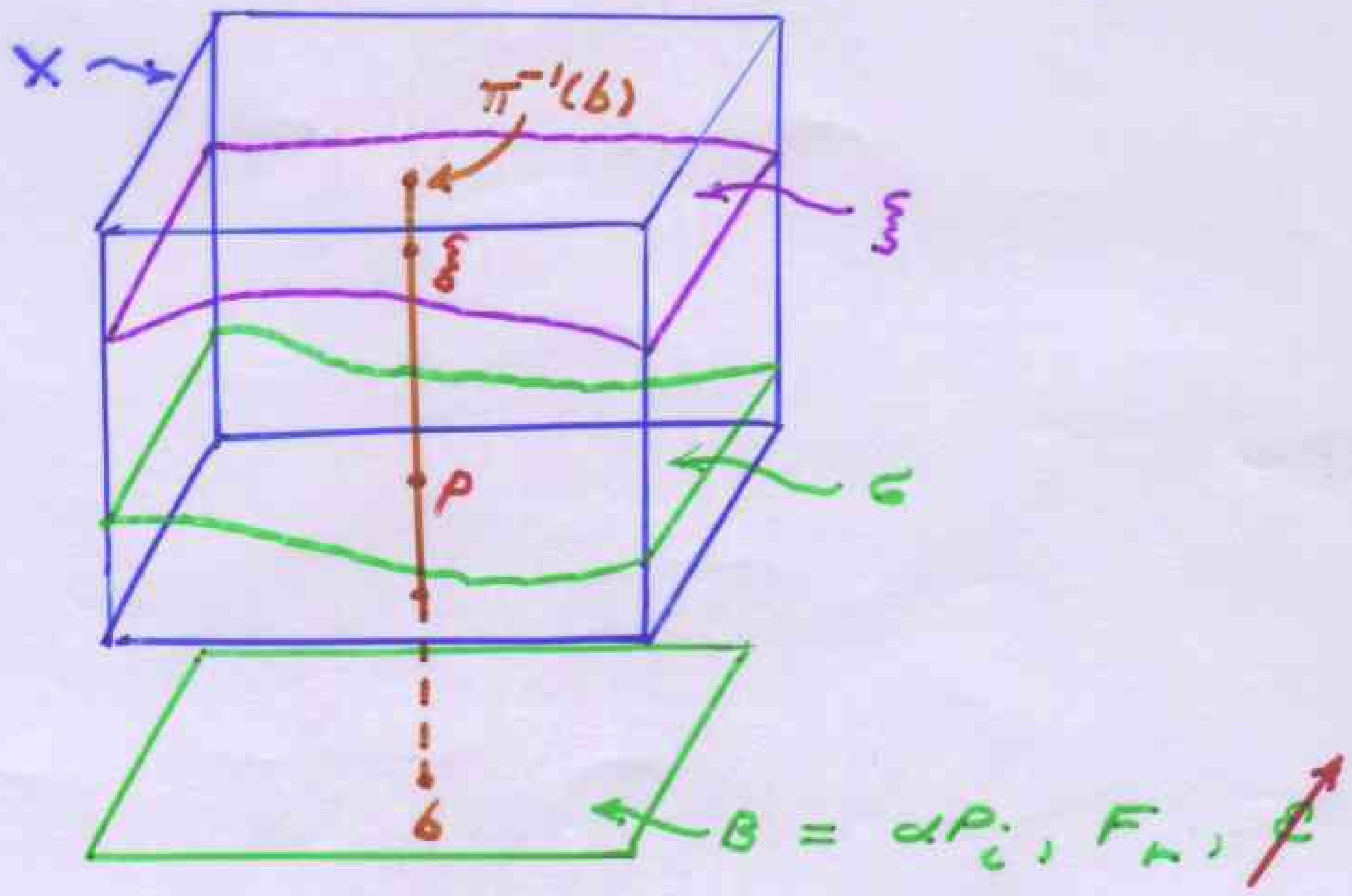
$\pi_1(Z) = \mathbb{Z}_2$ CONSTRUCTIONS : DONAGI, BAO, PANTEV, WALDRAM

CONSIDER ELLIPTICALLY FIBERED CY X WITH

$$\pi_1(X) = \mathbb{1}$$

AND

TWO GLOBAL SECTIONS – σ AND ξ



ASSUME

$$\xi + \xi = p$$

DEFINE $t_\xi : \pi^{-1}(b) \to \pi^{-1}(b)$ BY

$$t_\xi(x) = x + \xi$$

NOTE $t_\xi^2 = id$ AND

$$t_\xi(p) = \xi, \quad t_\xi(\xi) = p$$

NOW CONSIDER $\tau_B : B \to B$ SUCH THAT

$$\tau_B{}^2 = id \quad \text{(INVOLUTION ON } B\text{)}$$

τ_B <u>CAN</u> HAVE FIXED POINTS

$$\mathcal{F} = \{ b \in B \mid \tau_B(b) = b \}$$

LIFT TO $\alpha : X \to X$ SUCH THAT

$$\alpha(\sigma) = \sigma \,, \quad \alpha(\xi) = \xi$$

$$\alpha \circ t_\xi = t_\xi \circ \alpha$$

DEFINE $\tau_X : X \to X$ BY

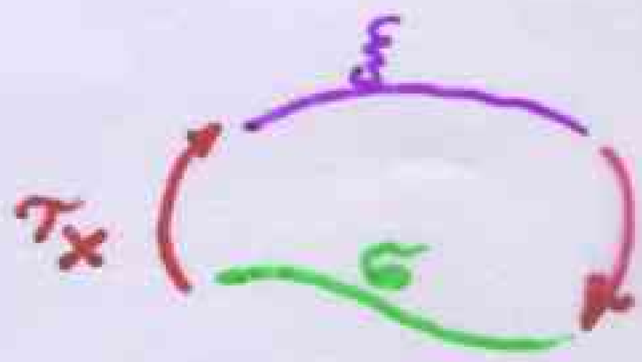
$$\tau_X = \alpha \circ t_\xi$$

CLEARLY

$$\tau_X{}^2 = id \quad \text{(INVOLUTION ON } X\text{)}$$

NOTE

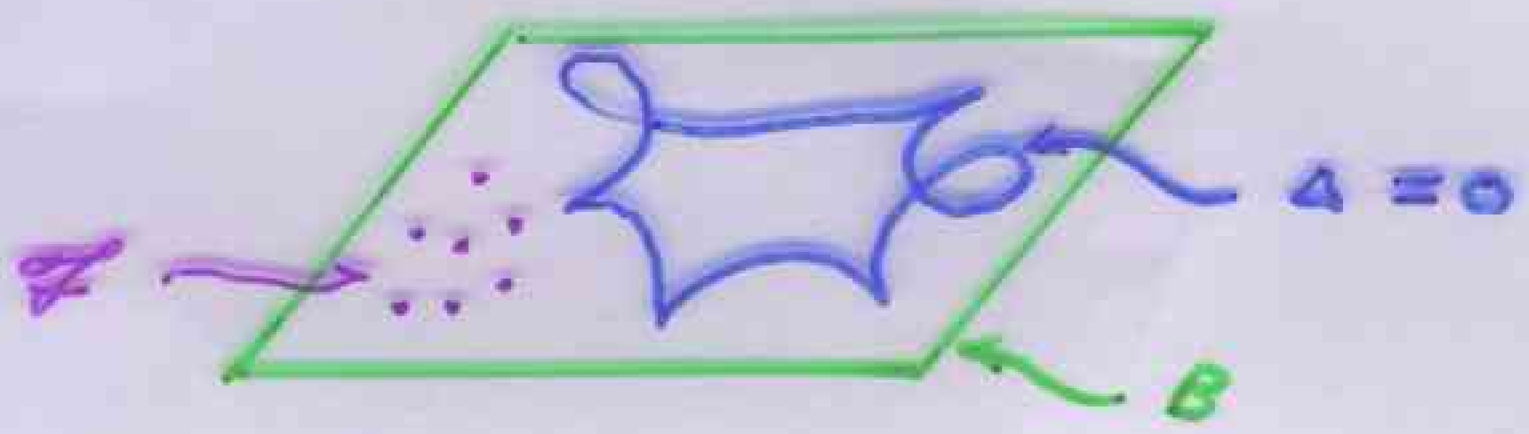
$$\tau_X(\sigma) = \xi \,, \quad \tau_X(\xi) = \sigma$$



WE CAN SHOW τ_X ACTS <u>FREELY</u> ON X iff

a) $\mathcal{F} = \{ \text{DISCRETE POINTS} \}$

6) $\mathcal{F} \cap \{\Delta = 0\} = \phi$



IN THIS CASE

$$Z = X / \tau_X$$

IS A TORUS-FIBERED CY OVER BASE

$$S = B / \tau_B$$

WITH <u>NO</u> SECTION AND

$$\pi_1(Z) = \mathbb{Z}_2$$

WE WILL CONSTRUCT PARTICLE PHYSICS MODELS OVER Z.

HOWEVER, IT IS EASIEST TO WORK ON X AND <u>PROJECT</u> ONTO Z.

RESULTS:

A) INDEPENDENT CLASSES OF CURVES ARE

1. $\sigma \cdot \pi^* \beta \quad , \quad \beta \in H_2(B, \mathbb{Z})$
2. $F - N$
3. N

BLOW-UP

B) $c_2(TX) = 12\,\sigma\,\pi^* c_1(B) + \left(c_2(B) + 11\, c_1(B)^2\right)(F - N)$

$$- \left(c_2(B) - c_1(B)^2\right) N$$

VECTOR BUNDLES OVER X ($\Rightarrow$ OVER Z):

RESULTS:

A) $\quad C = n\sigma + \pi^* \eta \qquad (\underline{\underline{\text{NO}}}\ \xi)$

B) $\quad c_1(N) = n\left(\frac{1}{2} + \lambda\right)\sigma + \left(\frac{1}{2} - \lambda\right)\pi_C^* \eta + \left(\frac{1}{2} + n\lambda\right)\pi_C^* c_1(B)$

$$+ \kappa\left(\sigma - \xi + \pi_C^* c_1(B)\right)$$

WHERE

$$n \text{ ODD} \Rightarrow \lambda - \frac{1}{2},\ \kappa \in \mathbb{Z}$$

C) $\quad c_2(V) = \sigma\eta - \frac{1}{24} c_1(B)^2 (n^3 - n) F + \frac{1}{2}\left(\lambda^2 - \frac{1}{4}\right) n \eta \left(\eta - n c_1(B)\right) F$

$$+ \kappa^2 \eta\, c_1(B) F + 2\kappa\, \eta\, c_1(B) N$$

$$c_3(V) = 2\lambda\sigma\eta\left(\eta - n c_1(B)\right)$$

FURTHERMORE, WE MUST RESTRICT TO BUNDLES THAT ALSO LIVE ON $Z = X/\tau_X$. THAT IS

$$\tau_X V = V$$

THIS HOLDS iff

$$\tau_B \eta = \eta$$

RETURN TO THE ANOMALY CONDITION

$$c_2(V_1) + c_2(\cancel{V_2}) + W = c_2(TX)$$

WRITE

$$W = W_B + a_F(F-N) + a_N N$$

INSERTING ABOVE $\Rightarrow$

$$W_B = c_2(TX) - \pi^*(12c_1(B) - \eta)$$

$$W_B = 6\pi^*(12c_1(B) - \eta)$$

$$a_F = c_2(B) + 16c_1(B)^2 - \frac{n}{2}\left(\lambda^2 - \frac{1}{4}\right)\eta(\eta - nc_1(B))$$

$$a_N = a_F - 12c_1(B)^2 - 2K\eta c_1(B)$$

FURTHERMORE

$$N_{gen} = \lambda 6\eta(\eta - nc_1(B))$$

PARTICLE PHYSICS <u>RULES</u>:

1. n ODD $\Rightarrow$ $\lambda - \frac{1}{2}$, $K \in \mathbb{Z}$

2. $\tau_B \eta = \eta$

3. W MUST BE EFFECTIVE $\Leftrightarrow$ W_B EFFECTIVE, $a_F \geq 0$, $a_N \geq 0$

$(\Rightarrow \varrho W$ EFFECTIVE ON $Z = X/\tau_X)$

4. $N_{gen} = 6$ $\quad (\Rightarrow N_{gen} = 3$ ON $Z = X/\tau_X)$

EXAMPLE :

CHOOSE STRUCTURE GROUP

$$G = SU(5) \quad (\Rightarrow n = 5)$$

$\Rightarrow$ GUT GROUP IS

$$H = SU(5)$$

n ODD $\Rightarrow$ $\lambda - \frac{1}{2}$, $k \in \mathbb{Z}$. CHOOSE

$$\lambda = -\frac{1}{2}, \quad k = 0, 1$$

CHOOSE BASE

$$B = dP_3$$

$dP_3 \approx \mathbb{CP}^2$ BLOWN-UP AT 3 POINTS $\Rightarrow$ COORDINATES ARE

$$(x, y, z)$$

AN EFFECTIVE BASIS OF $H_2(dP_3, \mathbb{Z})$ IS

$$\ell, E_i \quad i = 1, 2, 3$$

AND

$$c_1(B) = 3\ell - E_1 - E_2 - E_3$$

$$c_2(B) = 6$$

OTHER DEPENDENT EFFECTIVE CLASSES ARE

$$\ell - E_1 - E_2 \ , \quad \ell - E_2 - E_3 \ , \quad \ell - E_3 - E_1$$

DEFINE INVOLUTION $\tau_B : B \longrightarrow B$ BY

$$\tau_B(x, y, z) = (x^{-1}, y^{-1}, z^{-1})$$

τ_B HAS FOUR DISCRETE FIXED POINTS

$$\mathcal{F} = \{ (1, \pm 1, \pm 1) \}$$

WE CAN SHOW

$$\mathcal{F} \cap \{ \Delta = 0 \} = \emptyset$$

CONSIDER THE THREE EFFECTIVE CLASSES

$$a_1 = \ell + E_1 - E_2 - E_3$$

$$a_2 = \ell - E_1 + E_2 - E_3$$

$$a_3 = \ell - E_1 - E_2 + E_3$$

WE CAN SHOW

$$\tau_B(a_i) = a_i \quad i = 1, 2, 3$$

AND THEY GENERATE ALL SUCH INVARIANT CLASSES.

THEN THE CONDITION $\tau_B \eta = \eta \Rightarrow$

$$\eta = c_1 a_1 + c_2 a_2 + c_3 a_3$$

NOW CHOOSE

$$\eta = 11 a_1 + 5 a_2 + 5 a_3$$

$\Rightarrow \tau_B \eta = \eta$ AND η IS EFFECTIVE IN B. INSERTING ABOVE $\Rightarrow$

$$W = W_B + a_F(F - N) + a_N N$$

WHERE

$$W_B = 6\pi^*(a_1 + 7a_2 + 7a_3)$$

a_i EFFECTIVE $\Rightarrow a_1 + 7a_2 + 7a_3$ EFFECTIVE $\Rightarrow W_B$

IS EFFECTIVE IN B. ALSO

1. $\lambda = -\frac{1}{2}$, $K = 0 \quad \Rightarrow \quad a_F = 102$, $a_N = 30$

2. $\lambda = -\frac{1}{2}$, $k = 1 \implies a_F = 60$, $a_N = 72$

IN EITHER CASE $\implies$

W IS AN EFFECTIVE CURVE

FURTHERMORE, INSERTING ABOVE $\implies$

$$N_{gen} = 6$$

FOR BOTH $k = 0, 1$.

<u>CONCLUSION</u>:

ON $Z = X/\tau_X$ WE HAVE TWO NON-PERTURBATIVE VACUA EACH WITH 3-FAMILIES, GUT GROUP $H = SU(5)$ AND FIVE-BRANE CLASSES

$$W = 15\ell - 13E_1 - E_2 - E_3 + 102(F-N) + 30N \qquad (\lambda = -\tfrac{1}{2},\ k = 0)$$

$$W = 15\ell - 13E_1 - E_2 - E_3 + 60(F-N) + 72N \qquad (\lambda = -\tfrac{1}{2},\ k = 1)$$

NOW, HOWEVER

$$\pi_1(Z) = \mathbb{Z}_2$$

AND WILSON LINE GENERATOR

$$g = \left(\begin{array}{ccc|cc} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ \hline & & & -1 & \\ & & & & -1 \end{array}\right)$$

WILL SPONTANEOUSLY BREAK

$$SU(5) \longrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y$$

RELATED WORK : ANDREAS, CURIO, KLEMM