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PLACE: CALTECH (PASADENA, CALIFORNIA)

DATES: JAN. 12-15, 2000

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REMARKS ON NON-BPS D-BRANES

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PLAN:

- Review Sen's constructions of non-BPS D-branes as kinks in tachyon fields of $Dp - \overline{Dp}$ systems
- Review the K-theory classification of D-brane charges for type IIB and type I
- Discuss issues concerning the interpretation of the proposed non-BPS D8-brane in type I

D-brane + anti-D-brane systems

A coincident system of N D_p -branes and N' \bar{D}_p -branes is not BPS and is unstable. The $P\bar{P}$ open strings have reversed GSO projection and their ground states are tachyonic.

The world-volume gauge fields A, A' and the bifundamental tachyon T can be written together as a "superconnection"

$$Q = \begin{pmatrix} A & T \\ \bar{T} & A' \end{pmatrix}$$

If $N=N'$, $E \sim E'$, and T is top. trivial, then complete annihilation is possible. Otherwise something survives.

For example, consider $D2 + \overline{D2}$ on T^2 .

The WZ term of the D2 is

$$\int (C e^F)_3 = \int dt \int_{T^2} (C_3 + C_{1 \wedge} F)$$

Thus, magnetic flux $\int_{T^2} F$ is a source for C_1 — it carries D0-brane charge.

If the D2 has one unit of D0 charge and the $\overline{D2}$ has none, then

$$D2 + \overline{D2} \leftrightarrow D0$$

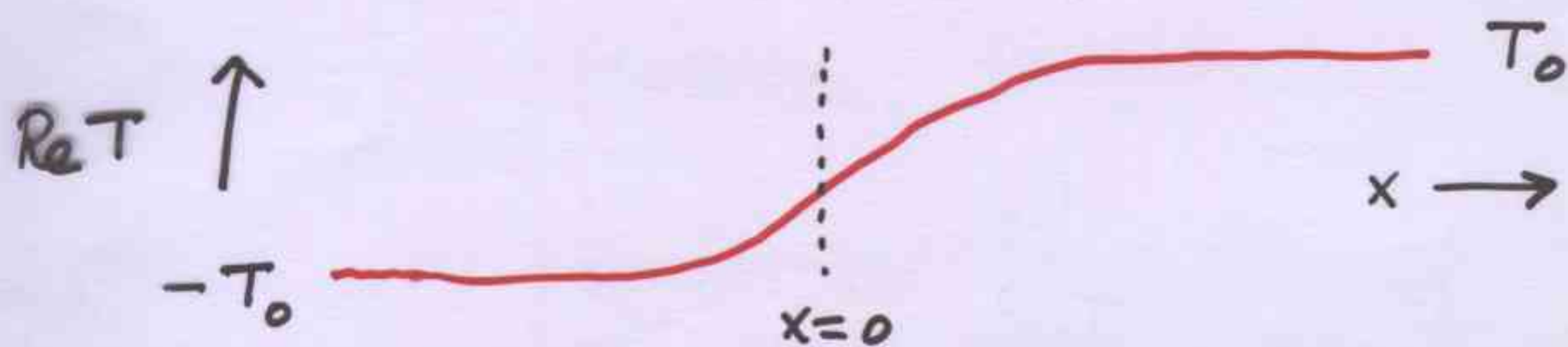
Which system has lower mass depends on the area of the torus.

A $Dp + \overline{Dp}$ system has a tachyon potential $V(T)$. Sen argues that when $N = N'$ it has a minimum for $T = T_0 \mathcal{U}$ where $\mathcal{U} \in U(N)$, such that

$$V(T_0 \mathcal{U}) + 2N T_{Dp} = 0.$$

Thus when \mathcal{Q} is trivial, the $D_p + \overline{D}_p$ system is equivalent to pure vacuum. What happens to the gauge groups is not completely understood.

Take $N = N' = 1$ and consider a kink



This describes a $D(p-1)$ -brane localized around $x=0$. Since the vacuum manifold $|T|=T_0$ is a circle, and $\pi_0(S^1)$ is trivial, this D-brane has a tachyon in its world volume and is *unstable*. It carries no conserved charge. Such D-branes can be constructed for all "wrong" values of p in Type II theories.

Non-BPS type I D0-branes

The type I D-string is the $spin(32)/\mathbb{Z}_2$ heterotic string continued to strong coupling. A system of N coincident D-strings has world-volume gauge group $O(N)$. In particular, for a single D-string it is $O(1) = \mathbb{Z}_2$. Thus a D-string wrapped on a circle has possible Wilson lines $W = \pm 1$.

The 32 left-moving fermion fields λ^A on the D-string world-sheet arise as zero-modes of D1-D9 open strings. The Wilson line encodes their periodicity:

$$\lambda^A(x + 2\pi R) = W \lambda^A(x)$$

Thus, for $W = 1$, λ^A has zero modes, and D-string quantum states are gauge group spinors.

Now consider a $D1 + \bar{D1}$ pair wrapped on a circle. If one string has $W = +1$ and the other has $W = -1$, the overall state is a gauge group spinor. Complete annihilation is not possible. The common world volume has an antiperiodic real "tachyon"

$$T = \sum T_{n+1/2}(t) \exp\left[i\left(\frac{n+1/2}{R}\right)x\right]$$

The mass of $T_{n+1/2}$ is

$$M_{n+1/2}^2 = (n+1/2)^2 / R^2 - \frac{1}{2}$$

- For $R < 1/\sqrt{2}$, there is no true tachyon and the wrapped $D1 + \bar{D1}$ pair is stable.
- For $R > 1/\sqrt{2}$, $T_{\pm 1/2}$ is tachyonic and the strings can annihilate to give a **stable non-BPS D0-brane**, which is a gauge group spinor. It carries a conserved \mathbb{Z}_2 charge.

In this case, the \mathbb{Z}_2 corresponds to the center of the $\text{spin}(32)/\mathbb{Z}_2$ gauge group.

At $R=R_c = 1/\sqrt{2}$ and small g

$$M_{D0} \sim 2 \cdot 2\pi R_c \cdot T_{D1} = \sqrt{2}/g$$

Sen argues that this is the leading small g value of the D0-brane mass for all R .

In the S-dual heterotic theory, the lightest gauge group spinor occurs at the first excited level in the perturbative spectrum. Presumably, the non-BPS D0-brane is this state continued to strong coupling.

K theory classification of D-branes

Recall that a $D_p + \bar{D}_p$ system is characterized by a pair of bundles (E, E') and the tachyon T , which is a section of $E^* \otimes E'$. Complete annihilation should be possible iff $E \sim E'$. This requires $N=N'$ and is described by $T = T_0 \mathcal{U}$ and

$$V(T_0 \mathcal{U}) + 2N T_{D_p} = 0$$

Equivalence classes of pairs (E, E') that can be related by brane annihilation and creation correspond to K-theory classes. So they are the mathematical objects that correspond to conserved D-brane charges.

For example, D-brane charges of the type IIB theory on \mathbb{R}^{10} are given by

$$\tilde{K}(S^{9-p}) = \begin{cases} \mathbb{Z} & p = \text{odd} \\ 0 & p = \text{even} \end{cases}$$

This accounts for the RR charges of all stable type IIB D-branes. Note that the unstable D-branes (for $p = \text{even}$) carry no conserved charges and do not show up in this classification.

In the case of type I, E is an $O(N+32)$ bundle and E' is an $O(N)$ bundle, so that the total RR 9-brane charge is 32. The relevant K-theory groups for \mathbb{R}^{10} in this case are denoted $\tilde{K}O(S^{9-p})$.

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The results are as follows:

- $\tilde{K}O(S^{9-p}) = \mathbb{Z}$ for $p = 1, 5, 9$

These are the three kinds of BPS Dp-branes of type I.

- $\tilde{K}O(S^{9-p}) = \mathbb{Z}_2$ for $p = -1, 0, 7, 8$

$p = -1$ is the type I D instanton.

$p = 0$ is the non-BPS D0-brane, which we have discussed.

$p = 7, 8$ are additional non-BPS D-branes proposed by Witten.

- $\tilde{K}O(S^{9-p}) = 0$ for $p = 2, 3, 4, 6$

There are no conserved D-brane charges in these cases.

The K theory classification suggests two new type I D-branes not discussed previously:

$$D7 + D8$$

each of which is supposed to carry a conserved Z_2 charge.

As noted by Frau et al there is a tachyon in the spectrum of $D7-D9 + D8-D9$ open strings.

Therefore neither of these D-branes is stable.

The important question, which I will return to later, is whether the conserved Z_2 charge survives when they dissolve in the background D9-branes.

The following is based on discussions with Bergman + Sen.

Witten has argued in support of the D8-brane as follows:

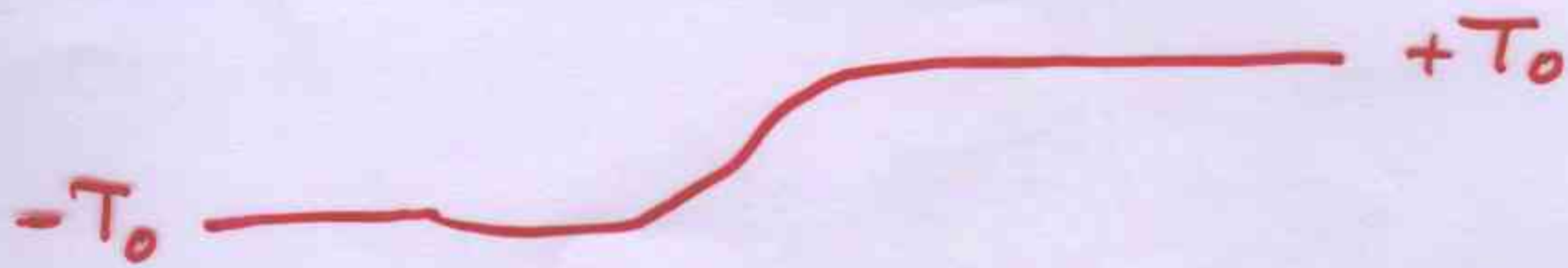
The (-1)-brane instanton can ~~make~~ ^{connect} two different "vacua", distinguished by the sign of the (-1)-brane amplitudes. This is a Z_2 analog of the θ -angle in QCD. One should expect that there is a domain wall connecting the two vacua — the D8-brane.

The sign charge would mean that the D-instanton is the EM dual of the D8-brane. Note that $p+p'=7$, whereas EM dual BPS D-branes in 10d have $p+p'=6$.

This was investigated by Gubov.

One could argue that a localized D8-brane should not exist on a circle, since this requires identifying the two distinct vacua. So let's consider the case in R^{10} first.

Sen constructed the DO-brane as a kink of a D1-D1 system. The system has one real tachyon field T and the potential $V(T)$ is even because of the Z_2 gauge symmetries. This led to a topologically stable kink



describing the DO-brane. The vacuum manifold is S^0 and

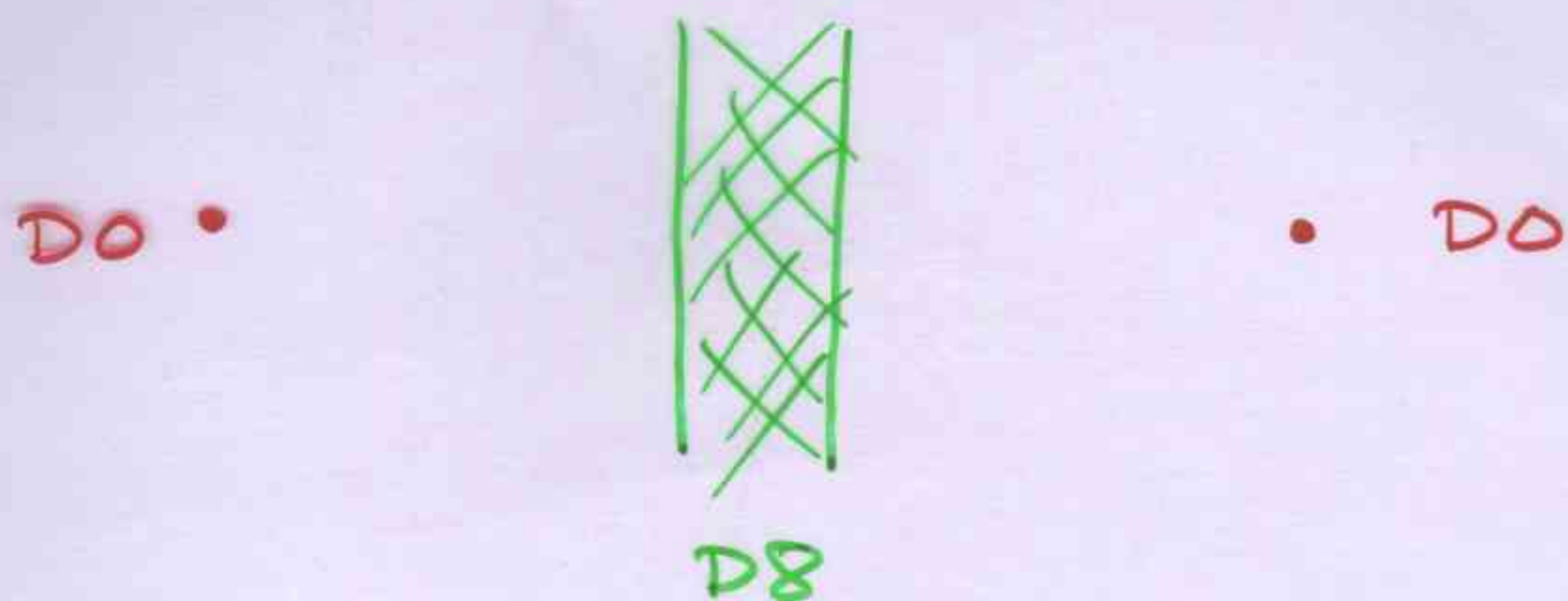
$$\pi_0(S^0) = Z_2$$

By analogy, consider 33 D9-branes and one $\overline{D9}$ -brane filling the entire R^{10} spacetime. There are 33 real tachyon fields \vec{T} in the fundamental rep of $SO(33)$. The potential $V(\vec{T})$ must have $SO(33)$ symmetry, so the minimum at $|\vec{T}| = T_0$ describes an S^{32} vacuum manifold. This is connected so there is no topologically stable kink. For this argument, it doesn't matter whether one uses $O(N)$ or $SO(N)$.

I claim that the triviality of $\pi_0(S^{32})$ means that there is no conserved D8-brane charge.

One sees the 32 tachyonic modes associated to D8-D9 strings.

We can now consider once more
an unstable D8-brane domain wall



The two vacua are distinguished
by their spin(32) chirality!

What happens to the two pictured
D0-branes when the D8-brane decays?
Clearly, they must end up with the
same chirality. This is possible
because the group is broken inside the D8-brane.

CONCLUSION

The spectrum of BPS D-branes is always easy to identify. They couple electrically or magnetically to the RR gauge fields in the supergravity multiplet.

In addition there can be non-BPS D-branes that do not couple to RR gauge fields. They can carry conserved charges (Z_2 in our examples) and be stable or carry no conserved charges and be unstable.

High dimension non-BPS D-branes tend to be unstable due to tachyonic modes of open strings connecting them to background space-time filling D-branes.