"Baldness / Delocalization in Intersecting Brane Systems"

Based on hep-th/9903213

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Thanks to J. Polchinski

Work done at ITP, UCSB

during Program on Strong Gravitational Fields

Strings '99

Baldness and Delocalization

- By baldness I simply mean no-hair theorem

  e.g. bring charge up to RN black hole, find smooth approach to spherical symmetry.

- Localized SUGRA p-brane solutions hard to find

e.g. BPS D3 has $SO(1,3) \times SO(6)$ symmetry.

Harmonic function of SUGRA solution falls off as

$$H_3 = 1 + \frac{\#}{r^4} \leftarrow SO(6) \text{ in } d=10$$

If use Büscher T-duality to get "D2", find only $SO(6)$-symmetric solution $\Rightarrow$ smeared D2 sol$^{\underline{n}}$.

Intersecting Brane Systems

* Prototype will be $D0 \parallel D4$

T, S- dualities give also

$Dp \parallel Dp{+}4$

$Dm \perp Dn\,(p)$, $m+n = p+4$ e.g. $D2 \perp D2(0)$

$F1 \perp Dp\,(0)$

$Dp \perp NS5\,(p-1)$

etc.

General Set-Up

Consider e.g. 2 D-brane types A, B (no relation to IIA, B !)

	t, z_I	z_a	z_b	$x_\perp$
DA	×	×		
DB	×		×	

See also

- Ansatz: "Harmonic function rule" (Tseytlin)

$$ds_{10}^2 = \frac{(-dt^2 + d\vec{z}_I^{\,2})}{\sqrt{H_A H_B}} + \sqrt{\frac{H_B}{H_A}}\, d\vec{z}_a^{\,2} + \sqrt{\frac{H_A}{H_B}}\, d\vec{z}_b^{\,2} + \sqrt{H_A H_B}\, d\vec{x}_\perp^{\,2} ;$$

$$e^{\Phi}/g_s = H_A^{(3-A)/4}\, H_B^{(3-B)/4} \; ; \; \text{"} C_{01\cdots\binom{A}{B}} \text{"} = \tfrac{1}{2}\left(H_{\binom{A}{B}}^{-1} - 1\right) ;$$

plugging into SUGRA e.o.m.'s gives

$$[\vec{\partial}_\perp^{\,2} + H_B \vec{\partial}_b^{\,2}]\, H_A(\vec{x}_\perp, \vec{z}_b) = q_A\, \delta(A)$$

$$[\vec{\partial}_\perp^{\,2} + H_A \vec{\partial}_a^{\,2}]\, H_B(\vec{x}_\perp, \vec{z}_a) = q_B\, \delta(B)$$

$$(\vec{\partial}_b H_A)(\vec{\partial}_a H_B) = 0$$

- Constraint ⇒ one (A) delocalized in other (B)'s worldvolume !

$$\vec{\partial}_\perp^{\,2} H_A(\vec{x}_\perp) = q_A\, \delta(A) \Rightarrow H_A \text{ as usual}$$

but $\boxed{(\vec{\partial}_\perp^{\,2} + H_A \vec{\partial}_a^{\,2})\, H_B(\vec{x}_\perp, \vec{z}_a) = q_B\, \delta(B)}$ different

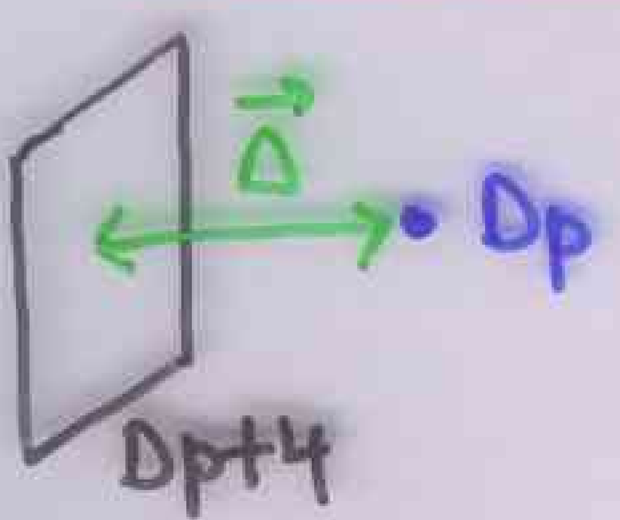
Some Intersecting Brane People

(– apologies to those I failed to include!)

R. Khuri	G. Papadopoulos
P. Townsend	A. Tseytlin
I. Klebanov	K. Behrndt
E. Bergshoeff	B. Janssen
J. Gauntlett	D. Kastor
J. Traschen	M. de Roo
E. Eyras	J. van der Schaar
M. Cvetič	J. Breckenridge
R. Michaud	R. Myers
V. Balasubramanian	F. Larsen
R. Leigh	J. Edelstein
L. Tataru	R. Tatar
I. Aref'eva	M. Ivanov
O. Rytchkov	I. Volovich
K. Stelle	H. Yang
A. Fayyazuddin	D. Smith
A. Loewy	N. Itzhaki
S. Yankielowicz	D. Youm
A. Hashimoto	C. Chu
P. Ho	Y. Wu
M. Bianchi	M. Green
S. Kovacs	G. Rossi
I. Kogan	G. Luzon
S. Surya	D. Marolf

(apologies also for spelling errors.)

Localization vs. Separation



Our question: (in AdS/CFT context)

$\boxed{\text{Can Dp be localized // Dp+4 as } \vec{\Delta} \to \vec{0} \text{ in SUGRA regime?}}$

- Simple & familiar example: D-1 and D3

Solution elementary near-D3-horizon (BKGR)

$$ds_{10}^2 = \sqrt{H_{-1}}\left\{AdS_5 \times S^5\right\} \quad ; \quad e^{\Phi}/g_s = H_{-1} \; ;$$

$$H_{-1} = 1 + \frac{3N_{-1}}{16\pi^6 N_3^2 g_s^2} \frac{\rho^4 \rho_0^4}{[\rho_0^2 + (x-x_0)^2]^4} \quad \leftarrow \text{D-1 @ } (\underset{4}{\vec{x}_0}, \underset{6}{\vec{\rho}_0})$$

"Scale-radius duality" (IR/UV relation):

$$\rho_0 = \frac{\sqrt{4\pi g_s N_3}}{U_0} \quad , \quad U_0 = \frac{r_0}{\ell_s^2}$$

(Recall: instanton localized at boundary of AdS_5)

N.B.: As $U_0 \to 0$, $H_{-1} \to 1 \Rightarrow$ No D-1 hair on D3

General Solution-ology

- Solutions depending only on $\vec{x}_\perp$ easy $\left\{\begin{array}{l}\text{e.g. } H_{1,5} \sim \#/r^2 \\ \text{in } AdS_3/CFT_2\end{array}\right.$
- some partially localized (T-!)
- some only near-horizon for bigger brane
- from field eqns, find can trade some $\perp$ localization for worldvolume localization (not relevant for us)

Surya-Marolf Method (9805121)

Allows for transverse separation $\vec{\Delta}$

- Note: here we discuss only solutions which are (or can be matched to) asymptotically flat

  $\Rightarrow p \leq 6$, and p-brane on $\mathbb{R}^p$

D0, D4

- Big-brane e.o.m. $\vec{\partial}_\perp^2 H_4 = q_4 \, \delta(\vec{x}_\perp)$

$$\Rightarrow H_4 = 1 + \frac{c_4 g_s N_4 \ell_s^3}{|\vec{x}_\perp|^3}$$

- Wee-brane: $\left[\vec{\partial}_\perp^2 + H_4(\vec{x}_\perp) \underset{\uparrow \, 4\text{of}}{\vec{\partial}_a^2}\right] H_0(\vec{x}_\perp, \vec{z}_a) = q_0 \, \delta(\vec{z}_a) \, \delta(\vec{x}_\perp - \underset{\uparrow \, \perp \text{ separation}}{\vec{\Delta}})$

Method

(1) Fourier decompose on $\vec{z}_a$

(2) To find small-$|\vec{\Delta}|$ behavior, can show that we need only track radial modes (spherical shells of D0)

(3) Get 2nd order ODE for each Fourier mode

Solve; match across δ-function

Fourier series converges absolutely (except @ δ-fn, of course)

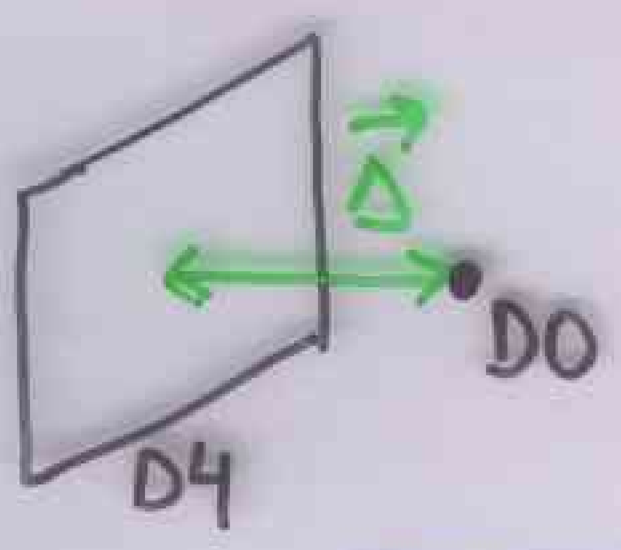
Results

- Eqns have different character depending on p

Find $\boxed{\text{hair for } p=2}$ (6-brane $1/r$ potential) ← c.f. ITY

! $\boxed{\text{baldness for } p=1,0,-1}$ (stronger potentials)

SUGRA delocalization rates



★ Sit out here @ fixed distance
e.g. characteristic scale
$R_4 \sim \ell_s (g_s N_4)^{1/3}$

Find

$\underline{p=0}$	$\frac{\delta x}{\ell_s} \sim \sqrt{g_s N_4} \sqrt{\frac{\ell_s}{\Delta}}$
$\underline{p=-1}$	$\frac{\delta x}{\ell_s} \sim \sqrt{g_s N_3} \left(\frac{\ell_s}{\Delta}\right)$
$\underline{p=+1}$	$\frac{\delta x}{\ell_s} \sim \sqrt{g_s N_5} \sqrt{\log\left(\frac{\sqrt{g_s N_5}\,\ell_s}{\Delta}\right)}$

← Field theory explanation?

Correspondence à la Maldacena (IMSY)

$$g_{YM}^{2\,(Dp)} = (2\pi)^{p-2} g_s \ell_s^{p-3} \qquad \text{for D0 \underline{and} D4}$$

- Low-energy limit "$\ell_s \to 0$" : $(E\ell_s) \to 0$ E = energy
  e.g. $E = U \equiv r/\ell_s^2$; Δ/ℓ_s^2
- Gauge coupling (and U) "fixed"
  - form dimensionless coupling $g_{YM}^{2\,(Dp)} E^{p-3}$

$\Rightarrow$ Dimensionless ratio "$\frac{Dp}{Dp+4}$" $= \frac{1}{(2\pi E \ell_s)^4} \to \infty$

$\Rightarrow$ big-brane dynamics relatively frozen-out

fix $g_{YM}^{2\,(big)}$, then $g_{YM}^{2\,(wee)} \to \infty$ (fixed E)

Other Parameters

- N_0, N_4 large, but no big hierarchy
- V_4 "fixed" (not scaled as ℓ_s^4)

  Mostly interested in "$V_4 \to \infty$" : D4 worldvol is $\mathbb{R}^4$

  but large V_4 is also O.K.

Doing Malda to SUGRA

- Holding $g_{YM}^{2\,(D4)}$ fixed at low-energy gives H_4 sans 1

$$H_4 = \frac{c_4}{2\pi} \frac{(g_{YM}^{2\,(D4)} U)}{(\ell_s U)^4} \sim \frac{1}{U^3} \quad , U = \frac{r}{\ell_s^2}$$

- H_0: Find, at D4 core, $H_0 = 1 + \frac{c_0 g_s N_0 \ell_s^7}{V_4 \Delta^3}$; so

$$H_0\Big|_{core} = 1 + \frac{c_0}{2\pi} \frac{N_0}{N_4} \frac{(g_{YM}^{2(D4)} N_4)}{(V_4)} \left(\frac{\ell_s^2}{\Delta}\right)^3$$

$\therefore$ for "fixed" $V_4, g_4, U_0 \equiv \frac{\Delta}{\ell_s^2}$, $V_4 \to \infty$ gives $H_0\Big|_{core} = 1$!

$\Rightarrow$ don't drop 1 in H_0 (see also Martinec-Sahakian 9906)

This is precisely analogous to D1, D3 behavior.

"Halfway between" probe ($H_0 = 1$) and DLCQ (H_0 sans 1) ...

If check on SUGRA curvature, dilaton, find only a wee bubble of 11-d very near D0.

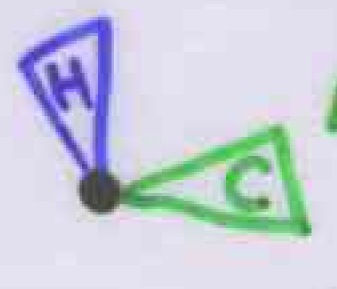
$V_4 \to$ finite gives old smeared solution at $\vec{\Delta} \to \vec{0}$

i.e. $H_0, H_4 \sim \#/U^3$.

Coulomb / Higgs

- If bulk/brane decoupling really is C/H decoupling, i.e. near-horizon geometry $\leftrightarrow$ Higgs branch NLσM, what happened to 0-4 strings, like $U_0 \equiv \frac{\Delta}{\ell_s^2}$? (Gone...)

M. Berkooz, H. Verlinde: were $\int$ out to get NLσM,
(9907100) still hiding inside ...

see also Seiberg-Witten (9903):
$\partial(AdS_5)$, $\rho \to 0$ $\leftrightarrow$ 

...

Field Theory view of delocalization

- D0 appears as instanton of D4 theory; Collective modes live in $d=0+1$ - NLσM
instanton scale size ρ + orientation, CM position
- $d=0+1$ theory very strongly coupled (SUGRA regime);
Coleman-Mermin-Wagner theorem: no superselection sectors.
Hence have wild IR fluctuations for ρ. ∞ at $\Delta=0$

* Treat $\frac{\Delta}{\ell_s^2} \equiv U_0$ as IR cutoff Λ_{IR} as $\vec{\Delta} \to \vec{0}$

Let us compute r.m.s. "size of ρ" $\rightarrow$

→

Scaling argument

dimensions

D0 $\langle \rho^2 \rangle \sim N_4 \cdot g_s \ell_s \cdot \left(\Lambda_{IR}^{-1} = \frac{\ell_s^2}{\Delta}\right)$

\# possible orientations

normalization from action (moduli space flat only for $N_0 = 1$; assume nothing goes badly wrong for $N_0 > 1$; expect admixture of X_{cm} down by $1/N$)

Thus $$\boxed{\frac{\sqrt{\langle \rho^2 \rangle}}{\ell_s} \underset{D0}{\sim} \sqrt{g_s N_4} \left(\frac{\ell_s}{\Delta}\right)^{\frac{1}{2}}}$$ ✓ (see also Berkooz-Verlinde ← (!))

D1 $\langle \rho^2 \rangle \sim N_5 \cdot g_s \ell_s^2 \cdot \log\left(\frac{\Lambda_{UV}}{\Lambda_{IR}}\right)$

Take $\Lambda_{UV} \sim 1/\ell_s$; then

$$\boxed{\frac{\sqrt{\langle \rho^2 \rangle}}{\ell_s} \underset{D1}{\sim} \sqrt{g_s N_5} \sqrt{\log\left(\sqrt{g_s N_5} \frac{\ell_s}{\Delta}\right)}}$$ ✓

D-1 A bit degenerate, but taking $\langle \rho^2 \rangle \sim N_3 \, g_3 \, \Lambda_{IR}^{-2}$ gives $$\boxed{\frac{\sqrt{\langle \rho^2 \rangle}}{\ell_s} \underset{D-1}{\sim} \sqrt{g_s N_3} \left(\frac{\ell_s}{\Delta}\right)}$$ ✓

i.e. <u>baldness for $p = 1, 0, -1$</u>

Of course, $\boxed{\text{hair for } p=2}$ ∵ have superselection sectors ✓

(For V_4 finite, localization for <u>long</u> timescale)

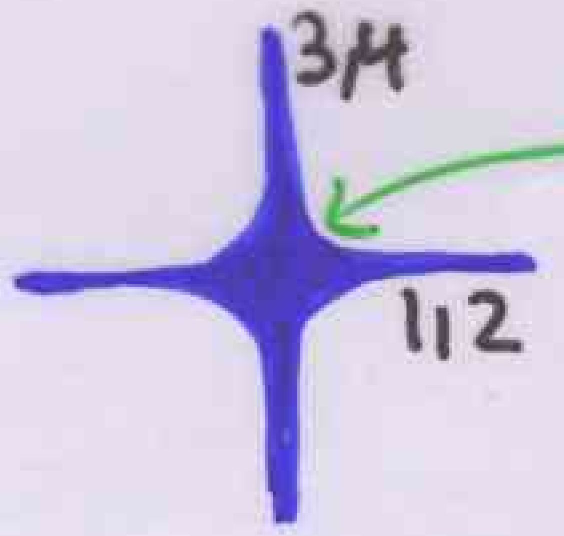
$\perp$ Intersections

- Near-horizon solutions obtained using S-M method naturally matched to asymptotically flat ones

- Field theory side suggests for near-horizon part:

  baldness as $\Delta \to 0$ for $p = -1, 0, 1$ -dim intersections
  hair " " " $p \geq 2$ " "

- e.g. D2 $\perp$ D2(0)



delocalization out to $\sim R_2$
$\rho \leftrightarrow$ blow-up
$(z_1 + i z_2)(z_3 + i z_4) \sim \rho^2$

- Predict SUGRA solutions with fat necks for small p

Gomberoff, Kastor, Marolf, Traschen (9905094):

- Use F-S ansatz with DBI source terms
- (Find off-diagonal pieces in metric <u>un</u>like F-S solutions)
- Perturbation theory gives
  - divergence @ 2nd-order for small-p even when sources treated carefully
  - sensible piece for big-p

as expected from our arguments ✓