DLCQ of $d=4$, $N=4$ SUSY Yang-Mills

with Simeon Hellerman

Goal: To explicitly demonstrate S-duality.

Idea: Use Discrete Light-Cone Quantization (DLCQ), which reduces QFT to QM.

Problem: Need a version of DLCQ which <u>preserves</u> S-duality.

Advantage of DLCQ: In light-cone

x^+ is 'time' so p_- is a 'spatial' momentum. In free field theory,

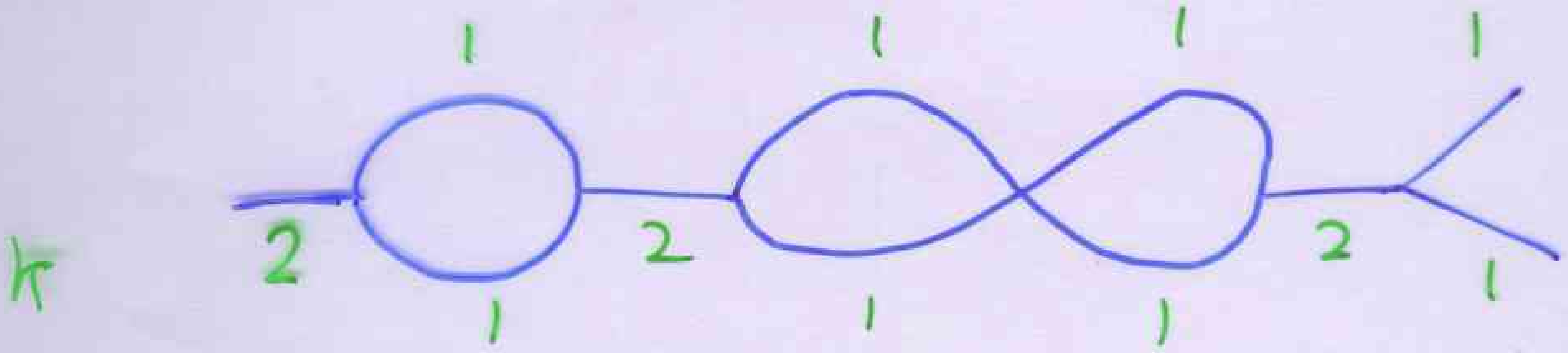
$$p_- = \sqrt{\vec{p}^{\,2}+m^2} + p_3$$

is positive (except for set of measure zero, $m = \vec{p} = 0$). In discrete LC, it is quantized, $p_- = k/R \quad k \in \mathbb{Z}$.

A sector of given total k can contain only a finite number of quanta.

E.g. total $k = 2 \Rightarrow$ one particle with $k=2$ or two, each with $k=1$.

p_- is conserved by interactions, so only a finite number of particle states mix*. E.g. for total $k=2$,



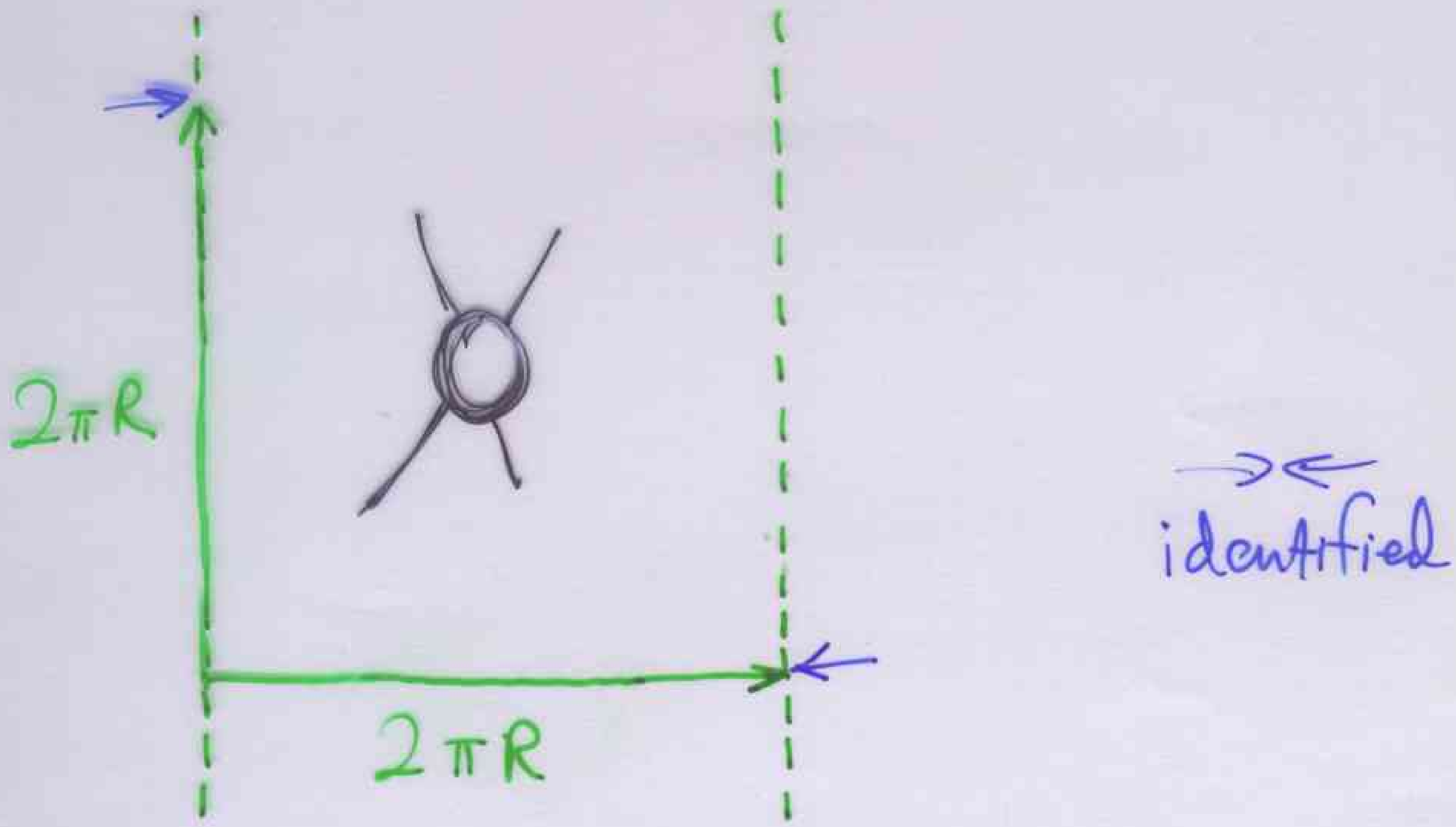
Geometric series. Goal: sum series and see S-duality at finite k.

k ≡ 'harmonic resolution'

* Thorn, Casher, Maskawa + Yamawaki, Brodsky + Pauli

Another application of DLCQ:

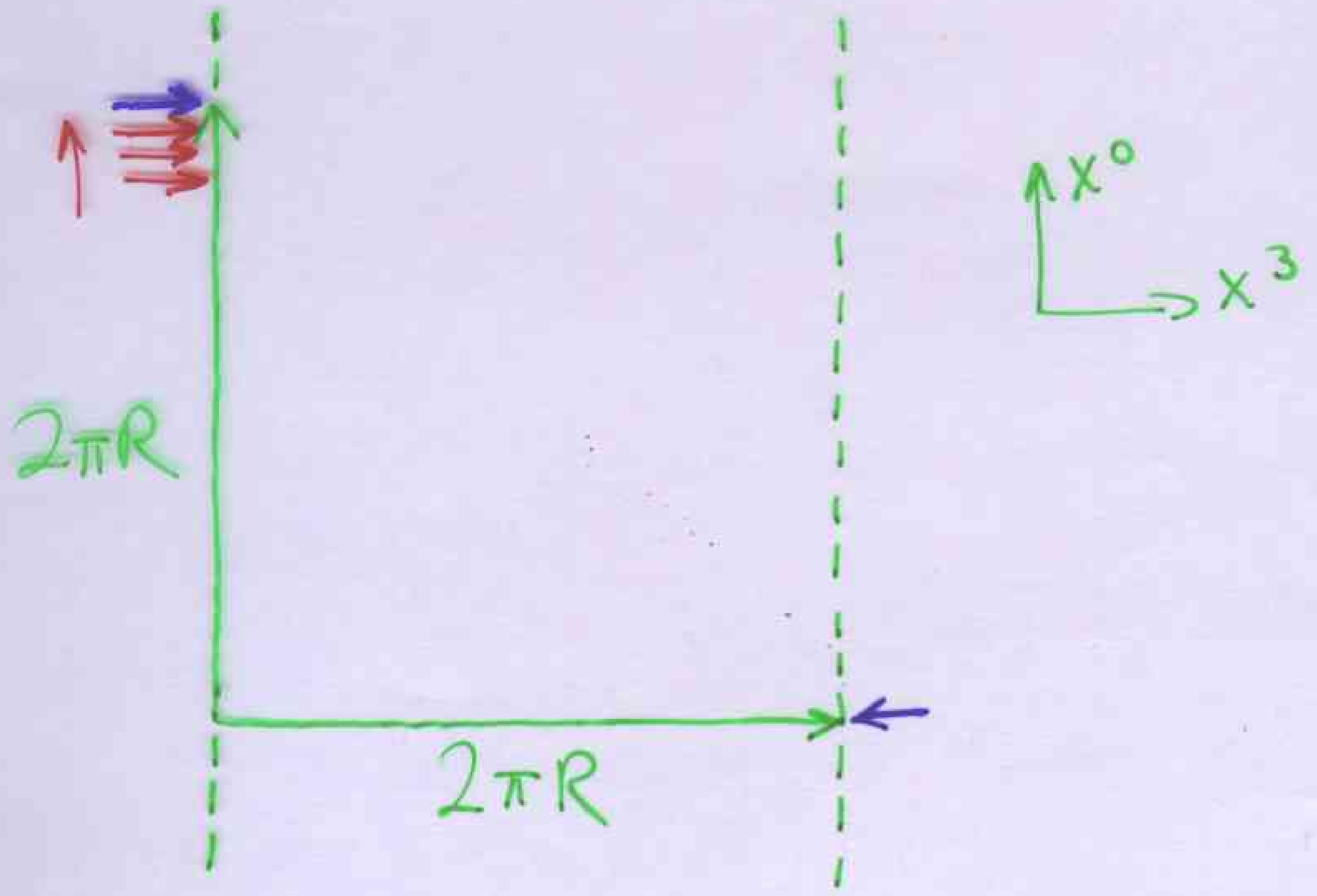
Take $R \to \infty$ holding scattering fixed: (requires $k \to \infty$)



As $k \to \infty$, we 'should' recover ordinary bulk physics.

We wish to preserve duality at finite k $(P_- = k/R)$ $\Rightarrow$ define DLCQ as a limit of spacelike compactification (presumed to preserve duality)

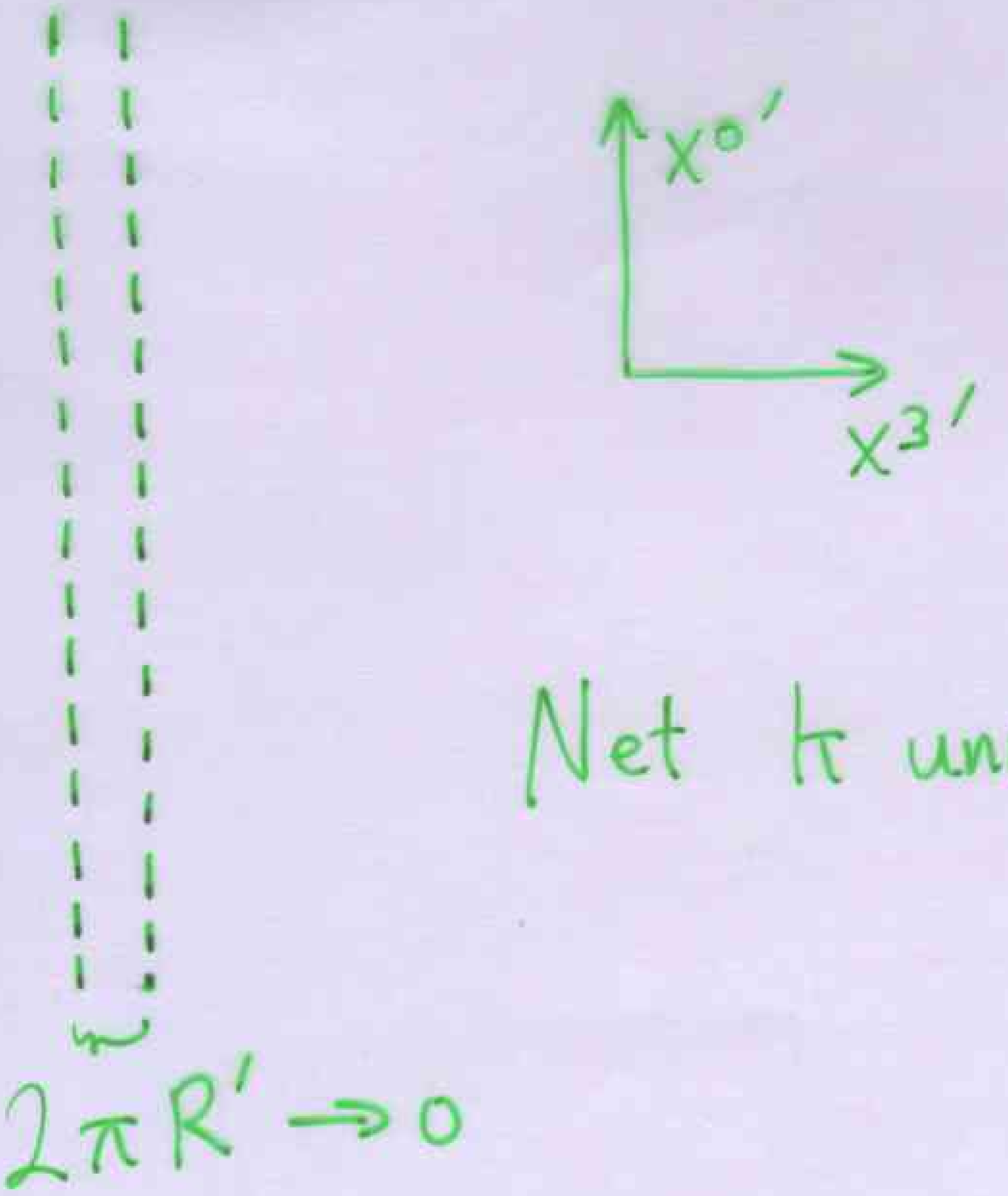
$k \equiv$ "harmonic resolution"



Light-like limit (L^3).

Will compare with other versions of DLCQ, and discuss large-k limit, later.

Analyze in frame where periodicity is spacelike:



Net k units of momentum.

Energy $(H' + p_3')$ scales in free theory:

Left-movers: R'^{-1}

Zero modes $(p_3 = 0)$: R'^{0}

Transverse motions of right-movers: R'^{1}.

$$\sqrt{\frac{\ell^2}{R^2} + \vec{p}^{\,2}} - \frac{\ell}{R} = O(R)$$

$$\uparrow H' + p_3' \equiv E$$

$O(R'^{-1})$ left-movers

$O(R'^{0})$ zero modes

$O(R')$ } DLCQ quantum mechanics (= states with finite energy in lightlike limit of original frame).

Versions of DLCQ:

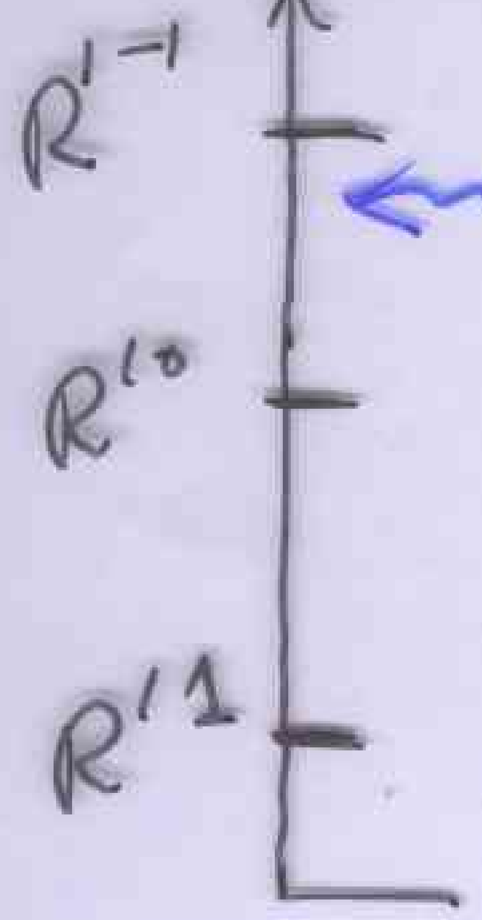
- Naive reduction: ignore zero modes and left movers.
- Light cone canonical: left movers decouple, zero modes treated classically.
- L^3: full Wilsonian problem.

For $k \to \infty$, first two probably suffice, e.g. Antonuccio, Hashimoto, Lunin, Pinsky

Finite $k \to L^3$

Complication with L^3: zero modes $\equiv$ 2+1 field theory with coupling $g_3^2 = g_4^2/R'$

$\Rightarrow$ strong dynamics at $E = g_3^2$



strong dynamics of zero modes
$\rightarrow$ radically changes lower energy physics.

Evaded in:

- (2,0) theory: $g_5^2 = g_6^2 R'$!
  (Aharony, Berkooz, Seiberg).
- Theories with gravity ($R' \to 0$ only at ∞ (BKL/HKS)).
- Gauge theories, if we turn on a Wilson line A_3, so zero modes are Abelian.

With Wilson line can carry out reduction to quantum mechanics. E.g. for SU(2),

$$L = \sum_i \frac{1}{2p_{3'i}} v_i^2$$

$$+ \frac{g^2}{16\pi} \sum_{i,j} q_i q_j (v_i - v_j)^2 \ln \frac{|x_i - x_j|^2}{L^2}$$

$$+ \text{fermionic}$$

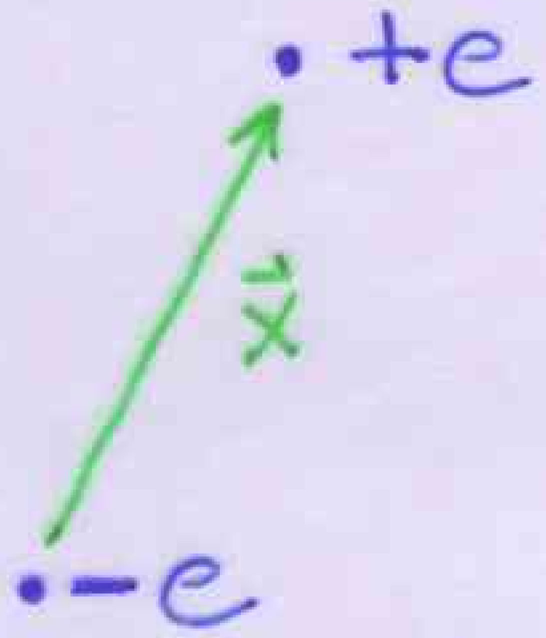
- i labels particles; $q_i = \pm 1$; $\sum_i q_i = 0$
- L = IR cutoff (net current costs ∞ energy due to 2+1-dim. logs) $\Rightarrow$

$$\sum_i q_i \vec{x}_i = \text{constant}$$

Aside: IR divs $\Rightarrow k \to \infty$ limit does not reproduce bulk physics.

Aside: New SUSY QM models (Hellerman + Polchinski, Golfand Memorial)

Due to IR divs, 2-body sector is trivial:



Pointlike + and − electric charges at fixed separation, independent of g.

$\left[\, p_- = \frac{\theta}{R},\ \frac{1-\theta}{R} \Rightarrow k = 1 \,\right]$

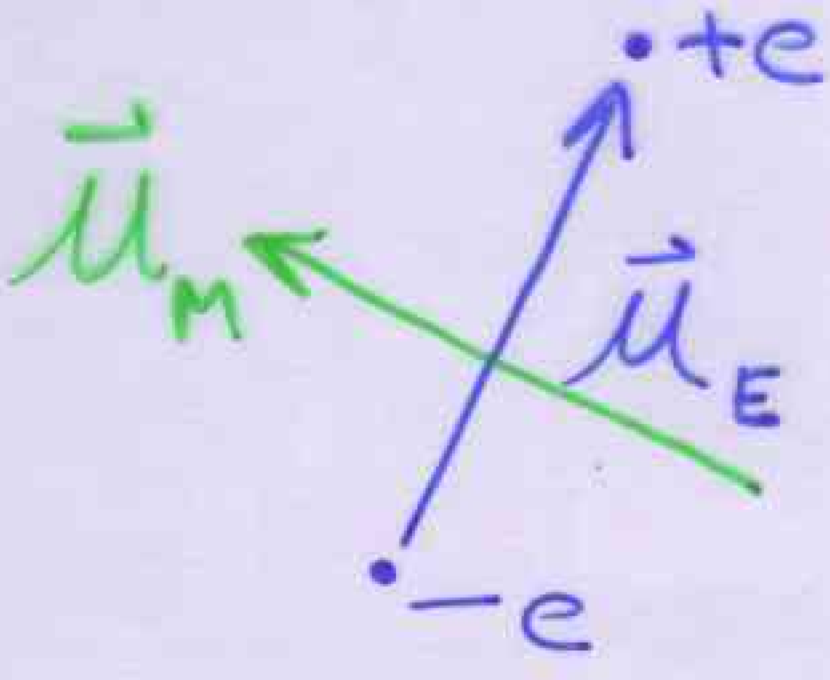
Where are the corresponding magnetic states, required by S-duality?

Should the low energy effective theory include explicit pointlike magnetic sources? No way to get them.

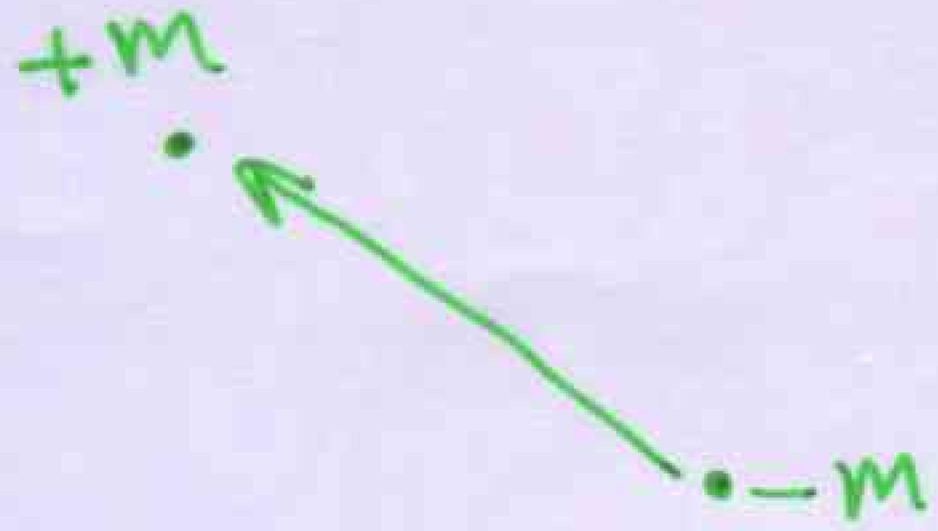
(No such magnetic configurations

More subtle breakdown of DLCQ.

Note first that the 2-body state has both electric and magnetic dipole moments:



Claim that at strong coupling it evolves into

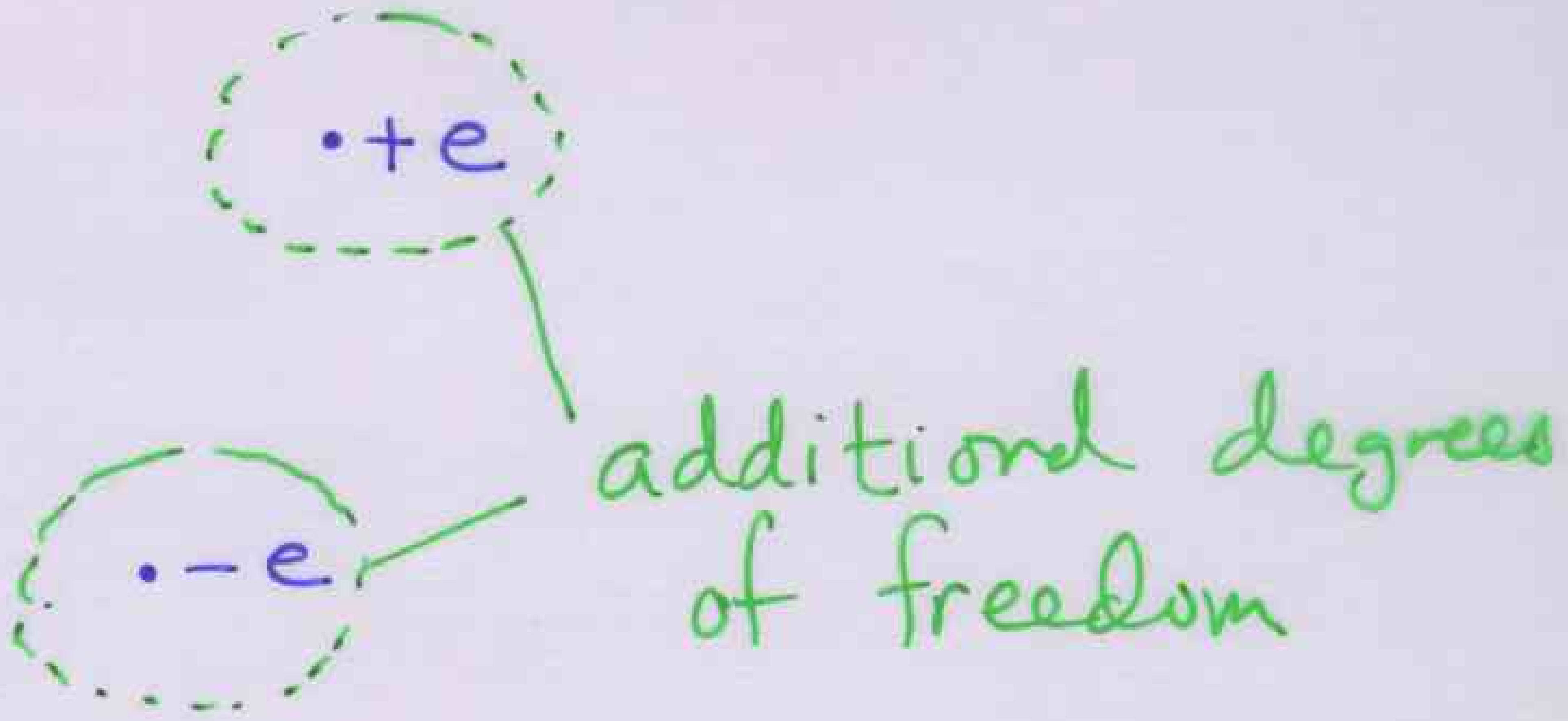


Another point: Wilson line is position-dependent:

$$A_{3'}(z) = A_{3'}(\infty) - \frac{g^2}{4\pi R'} \ln\left|\frac{z - z_+}{z - z_-}\right|$$

$$z = x + iy$$

The point is that the Wilson line vanishes along curves:



Radii $\sim e^{-1/g^2}$ grow with coupling.

Consistent picture: charges spread with g, at large g become pointlike magnetic.

Hard dynamical problem.

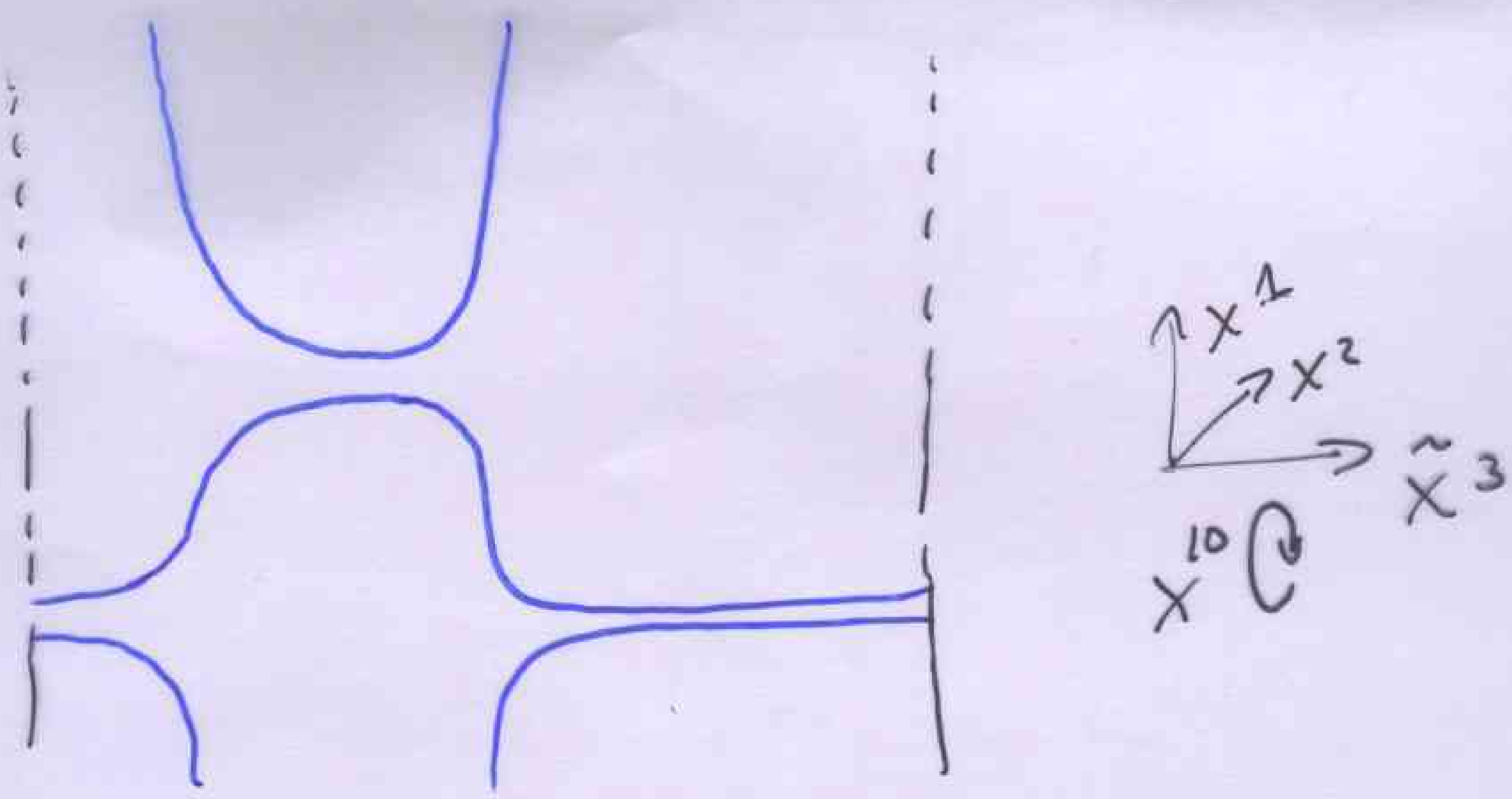
Different approach (Ganor + Sethi, 9712...): embed the problem in gauge theory — D3-branes in IIB — and keep only massless open string sector of DLCQ.

- $R' \to 0$ requires T-duality to $\tilde{X}_3$,

  $D3 \to D2 \qquad P_3 \to F1$
- T-dual $g' \propto 1/R'$ so we get M-theory.

  $$D2 \to M2 \qquad F1 \to M2$$

- DLCQ = QM of holomorphic maps, $X^1 + iX^2 \to \tilde{X}^3 + iX^{10}$



Appears to interpolate smoothly as a function of $g^2 = \frac{R_{10}}{R_{\tilde{3}}}$.

Is this the correct way to think about gauge theory, as QM of holomorphic curves?