NON-BPS D-BRANES IN STRING THEORY

- Construction of unstable non-BPS D-branes in type II string theories.

- Stable type I D-particle

- Type II D-branes wrapped on non-supersymmetric cycles of K3

  - Construction

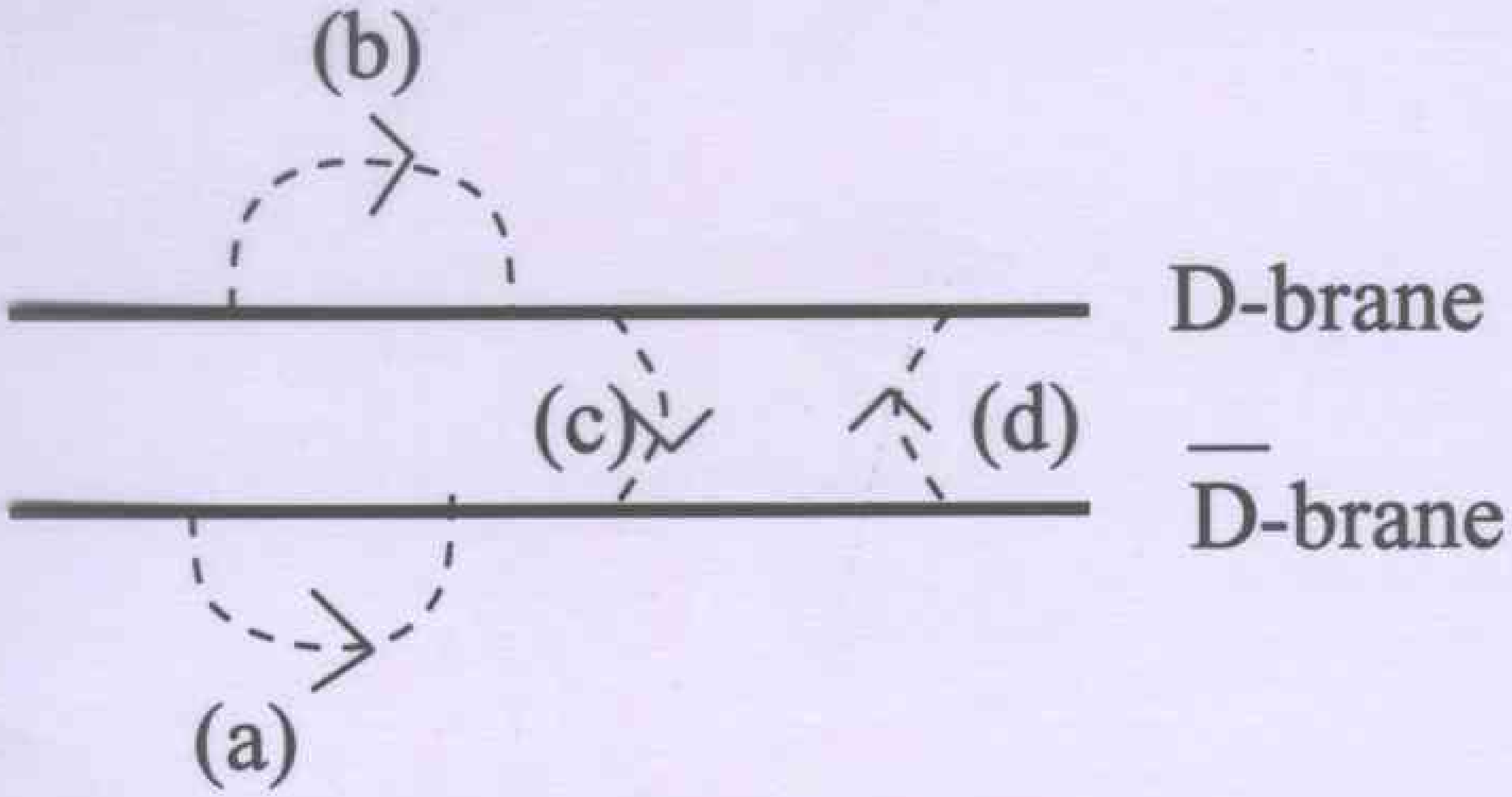
  - Phase diagram and behaviour near the critical point

Convention: $\alpha' = 1$ (string tension$=(2\pi)^{-1}$).

Motivation

- testing duality conjectures beyond the BPS spectrum
- constructing non-supersymmetric string theories using non-supersymmetric branes
- providing theoretical challenges for non-perturbative formulations of string theory
- STUDY OF NON-SUPERSYMMETRIC GAUGE THEORIES VIA BRANES

We start with a system of coincident D-brane – $\bar{D}$-brane pair in type IIA/IIB string theory:



- Spectrum of open strings living on the world-volume contains four different sectors.

  → denoted by 2×2 Chan Paton (CP) factor:

  (a): $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$, (b): $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

  (c): $\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$, (d): $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

- GSO projection: Physical states in sectors (a) and (b) have $(-1)^F = 1$ whereas those in sectors (c) and (d) have $(-1)^F = -1$.

- The Neveu-Schwarz (NS) sector ground state has $(-1)^F = -1$

  Hence they survive in sectors (c) and (d) and give tachyonic excitations with

$$\boxed{m^2 = -(1/2)}$$

  Since the tachyon comes from two different sectors it is a complex field.

- Although individually the D-brane as well as the $\bar{\text{D}}$-brane is invariant under half of the space-time supersymmetry transformations, the combined system breaks all supersymmetries.

We shall now study the action of $(-1)^{F_L}$ on the coincident D-brane – $\bar{\text{D}}$-brane system.

$(-1)^{F_L}$: Acting on the closed string Hilbert space, it changes the sign of **all the states** on the left-moving Ramond sector, but does not change anything else.

$\rightarrow$ trivial action on the world-sheet fields.

Space-time fields originating in the RR-sector change sign under $(-1)^{F_L}$.

Since D-branes are charged under RR field, $(-1)^{F_L}$ must take a D-brane to a $\bar{\text{D}}$-brane.

$\rightarrow$ A D-brane – $\bar{\text{D}}$-brane system is invariant under $(-1)^{F_L}$ and it makes sense to study the action of $(-1)^{F_L}$ on the open strings living on this system.

Action of $(-1)^{F_L}$ on the open string living on the brane-antibrane system

We focus on the NS sector.

$(-1)^{F_L}$ has no action on the world-sheet fields.

Thus we need only to study the action on the CP factors.

Since $(-1)^{F_L}$ exchanges D-brane with D̄-brane, it acts on the CP matrix Λ as

$$\boxed{\Lambda \to S \Lambda S^{-1}}$$

where

$$\boxed{S = \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}$$

Thus states with CP factors I and σ_1 are even under $(-1)^{F_L}$, whereas σ_3 and $i\sigma_2$ are odd.

We are now ready to define a non-BPS D-$2p$-brane of type IIB string theory.

- We start with a D-$2p$ – $\bar{\text{D}}$-$2p$ pair in type IIA string theory and take the orbifold of this configuration by $(-1)^{F_L}$.

- In the bulk, modding out IIA by $(-1)^{F_L}$ gives IIB.

- Acting on the open strings living on the D-$\bar{\text{D}}$-brane world-volume, $(-1)^{F_L}$ projection keeps states with CP factors I and σ_1 and throws out states with CP factors σ_3 and $i\sigma_2$.

This defines a **non-BPS D-$2p$-brane** of type IIB string theory.

Similarly, starting from a D-$(2p+1)$-brane $\bar{\text{D}}$-$(2p+1)$ pair of IIB, we can define a non-BPS $(2p+1)$-brane of IIA.

A. S.
Bergman & Gaberdiel
Witten

Properties of the non-BPS D-$2p$-brane of IIB:

- Excitations on its world-volume are open strings with Dirichlet boundary condition on the $(9-2p)$ transverse directions, and Neumann boundary condition on $2p+1$ tangential directions (including time).

- These open strings carry Chan Paton factors I or σ_1.

- Physical states with CP factor I has $(-1)^F = 1$ and physical states with CP factor σ_1 has $(-1)^F = -1$.

  (Note: $(-1)^F$ denotes **world-sheet** fermion number)

  These rules are induced by the corresponding rules on D-$\bar{\text{D}}$ pair.

- The NS sector ground state from CP factor σ_1 has $(-1)^F = -1$ and hence is physical.

  $\Rightarrow$ a tachyonic mode with

  $$\boxed{m^2 = -\frac{1}{2}}$$

- Since tachyon comes from only one sector, it is a real field.

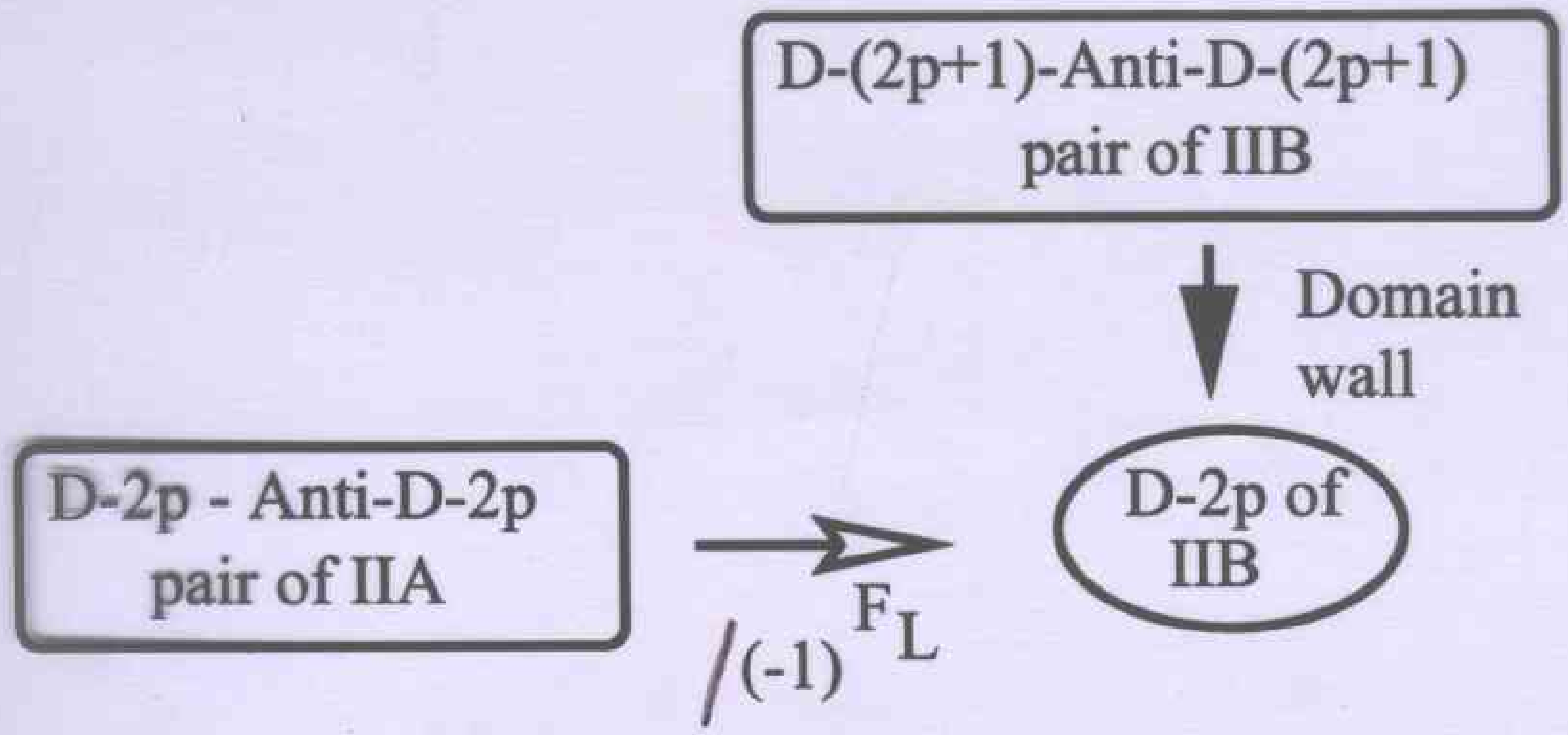
- The tension of the non-BPS D-$2p$-brane is given by:

  $$\boxed{(2\pi)^{-2p}(\sqrt{2}/g)}$$

  g: string coupling

One can also derive the spectrum of open strings with one end on the non-BPS brane and other end on a BPS brane, but we shall not discuss it here.

A non-BPS D-$2p$ brane of type IIB can also be viewed as a tachyonic domain wall on a D-$(2p+1)$ − $\bar{\text{D}}$-$(2p+1)$ brane system of IIB



Horizontal arrow: Effect of modding out by $(-1)^{F_L}$.

Vertical arrow: Effect of considering tachyonic domain wall solution.

The equivalence of these two constructions can be given using CFT techniques

Similar dual construction can be given for non-BPS D-$(2p+1)$ brane of IIA.

Stable non-BPS D-branes

Although we have constructed non-BPS D-branes in type IIA/IIB string theory, they are all unstable due to the presence of the tachyonic mode.

What is the use of such a D-brane?

Answer: Although they are unstable in type IIA/IIB, we may get **stable** non-BPS D-branes in certain orbifolds/orientifolds of IIA/IIB **if the tachyonic mode is projected out under this operation.**

Example I: Type I D-particle

- Start with the non-BPS D0-brane (D-particle) of type IIB.

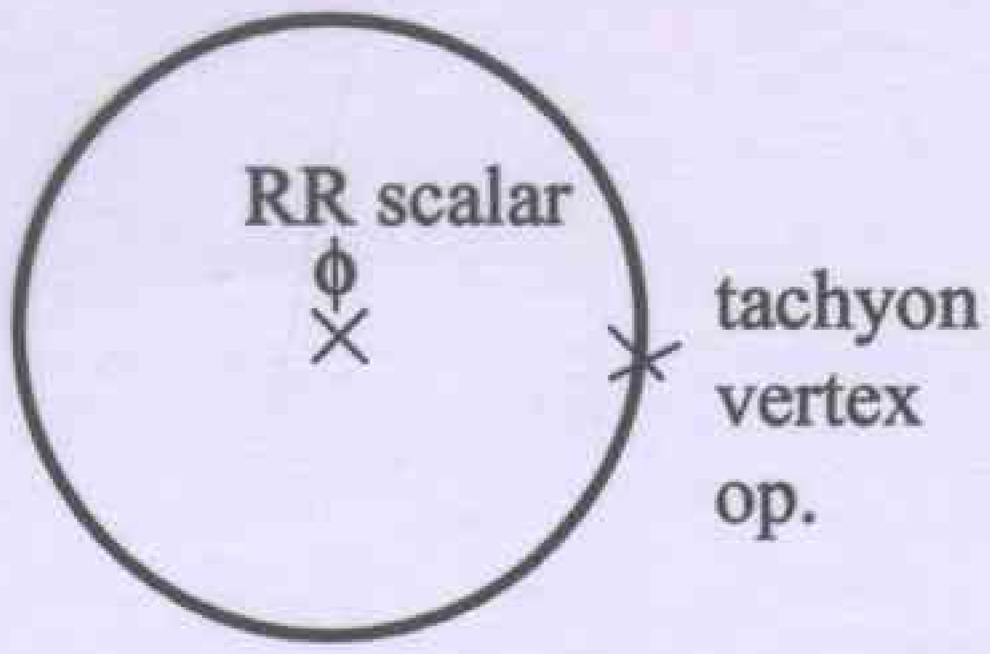
- Mod out the configuration by the world-sheet parity transformation Ω.

Result: A non-BPS D-particle of type I

Is this D-particle stable?

(Is the tachyonic mode on the type IIB D-particle odd under Ω?)

Observation: The two point function of the tachyonic mode on the D-particle world-volume and the RR sector scalar field ϕ of type IIB is non-vanishing.



ϕ is known to be odd under Ω

$\Rightarrow$ the tachyonic mode of the D-particle is also odd under Ω.

$\Rightarrow$ it is absent in type I D-particle

$\Rightarrow$ type I D-particle is stable.

It can be shown that this state transforms in the spinor representation of the type I gauge group SO(32).

Example II: D-branes wrapped on non-supersymmetric cycles

- Start with non-BPS D-string of type IIA on a circle along x^9 of radius R_9.
- Compactify three other directions x^6, x^7, x^8.
- Mod out the theory by a Z_2 transformation $\mathcal{I}_4$ which changes the sign of $x^6, \ldots x^9$:

$$\boxed{\mathcal{I}_4 : (x^6, x^7, x^8, x^9) \to (-x^6, -x^7, -x^8, -x^9)}$$

In the bulk this gives

IIA on an orbifold K3

Since the D-string lies along x^9, the tachyon field on its world-steet is a function of x^9 and time

Question: How does $T(x^9, t)$ transform under $\mathcal{I}_4$?

Answer:

$$\boxed{T(x^9, t) \rightarrow -T(-x^9, t)}$$

Expand $T(x^9, t)$ as

$$\boxed{T(x^9, t) = \sum_n T_n(t) e^{inx^9/R_9}}$$

Then under $\mathcal{I}_4$:

$$\boxed{T_n \rightarrow -T_{-n}}$$

Thus

- T_0 is projected out.
- For $n \neq 0$ the combination $T_n - T_{-n}$ survives the projection under $\mathcal{I}_4$.

Effective mass^2 of $T_n - T_{-n}$

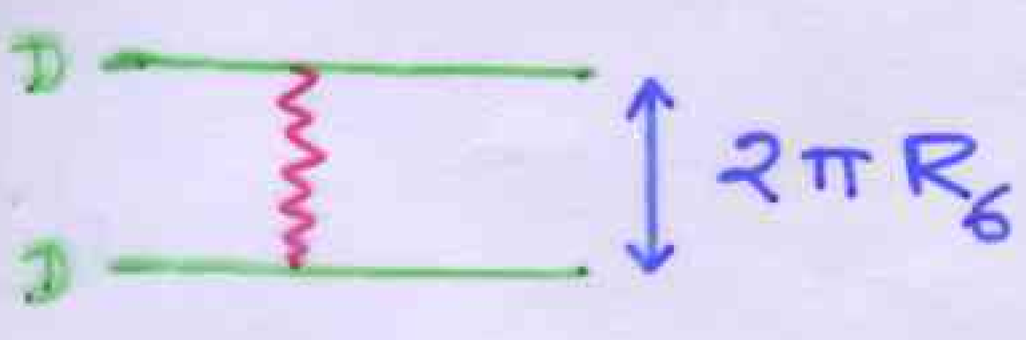
$$\boxed{m_n^2 = (n^2/R_9^2) - (1/2)}$$

Thus there is no tachyon in the spectrum for

$$\boxed{R_9 \leq \sqrt{2}}$$

Demanding that there are no tachyonic modes from open strings wound along x^6, x^7 or x^8, we also get

$$\boxed{R_8 \geq \frac{1}{\sqrt{2}}, \quad R_7 \geq \frac{1}{\sqrt{2}}, \quad R_6 \geq \frac{1}{\sqrt{2}}}$$



Result: A stable non-BPS state in type IIA on $T^4/\mathcal{I}_4$.

What is the interpretation of this state?

For this we need to study the physics at the critical radius $R_9 = \sqrt{2}$.

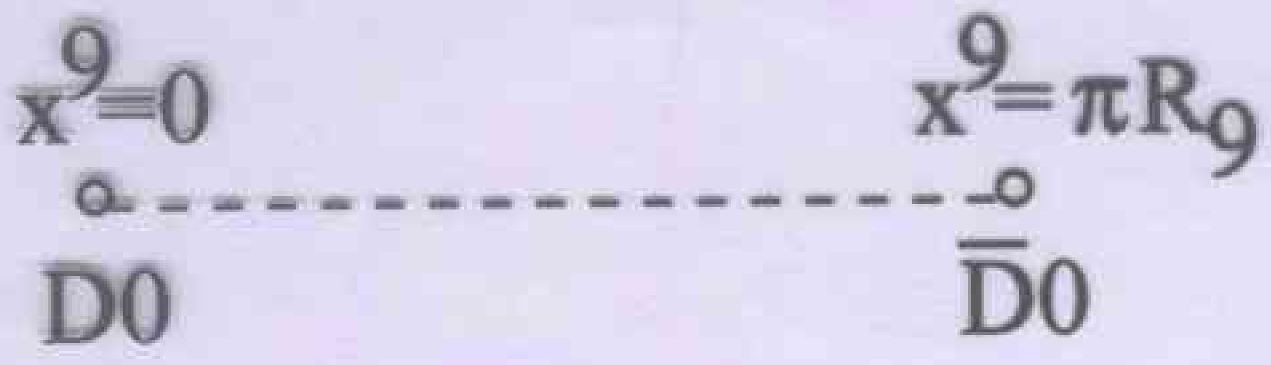
At this radius $(T_1 - T_{-1})$ is massless.

In fact one can show that the potential for $(T_1 - T_{-1})$ vanishes identically.

$\rightarrow$ $(T_1 - T_{-1})$ denotes an exactly marginal deformation of the boundary CFT describing the D-brane.

We can study this deformation using CFT techniques.

Result: This marginal deformation takes the non-BPS D-string of IIA to a D0-$\bar{\text{D}}$0-brane pair of IIA situated at the two fixed points $x^9 = 0$ and $x^9 = \pi R_9$ respectively.



What is the result of modding out this configuration by $\mathcal{I}_4$?

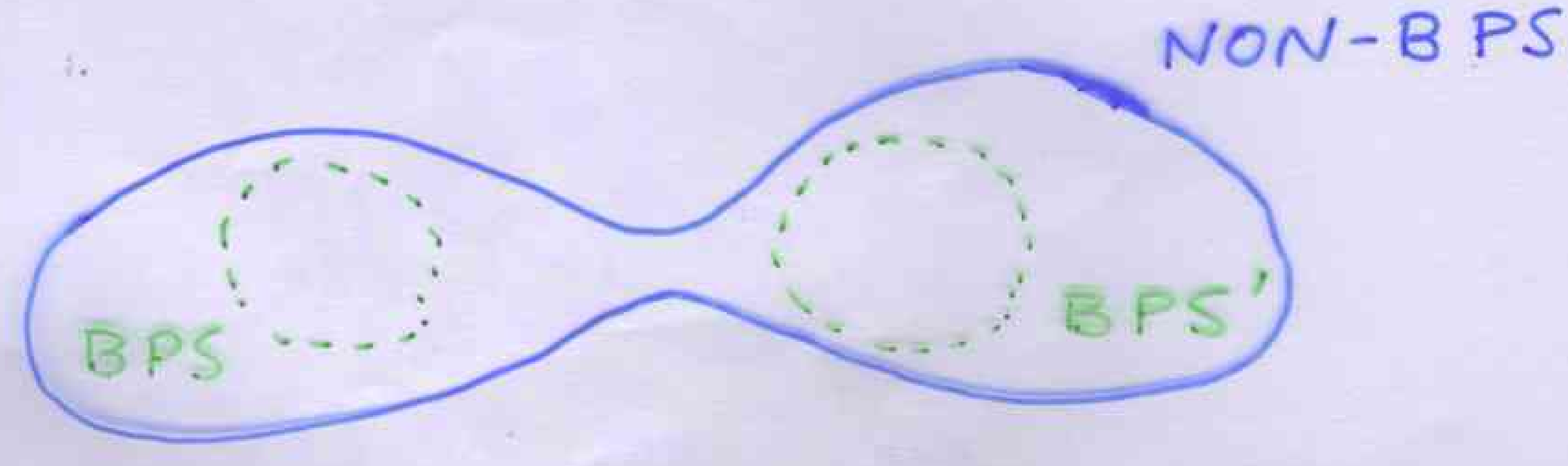
A pair of fractional D-particles at the orbifold singularities.
Douglas & Moore

Interpretation: A pair of D-2-branes of type IIA, wrapped on the 2-cycles associated with the two fixed points of $\mathcal{I}_4$ at $x^9 = 0$ and $x^9 = \pi R_9$.

This suggests that the original configuration is a D-2-brane of IIA wrapped simultaneously on both these 2-cycles.

A.S.
Bergman & Gaberdiel

$\rightarrow$ a non-supersymmetric 2-cycle.



- At the critical radius the D-2-brane wrapped on the non-supersymmetric cycle is degenerate with the pair of D-2-branes wrapped on the supersymmetric cycles.

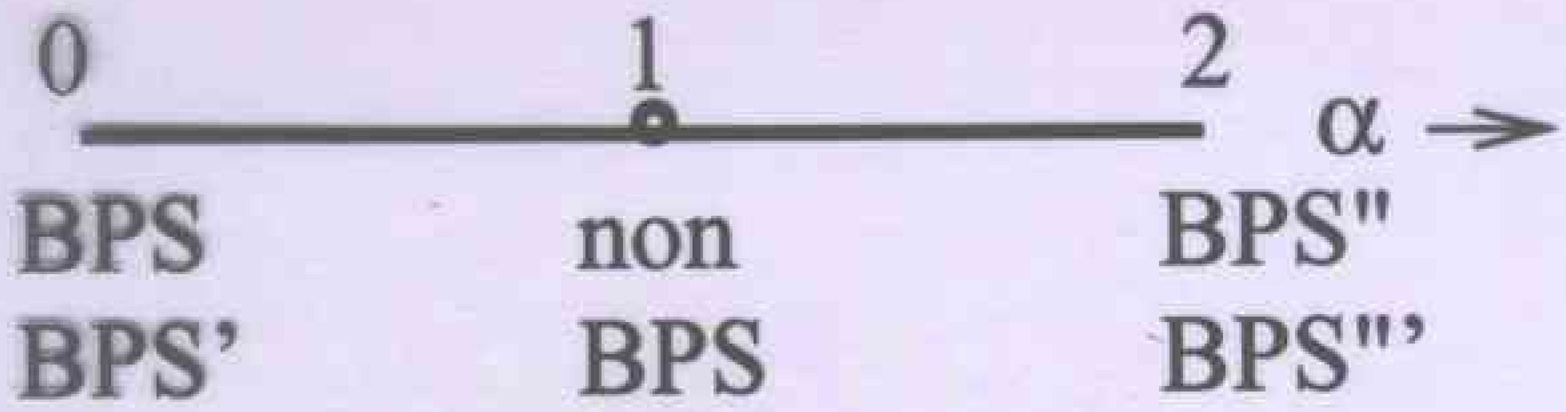
- Below the critical radius the D-2-brane wrapped on the non-SUSY cycle is lighter than the pair of D-2-branes wrapped on the two SUSY cycles.

  → this wrapped brane is stable.

- Above the critical radius the D-2-brane wrapped on the non-SUSY cycle is heavier than the pair of D-2-branes wrapped on the two SUSY cycles.

  → this wrapped brane is unstable against decay into a pair of supersymmetric brane configurations.

Denote the $(T_1 - T_{-1})$ vev by $(\alpha - 1)$.



At $R_9 = \sqrt{2}$, $V(\alpha) = 0$.

$\alpha \equiv 1$: D2-Brane wrapped on non-SUSY cycle

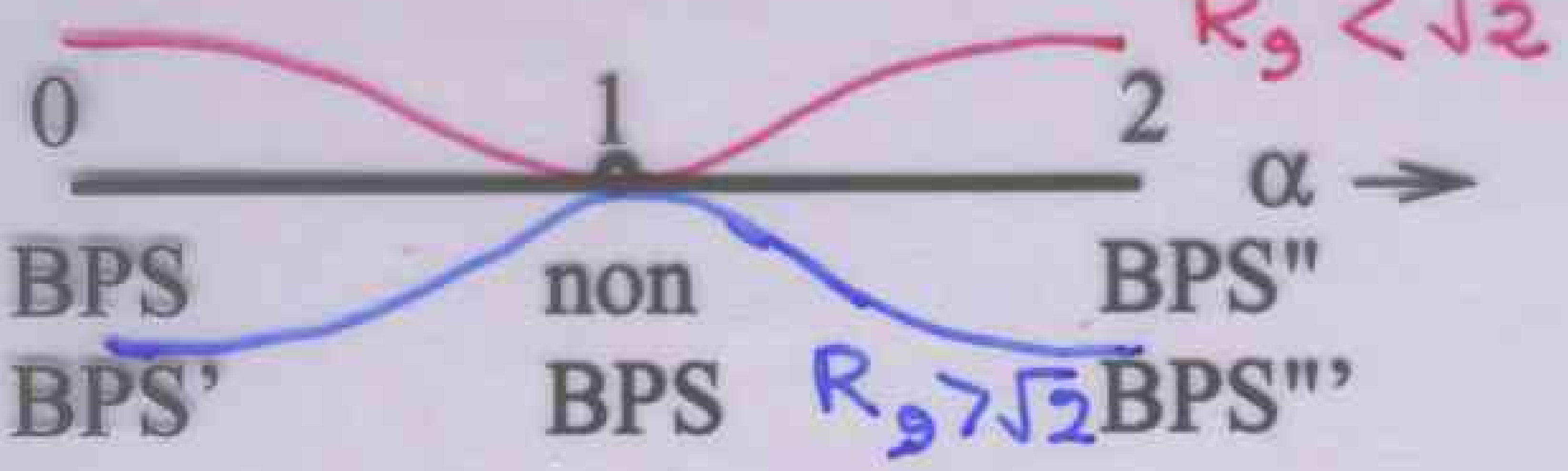
$\Rightarrow$ Minimum of $V(\alpha)$ for $R_9 (\equiv R) < \sqrt{2}$.

$\alpha \equiv 0$: Pair of D2-branes wrapped on SUSY cycles

$\alpha \equiv 2$: Pair of D2-branes wrapped on SUSY cycles, with their D0-brane charges reversed.

$\Rightarrow$ represent degenerate minima of $V(\alpha)$ for $R_9 > \sqrt{2}$.

Denote the $(T_1 - T_{-1})$ vev by $(\alpha - 1)$.



$\boxed{\text{At } R_9 \equiv \sqrt{2},\ V(\alpha) = 0.}$

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$\rightarrow$ represent degenerate minima of $V(\alpha)$ for $R_9 > \sqrt{2}$.

We want to see how this picture changes when we switch on twisted sector modes.

J. Majumder & A.S.

(Flux of anti-symmetric tensor field through the supersymmetric cycles).

ζ: Difference in flux through the two SUSY 2-cycles.

$\boxed{\alpha - 1 \equiv T_1 - T_{-1}}$

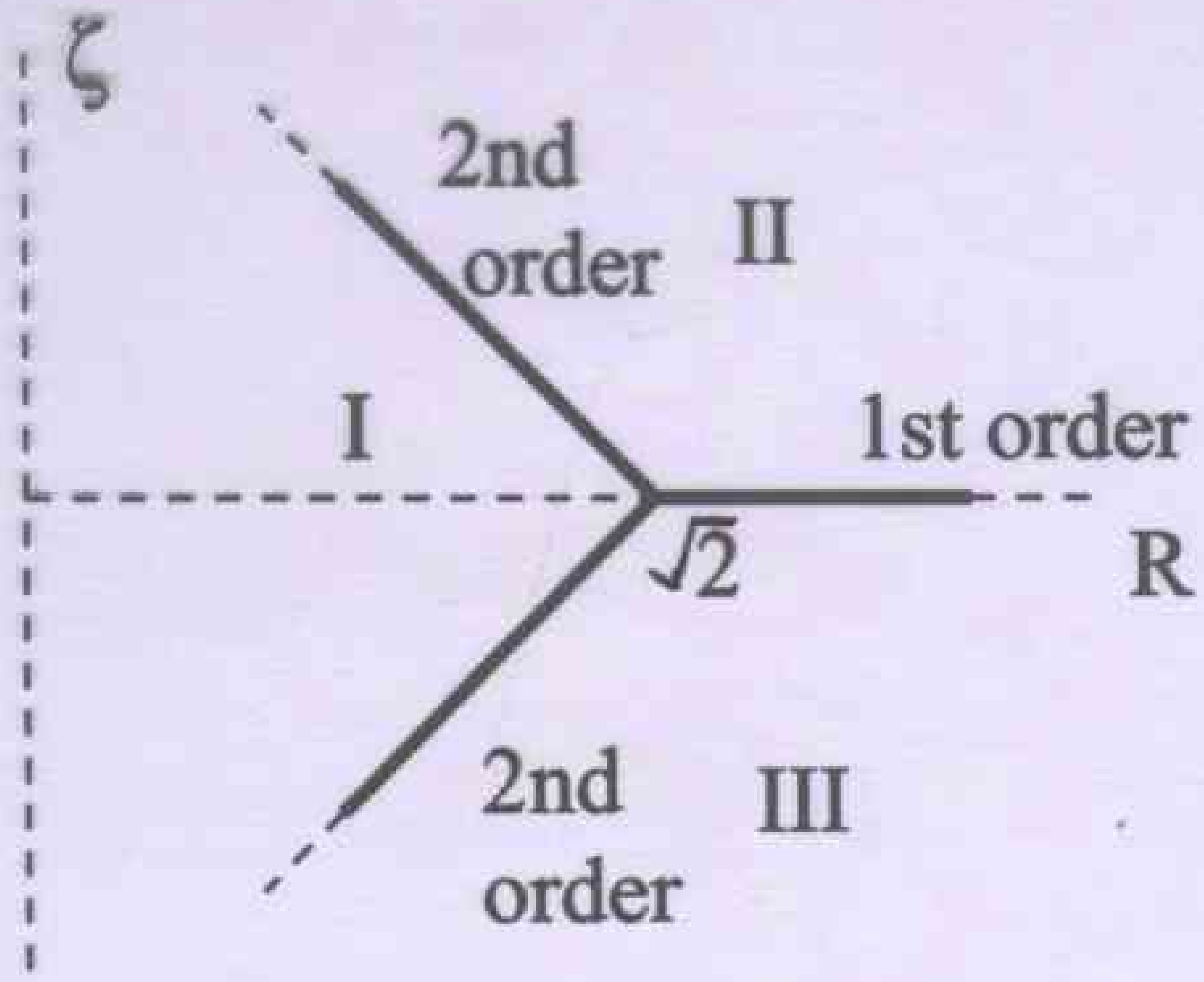
Tachyon potential to first order in $(R - \sqrt{2})$ and ζ:

$$\boxed{V(\alpha) = \frac{1}{4}(\sqrt{2} - R)\cos(\pi\alpha) + \zeta\cos(\frac{1}{2}\pi\alpha)\,.}$$

We can now study its minimum for different values of R and ζ.

Note: $\boxed{V(\alpha) = 0 \text{ at } (R = \sqrt{2},\ \zeta = 0)}$ as expected.

The phase diagram in the $R - \zeta$ plane:



Phase I: D2-brane on non-supersymmetric cycle.

$$\boxed{\alpha = \frac{2}{\pi} \cos^{-1} \frac{\zeta}{R - \sqrt{2}}}$$

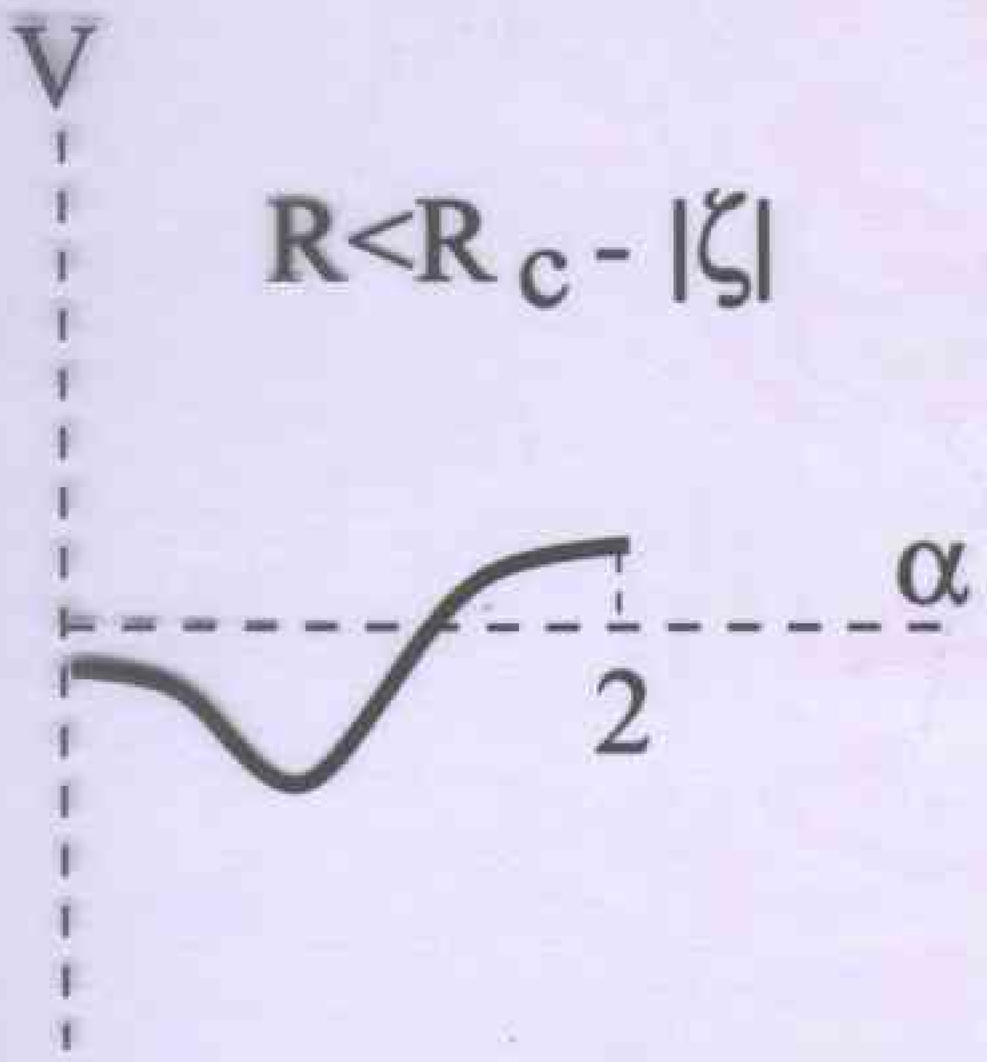
Phase II: A pair of D2-branes on SUSY cycles

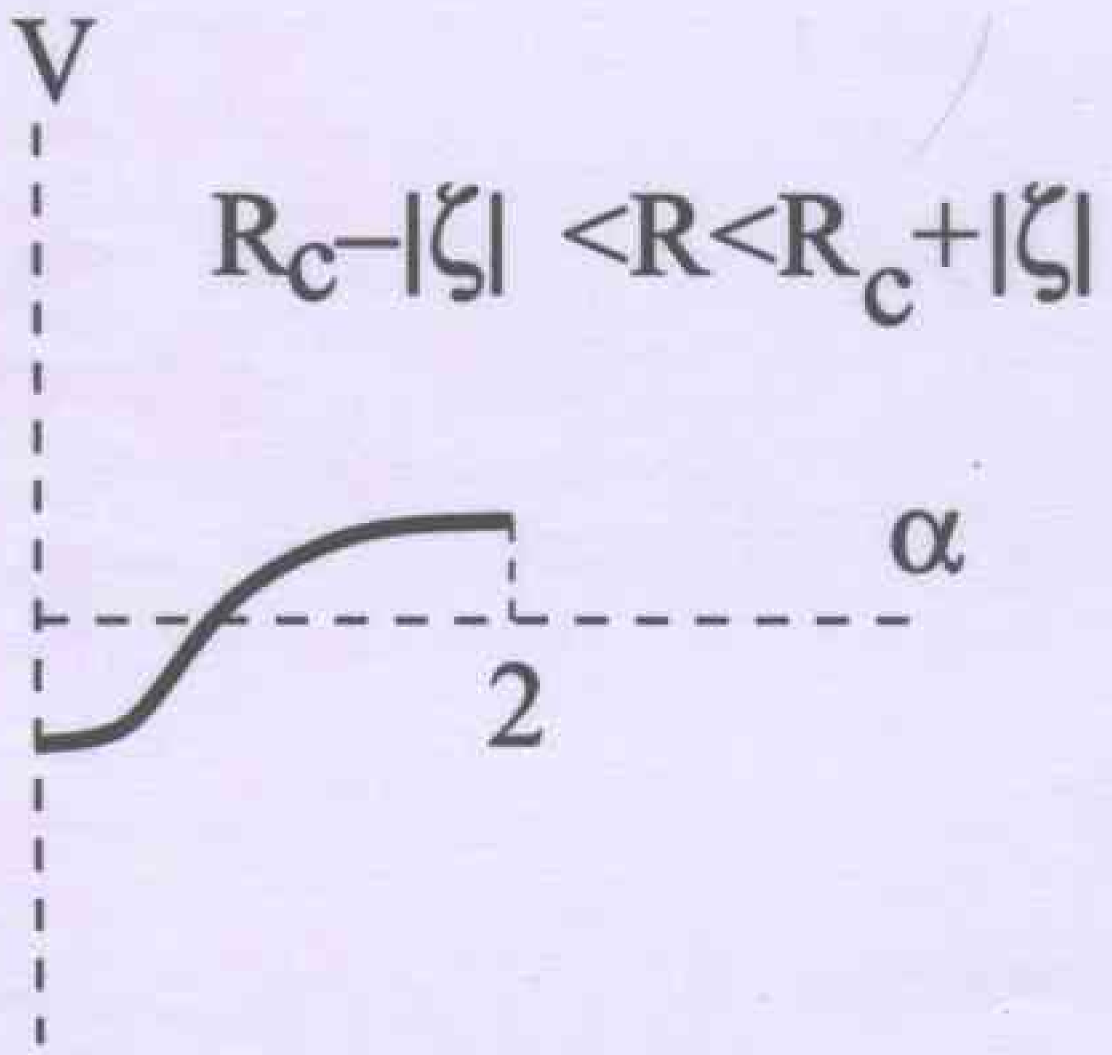
$$\boxed{\alpha = 2}$$

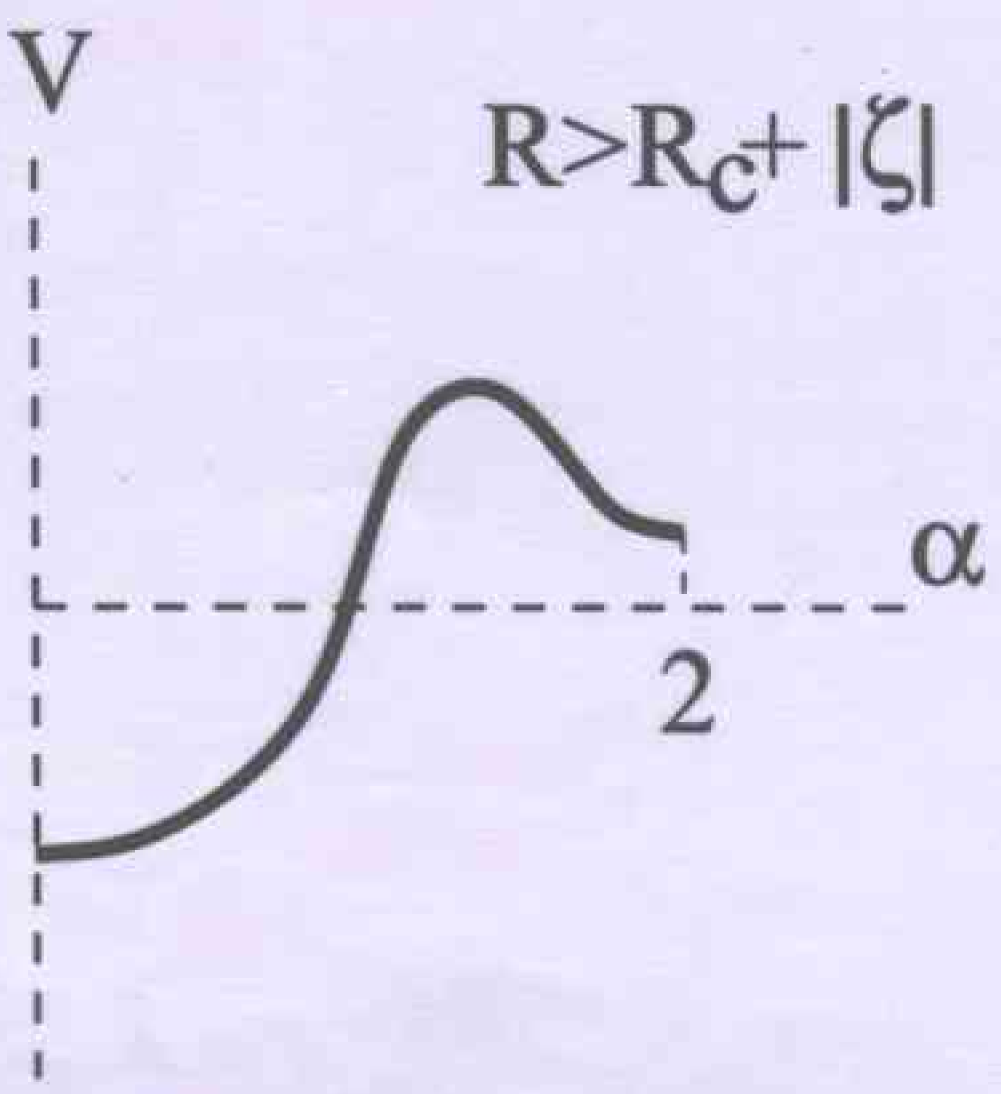
Phase III: A pair of D2-branes on SUSY cycles carrying opposite D0-brane charge

$$\boxed{\alpha = 0}$$

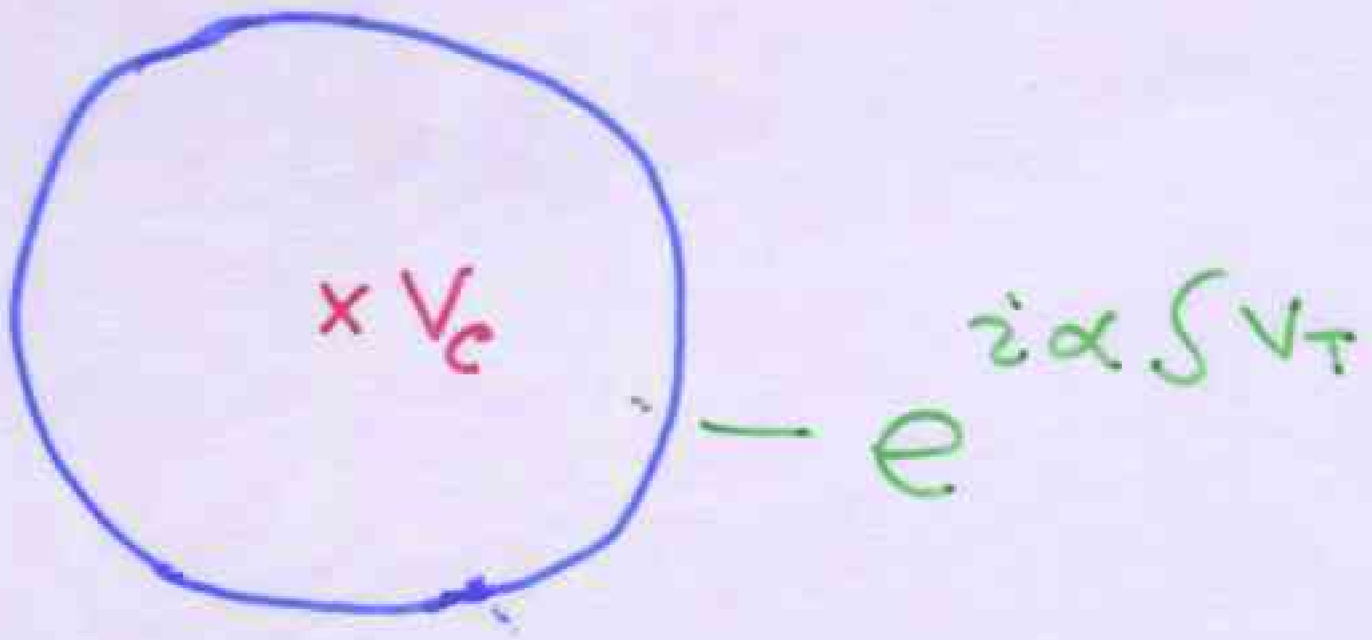
The potential $V(\alpha)$ for different values of R for fixed $\zeta < 0$:







COMPUTATION OF $V(\alpha)$:



V_T: TACHYON VERTEX OPERATOR

V_c: CLOSED STRING VERTEX OPERATOR CORRESPONDING TO R OR S DEFORMATION

Force between a pair of D2-branes wrapped on non-supersymmetric cycles

Can be calculated by computing the open string partition function.

M. Gaberdiel & A.S. (work in progress)

Recall the region of stability:

$$\boxed{R_6 \geq \frac{1}{\sqrt{2}}, \quad R_7 \geq \frac{1}{\sqrt{2}}, \quad R_8 \geq \frac{1}{\sqrt{2}}, \quad R_9 \leq \sqrt{2}}$$

Result: The force between a pair of wrapped branes is repulsive at all distance scale when any of the above condition is a strict inequality.

At

$$\boxed{R_6 = R_7 = R_8 = \frac{1}{\sqrt{2}}, \qquad R_9 = \sqrt{2}}$$

the force between a pair of particles vanishes at all distance scale.

Result of accidental degeneracy between bosonic and fermionic open string spectrum at all mass levels.

MANY OTHER NON-SUPERSYMMETRIC BRANE CONFIGURATIONS HAVE THIS FEATURE.

This construction can be generalized to describe a $(2p+2)$-brane ($(2p+1)$-brane) of IIA (IIB) wrapped on a non-supersymmetric cycle of K3.

Using this procedure one can also construct examples of D-branes wrapped on non-BPS 2- and 3-cycles of Calabi-Yau manifolds.

Summary

Type IIA (IIB) contains non-BPS D-branes.

These branes are related to brane-antibrane pair of type II string theories via orbifolding by $(-1)^{F_L}$ as well as via tachyonic kink construction.

However, these branes are unstable against decay into vacuum as indicated by the presence of tachyonic modes on these branes.

Upon modding out the theory by an appropriate group which projects out the tachyonic mode, these non-BPS D-branes may become stable and represent physically interesting states *e.g.*

- SO(32) spinors of type I
- D-branes wrapped on non-supersymmetric cycles of K3 and Calabi-Yau orbifolds

WORLD-VOLUME THEORY ON N COINCIDENT D-BRANES GIVES NON-SUPERSYMMETRIC SU(N) GAUGE THEORY WITH ADJOINT FERMIONS & SCALARS