SUSY Breaking

and

Open Strings

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Based on work with:

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hep-th/9807011, hep-th/9907yyy

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hep-th/9812118, hep-th/9907zzz

Plan

1. The Scherk-Schwarz mechanism
2. Oriented closed strings
3. Open and unoriented strings
4. Brane and bulk supersymmetry
5. Outlook

The Scherk-Schwarz mechanism

(Scherk and Schwarz, 1979)

Generalize ordinary Kaluza-Klein reduction by introducing a **suitable dependence** on internal coordinates.

A consistent ansatz requires that the dependence on internal coordinates disappear from the action, aside from an overall measure factor: **global internal symmetry**.

Supersymmetry breaking if different members of the same supermultiplet have different dependence on internal coordinates. One can obtain a **vanishing cosmological constant** at tree level.

HOWEVER (vacuum problem): typically a non-zero cosmological constant is produced by **quantum corrections**.

In Field Theory M (**gravitino mass**) is an arbitrary parameter. On the other hand, in String Theory $M \sim 1/R$, with R a typical **internal size**.

Conventional and Brane Kaluza-Klein

Conventional Kaluza-Klein: scale of supersymmetry breaking **linked** to K-K scale R.

Brane Kaluza-Klein: some (matter) sectors live on **brane islands**, while the gravitational sector spreads in the whole Kaluza-Klein space.

- Scherk-Schwarz mechanism **not affecting** (to lowest order) the **brane islands**?

SUSY breaking in bulk, with **SUSY islands**: matter sectors with **tree-level global SUSY**. String-induced SUSY breaking in these sectors.

- Scherk-Schwarz mechanism **not affecting** (to lowest order) the **bulk**?

SUSY breaking on branes. String-induced SUSY breaking in the bulk.

Both possibilities in perturbative type-I vacua

Scherk-Schwarz in closed strings:

$D = 9$

(Rohm, 1984)

(Ferrara, Kounnas, Porrati, Zwirner, 1989)

1. $SO(8)$ level-one characters: $(q = e^{2\pi i\tau})$

$$O_8 = \frac{\vartheta_3^4 + \vartheta_4^4}{2\eta^4} \qquad V_8 = \frac{\vartheta_3^4 - \vartheta_4^4}{2\eta^4}$$

$$S_8 = \frac{\vartheta_2^4 + \vartheta_1^4}{2\eta^4} \qquad C_8 = \frac{\vartheta_2^4 - \vartheta_1^4}{2\eta^4}$$

2. Type-IIB 10D superstring:

$$\mathcal{T} = |V_8 - S_8|^2$$

3. Circle reduction and lattice sums:

$$p_L = \frac{m}{R} + \frac{nR}{\alpha'} \qquad p_R = \frac{m}{R} - \frac{nR}{\alpha'}$$

$$Z_{mn} \equiv \sum_{m,n} \frac{q^{\alpha' p_L^2/4} \ \bar{q}^{\alpha' p_R^2/4}}{\eta\bar{\eta}}$$

4. Circle reduction of type-IIB to D=9:

$$\mathcal{T} = Z_{mn} |V_8 - S_8|^2$$

Scherk-Schwarz in closed strings:

$D = 9$

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1. Scherk-Schwarz breaking from **momentum shifts**:

$$\mathcal{T}_1 = Z_{m,2n}(V_8\bar{V}_8 + S_8\bar{S}_8) + Z_{m,2n+1}(O_8\bar{O}_8 + C_8\bar{C}_8) \\ - Z_{m+1/2,2n}(V_8\bar{S}_8 + S_8\bar{V}_8) - Z_{m+1/2,2n+1}(O_8\bar{C}_8 + C_8\bar{O}_8)$$

- Notice: **tachyon instability** for $R \leq \sqrt{\alpha'}$.

  Must be consistent in the **large-radius limit**.

2. Scherk-Schwarz breaking from **winding shifts**:

$$\mathcal{T}_2 = Z_{2m,n}(V_8\bar{V}_8 + S_8\bar{S}_8) + Z_{2m+1,n}(O_8\bar{O}_8 + C_8\bar{C}_8) \\ - Z_{2m,n+1/2}(V_8\bar{S}_8 + S_8\bar{V}_8) - Z_{2m+1,n+1/2}(O_8\bar{C}_8 + C_8\bar{O}_8)$$

- Notice: **tachyon instability** for $R \geq \sqrt{\alpha'}$.

  Must be consistent in the **small-radius limit**.

Scherk-Schwarz breaking and open strings

1. **For model 1 (momentum shifts):**

$$
\begin{aligned}
\mathcal{K}_1 &= \frac{1}{2}\,(V_8 - S_8)\;Z_m \\
\mathcal{A}_1 &= \frac{n_1^2 + n_2^2}{2}(V_8 Z_m - S_8 Z_{m+1/2}) \\
&+ \; n_1 n_2 (V_8 Z_{m+1/2} - S_8 Z_m) \\
\mathcal{M}_1 &= -\frac{n_1 + n_2}{2}(\hat{V}_8 Z_m - \hat{S}_8 Z_{m+1/2})
\end{aligned}
$$

- The tadpole conditions require:

$$n_1 + n_2 = 32$$

- SUSY is recovered in the limit $R \to \infty$. The vacuum channel contains winding modes, that become more and more spaced in the limit $R \to \infty$.

- As in the closed spectrum, for large R bosons and fermions have mass splittings $O(1/R)$.

Scherk-Schwarz breaking and open strings

2. **For model 2 (winding shifts)**, after imposing the **tadpole conditions** ($n_1 + n_2 = n_3 + n_4 = 16$):

$$
\begin{aligned}
\mathcal{K}_2 &= \frac{1}{2}\,(V_8 - S_8)\;Z_{2m} + \frac{1}{2}\,(O_8 - C_8)\;Z_{2m+1} \\
\mathcal{A}_2 &= \Big(\frac{n_1^2 + {n_2}^2 + {n_3}^2 + n_4^2}{2}(V_8 - S_8) \\
&+ (n_1 n_3 + n_2 n_4)(O_8 - C_8)\Big) Z_m \\
&+ \Big((n_1 n_2 + n_3 n_4)(V_8 - S_8) \\
&+ (n_1 n_4 + n_2 n_3)(O_8 - C_8)\Big) Z_{m+1/2} \\
\mathcal{M}_2 &= -\frac{n_1 + n_2 + n_3 + n_4}{2}\hat{V}_8 Z_m \\
&+ \frac{n_1 - n_2 - n_3 + n_4}{2}\hat{S}_8(-1)^m Z_m
\end{aligned}
$$

- SUSY is recovered in the limit $R \to 0$.
- **However:** as $R \to 0$ **new** tadpoles appear (**collapsing winding modes**): $n_2 = n_3 = 0$.

  (Polchinski and Witten, 1995)

- **SUSY massless spectrum** : $SO(16) \times SO(16)$

Winding shifts: M-theory breaking

- **Momentum shifts:** a rather conventional open spectrum. SUSY splittings $\sim 1/R$ at all levels.

- **Winding shifts:** Chan-Paton unpairing at alternate levels. **SUSY for massless modes**:

"Brane Supersymmetry"

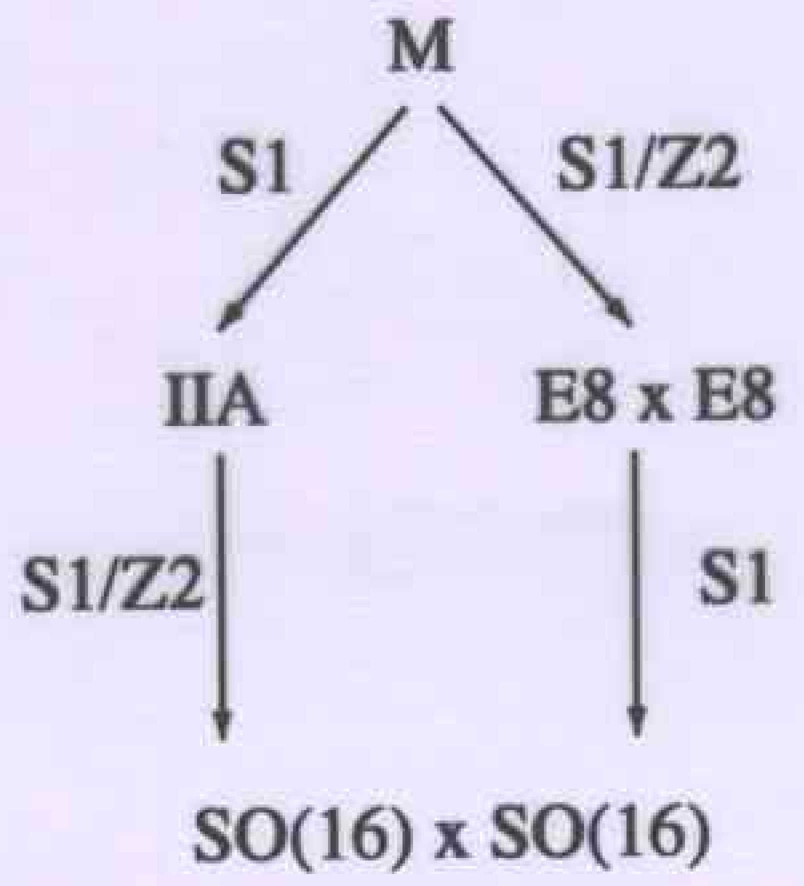
Can we understand this better?

- **String dualities** link the 5 10D strings to the 11D "M theory", and establish **relations** between corresponding **moduli**. String descriptions of low-energy supergravity differ by **field redefinitions**.

11D Scherk-Schwarz deformations

(Antoniadis and Quiros, 1997)
(Dudas and Grojean, 1997)

Compare 9D BPS States:



Reduction of M theory from 11D to 9D on $S1 \times S1/Z2$ of radii R_{11}, R_{10} can be seen in two ways:

(Horava and Witten, 1996)

- as reduction on $S1/Z2$ ($E_8 \times E_8$ string), and subsequent reduction on another $S1$.
- as reduction on $S1$ (IIA string), and subsequent reduction on an $S1/Z2$ orientifold. After a T duality this gives type I.

- K-K and membrane wrapping masses in M theory, type I, heterotic $SO(32)$ and heterotic $E_8 \times E_8$:

$$
\begin{aligned}
\mathcal{M}^2 &= \frac{l^2}{R_{11}^2} + \frac{m^2}{R_{10}^2} + n^2 R_{10}^2 R_{11}^2 M_{11}^6 \\
\mathcal{M}_I^2 &= l^2 R_I^2 M_I^4 + \frac{m^2 R_I^2 M_I^4}{{g_I}^2} + \frac{n^2}{R_I^2} \\
\mathcal{M}_{E_8}^2 &= \frac{l^2 M_H^2}{{g_{E_8}}^2} + \frac{m^2}{R_{E_8}^2} + n^2 R_{E_8}^2 M_H^4 \\
\mathcal{M}_H^2 &= l^2 \frac{R_H^2 M_H^4}{{g_H}^2} + m^2 R_H^2 M_H^4 + \frac{n^2}{R_H^2}
\end{aligned}
$$

- Scherk-Schwarz shifts along 11th dimension:

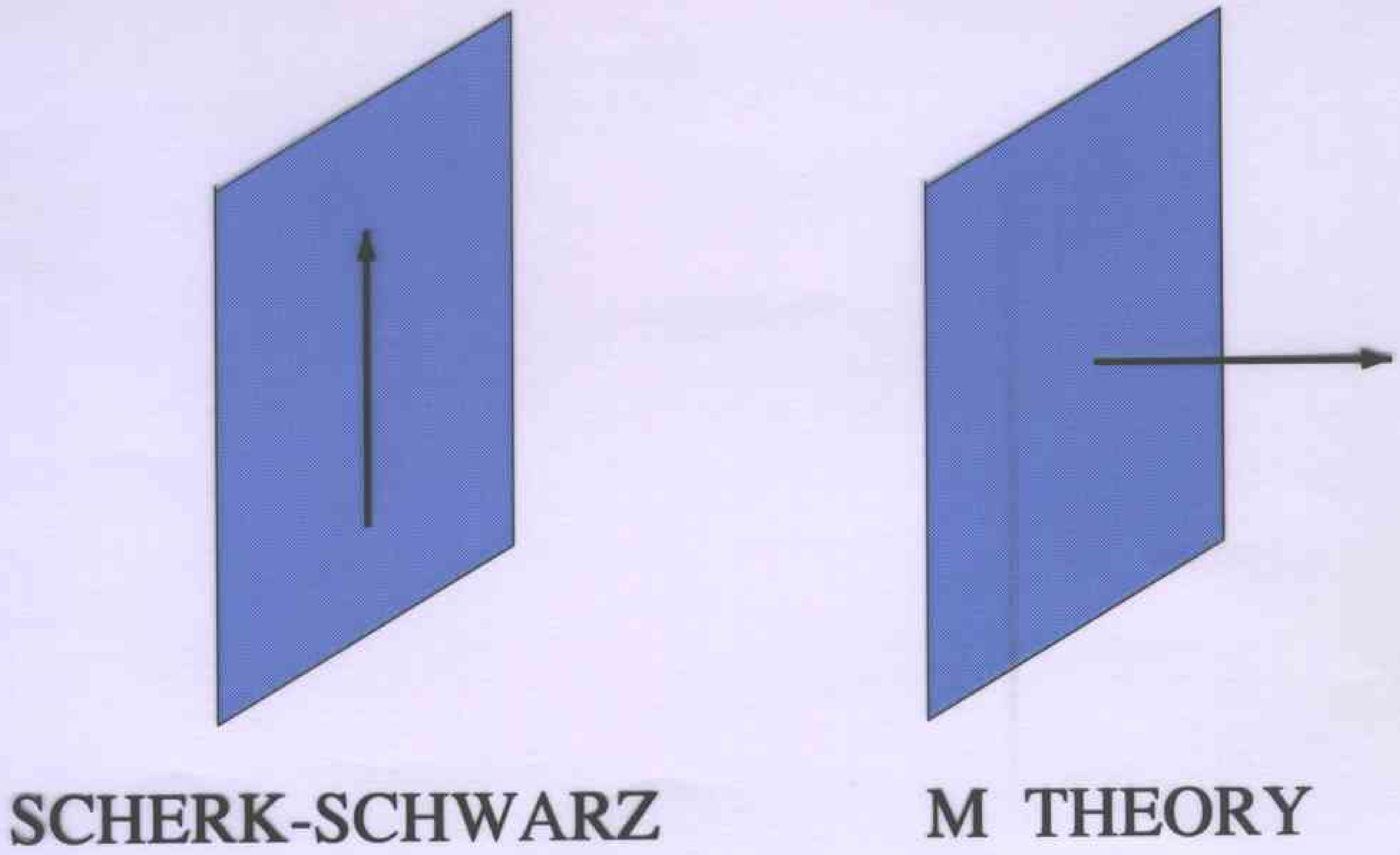
$$l \to l + s$$

- **Perturbative in type I.**

- **Non-perturbative in heterotic strings.**

- **In type I correspond to winding shifts.**

Scherk-Schwarz and M-Theory Breaking



- In Scherk-Schwarz breaking, the **shifts** are along directions **parallel** to the brane.

- In M theory breaking, the **shifts** are along directions **orthogonal** to the brane.

A $Z_2 \times Z_2$ shift orbifold

Consider a $Z_2 \times Z_2$ **"shift" orbifold**, where

$$g \equiv (1,-1,-1)\ ,\ f = (-1,p_2,-p_3)\ ,\ h = (-1,-p_2,p_3)$$

$$\begin{aligned}\mathcal{T} \equiv \frac{1}{4}\Big\{ & |T_{oo}|^2\Lambda_1\Lambda_2\Lambda_3 + |T_{og}|^2\Lambda_1|\frac{4\eta^2}{\theta_2^2}|^2 + |T_{of}|^2(-1)^{m_2}\Lambda_2|\frac{4\eta^2}{\theta_2^2}|^2 \\ & + |T_{oh}|^2(-1)^{m_3}\Lambda_3|\frac{4\eta^2}{\theta_2^2}|^2 + |T_{go}|^2\Lambda_1|\frac{4\eta^2}{\theta_4^2}|^2 + |T_{gg}|^2\Lambda_1|\frac{4\eta^2}{\theta_3^2}|^2 \\ & + |T_{fo}|^2\Lambda_2^{n_2+1/2}|\frac{4\eta^2}{\theta_4^2}|^2 + |T_{ff}|^2(-1)^{m_2}\Lambda_2^{n_2+1/2}|\frac{4\eta^2}{\theta_3^2}|^2 \\ & + |T_{ho}|^2\Lambda_3^{n_3+1/2}|\frac{4\eta^2}{\theta_4^2}|^2 + |T_{hh}|^2(-1)^{m_3}\Lambda_3^{n_3+1/2}|\frac{4\eta^2}{\theta_3^2}|^2\Big\}\end{aligned}$$

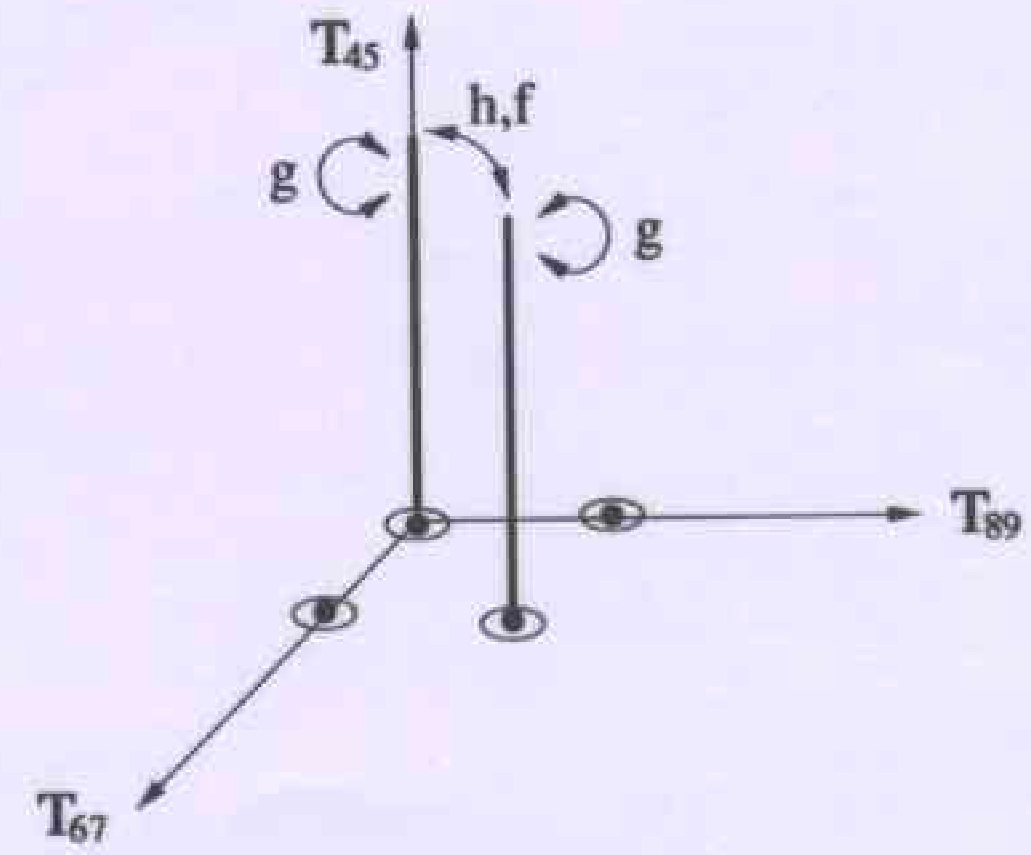
$$\begin{aligned}\mathcal{K} &= \frac{1}{8}\Big\{T_{oo}\big[P_1P_2P_3 + P_1W_2W_3 + W_1(-1)^{m_2}P_2W_3 \\ &\quad + W_1W_2(-1)^{m_3}P_3\big] + 2T_{go}\ P_1\left(\frac{2\eta}{\theta_4}\right)^2\Big\}\end{aligned}$$

$$\begin{aligned}\tilde{\mathcal{K}} &= \frac{2^5}{8}\Big\{T_{oo}\big[v_1v_2v_3W_1^eW_2^eW_3^e + \frac{v_1}{v_2v_3}W_1^eP_2^eP_3^e \\ &\quad + \frac{v_2}{v_1v_3}P_1^eW_2^oP_3^e + \frac{v_3}{v_1v_2}P_1^eP_2^eW_3^o\big] + 2T_{og}v_1W_1^e\left(\frac{2\eta}{\theta_2}\right)^2\Big\}\end{aligned}$$

Only $D9$ and $D5_1$ are left

A $Z_2 \times Z_2$ shift orbifold

$$
\begin{aligned}
\mathcal{A} \; = \; & \frac{1}{8}\Big\{ T_{oo}\Big[N^2 P_1 P_2 P_3 + \frac{D_1^2}{2} P_1 (W_2 W_3 + W_2^{n+1/2} W_3^{n+1/2}) \Big] \\
& + (G^2 + 2G_1^2) T_{og} P_1 \big(\frac{2\eta}{\theta_2}\big)^2 + F^2 \; T_{of} (-1)^{m_2} P_2 \big(\frac{2\eta}{\theta_2}\big)^2 \\
& + H^2 \; T_{oh} (-1)^{m_3} P_3 \big(\frac{2\eta}{\theta_2}\big)^2 + 2ND_1 T_{go} P_1 \big(\frac{\eta}{\theta_4}\big)^2 \\
& + \; 4GG_1 T_{gg} P_1 \big(\frac{\eta}{\theta_3}\big)^2 \Big\}
\end{aligned}
$$



- $U(16)_9 \times U(8)_5$ gauge group.
- N=1 supersymmetry in bulk and $D9$ sectors.
- (N=2) brane supersymmetry **at all mass levels** in $D5$ sector, as in some asymmetric orbifolds.

(Blumenhagen and Görlich, 1999)

(Angelantonj, Antoniadis and Förger, 1999)

Brane supersymmetry breaking

- There are SUSY models where it is problematic to solve all **tadpole conditions**.

1. 4D $Z_2 \times Z_2$ with **discrete torsion** ($\prod_i \epsilon_i = \epsilon$):

$$\mathcal{K} = \frac{1}{8}\Big\{ (P_1P_2P_3 + P_1W_2W_3 + W_1P_2W_3 + W_1W_2P_3)T_{oo}$$

$$+2\big[\epsilon_1(P_1 + \epsilon W_1)T_{go} + \epsilon_2(P_2 + \epsilon W_2)T_{fo} + \epsilon_3(P_3 + \epsilon W_3)T_{ho}$$

$$\tilde{\mathcal{K}}_0 = \frac{2^5}{8}\Big\{ \left(\sqrt{v_1v_2v_3} + \epsilon_1\sqrt{\frac{v_1}{v_2v_3}} + \epsilon_2\sqrt{\frac{v_2}{v_1v_3}} + \epsilon_3\sqrt{\frac{v_3}{v_1v_2}}\right)^2 \tau_{oo}$$
$$+ \left(\sqrt{v_1v_2v_3} + \epsilon_1\sqrt{\frac{v_1}{v_2v_3}} - \epsilon_2\sqrt{\frac{v_2}{v_1v_3}} - \epsilon_3\sqrt{\frac{v_3}{v_1v_2}}\right)^2 \tau_{og}$$
$$+ \left(\sqrt{v_1v_2v_3} - \epsilon_1\sqrt{\frac{v_1}{v_2v_3}} + \epsilon_2\sqrt{\frac{v_2}{v_1v_3}} - \epsilon_3\sqrt{\frac{v_3}{v_1v_2}}\right)^2 \tau_{of}$$
$$+ \left(\sqrt{v_1v_2v_3} - \epsilon_1\sqrt{\frac{v_1}{v_2v_3}} - \epsilon_2\sqrt{\frac{v_2}{v_1v_3}} + \epsilon_3\sqrt{\frac{v_3}{v_1v_2}}\right)^2 \tau_{oh} \Big\}$$

2. **Simpler example:** 6D T^4/Z_2 with "exotic" Klein.

- R-R tadpoles unpair NS and R : **anti-branes**.
- Anomaly free, **SUSY broken on branes**, $V > 0$.

Brane supersymmetry breaking

$$\mathcal{T} = \frac{1}{2}|Q_o + Q_v|^2 \Lambda + \frac{1}{2}|Q_o - Q_v|^2 \left|\frac{2\eta}{\theta_2}\right|^4 + \frac{1}{2}|Q_s + Q_c|^2 \left|\frac{2\eta}{\theta_4}\right|^4 + \frac{1}{2}|Q_s - Q_c|^2 \left|\frac{2\eta}{\theta_3}\right|^4$$

$$Q_o = V_4 O_4 - C_4 C_4 \quad , \quad Q_v = O_4 V_4 - S_4 S_4$$
$$Q_s = O_4 C_4 - S_4 O_4 \quad , \quad Q_c = V_4 S_4 - C_4 V_4$$

$$\mathcal{K} = \frac{1}{4}\Big\{(Q_o + Q_v)(P + W) + 2\,\epsilon \times 16(Q_s + Q_c)\Big\}$$

1. $\epsilon = 1$ (without B_{ab}): **1 tensor multiplet**.

- $U(16) \times U(16)$.

(Bianchi and A.S., 1990)
(Gimon and Polchinski, 1996)

2. $\epsilon = -1$ (without B_{ab}): **17 tensor multiplets**

$$\tilde{\mathcal{K}}_0 = \frac{2^5}{4}\left\{Q_o\left(\sqrt{v} + \epsilon\frac{1}{\sqrt{v}}\right)^2 + Q_v\left(\sqrt{v} - \epsilon\frac{1}{\sqrt{v}}\right)^2\right\}$$

- **$D9$ and $D\bar{5}$: tachyon-free chiral spectrum.**
- $[SO(16) \times SO(16)]_9 \times [USp(16) \times USp(16)]_5$.

Brane supersymmetry breaking

The annulus amplitude is:

$$\mathcal{A} = \frac{1}{4}\Big\{(Q_o + Q_v)(N^2P + D^2W) + 2ND(Q'_s + Q'_c)\left(\frac{\eta}{\theta_4}\right)^2$$
$$+ \; (R_N^2 + R_D^2)(Q_o - Q_v)\left(\frac{2\eta}{\theta_2}\right)^2$$
$$+ \; 2R_N R_D(-O_4S_4 - C_4O_4 + V_4C_4 + S_4V_4)\left(\frac{\eta}{\theta_3}\right)^2\Big\} \,.$$

while the Möbius projection is:

$$\mathcal{M} = -\frac{1}{4}\Big\{NP(\hat{O}_4\hat{V}_4 + \hat{V}_4\hat{O}_4 - \hat{S}_4\hat{S}_4 - \hat{C}_4\hat{C}_4)$$
$$- \; DW(\hat{O}_4\hat{V}_4 + \hat{V}_4\hat{O}_4 + \hat{S}_4\hat{S}_4 + \hat{C}_4\hat{C}_4)$$
$$- N(\hat{O}_4\hat{V}_4 - \hat{V}_4\hat{O}_4 - \hat{S}_4\hat{S}_4 + \hat{C}_4\hat{C}_4)\left(\frac{2\hat{\eta}}{\hat{\theta}_2}\right)^2$$
$$+ \; D(\hat{O}_4\hat{V}_4 - \hat{V}_4\hat{O}_4 + \hat{S}_4\hat{S}_4 - \hat{C}_4\hat{C}_4)\left(\frac{2\hat{\eta}}{\hat{\theta}_2}\right)^2\Big\}$$

Conclusions

- The Scherk-Schwarz mechanism allows one to induce **SUSY breaking** from generalized Kaluza-Klein reductions.
- In Field Theory the breaking scale is a **free** parameter.
- Main (unsolved) problem: in general, quantum corrections produce a large **cosmological constant**.
- Additional problem: in closed-string models the **scale** of SUSY breaking is linked to the **size of the internal dimensions**.
- In open-string models: **two** inequivalent ways to implement the Scherk-Schwarz breaking. In **M-theory breaking**, **brane islands** with (extended) SUSY for massless modes. SUSY breaking is **induced** by string corrections.
- SUSY breaking scale **not tied** to internal radii (to lowest order).
- Another possibility: brane supersymmetry breaking.