Wilson Loops

from

Supergravity and String theory

Cobi Sonnenschein string 99
Potsdam

based on work with Y. Kinar, E. Schreiber

and "old" work with

A. Brandhuber, N. Itzhaki, S. Yankielowicz

- The idea to describe the Wilson loop of QCD in terms of a string partition function dates back to the early eighty's. For instance Luscher, Symanzic and Weisz found that the potential of quark anti-quark separated at a distance L acquires a $\boxed{\frac{c}{L}}$ term (c is a universal constant) due to quantum fluctuations.

- Recently there has been a Renaissance to this idea in the framework of Maldacena's correspondence between large N gauge theories and string theory.

- The aim of this this talk is to address some of the questions from the old days in the context of the modern approach.

Outline

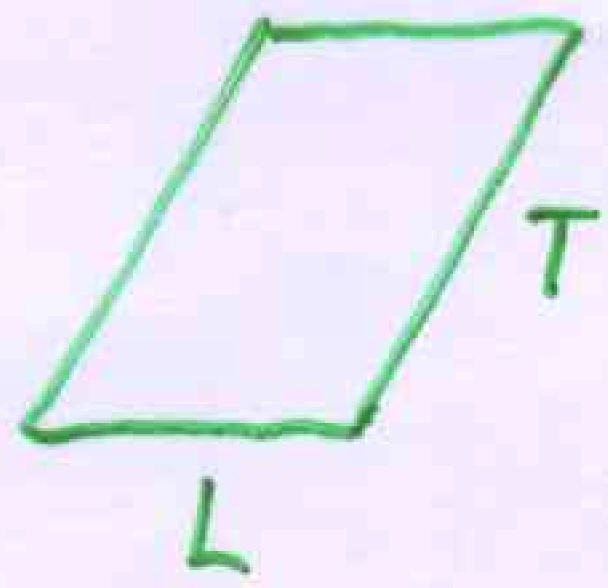
1. **Classical Wilson loops - general results**
2. **Applications to various models**
3. **Quantum Fluctuations**
4. **On the exact determination of Wilson loops**

Brief reminder — Wilson loops

- The Wilson loop operator we consider

$$W(c) = \frac{1}{N} \mathrm{Tr}\left[P e^{i\oint_c A} \right]$$

where here we take c to be an infinite strip



when $T \to \infty$

- This is a special case of the operator discussed by Ooguri

$$W(c) = \frac{1}{N} \mathrm{Tr}\left[P e^{\oint_c (i A_\mu \dot{x}^\mu + \theta^I \Phi^I \sqrt{\dot{x}^2})\, d\tau} \right]$$

with constant θ^I

- The quark anti-quark energy is extracted from $\langle W(c) \rangle$

$$\langle W(c) \rangle = A(L)\, e^{-T E(L)}$$

- The natural candidate for Wilson loop in string models

$$\langle W(c) \rangle \sim e^{-S_{NG}^{(ren)}}$$

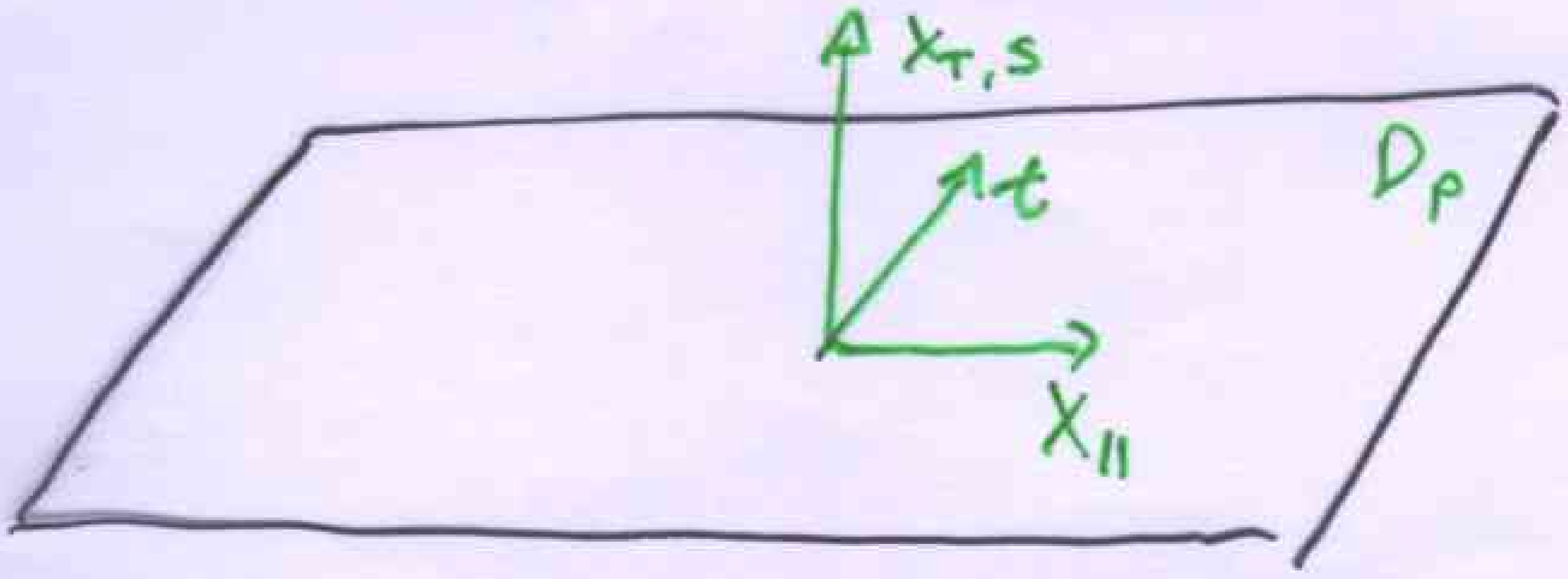
where $S_{NG}^{(ren)}$ is the renormalized Nambu-Goto action

Consider a 10d space-time metric

$$\begin{aligned} ds^2 &\equiv -G_{00}(s)dt^2 + G_{x_{||}x_{||}}(s)dx_{||}^2 \\ &+ G_{ss}(s)ds^2 + G_{x_T x_T}(s)dx_T^2 \end{aligned} \tag{1}$$

where $x_{||}$- p space coordinates on a D_p brane

s and x_T are the transverse coordinates



The corresponding Nambu-Goto action is

$$S_{NG} = \int d\sigma d\tau \sqrt{det[\partial_\alpha x^\mu \partial_\beta x^\nu G_{\mu\nu}]}$$

Upon using $\tau = t$ and $\sigma = x$, where x is one of the $x_{||}$ coordinates, the action for a static configuration reduces to

$$\boxed{S_{NG} = T \cdot \int dx \sqrt{f^2(s(x)) + g^2(s(x))(\partial_x s)^2}}$$

where

$$
\begin{aligned}
f^2(s(x)) &\equiv G_{00}(s(x))G_{x_{||}x_{||}}(s(x)) \\
g^2(s(x)) &\equiv G_{00}(s(x))G_{ss}(s(x))
\end{aligned}
$$

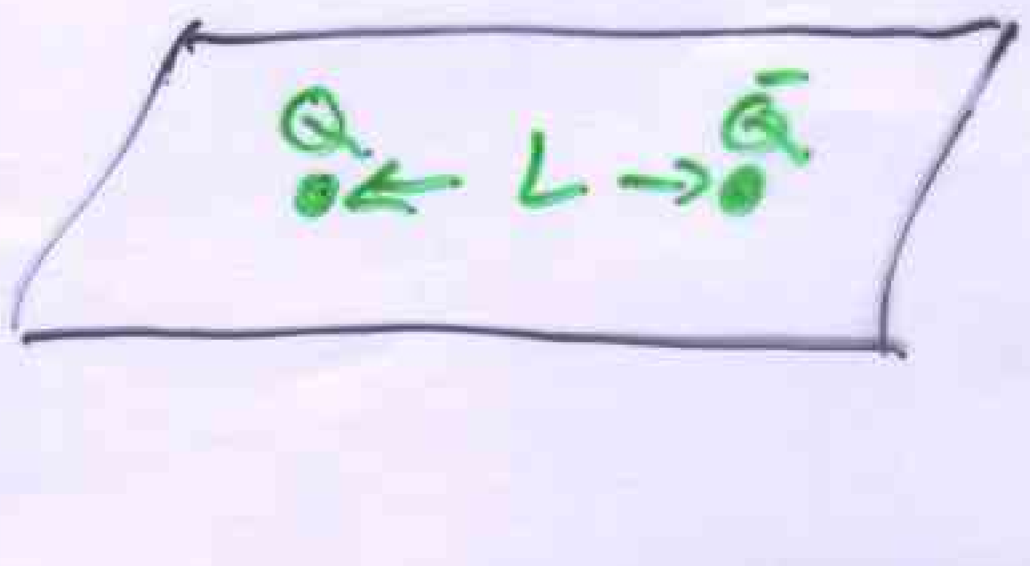
and T is the time interval.

The equation of motion (geodesic line)

$$
\frac{ds}{dx} = \pm\frac{f(s)}{g(s)} \cdot \frac{\sqrt{f^2(s) - f^2(s_0)}}{f(s_0)}
$$

For a static string configuration connecting "Quarks" separated by a distance

$$
L = \int dx = 2\int_{s_0}^{s_1} \frac{g(s)}{f(s)} \frac{f(s_0)}{\sqrt{f^2(s) - f^2(s_0)}} ds
$$



the NG action and corresponding energy $E = \frac{S_{NG}}{T}$ are divergent.

Maldacena

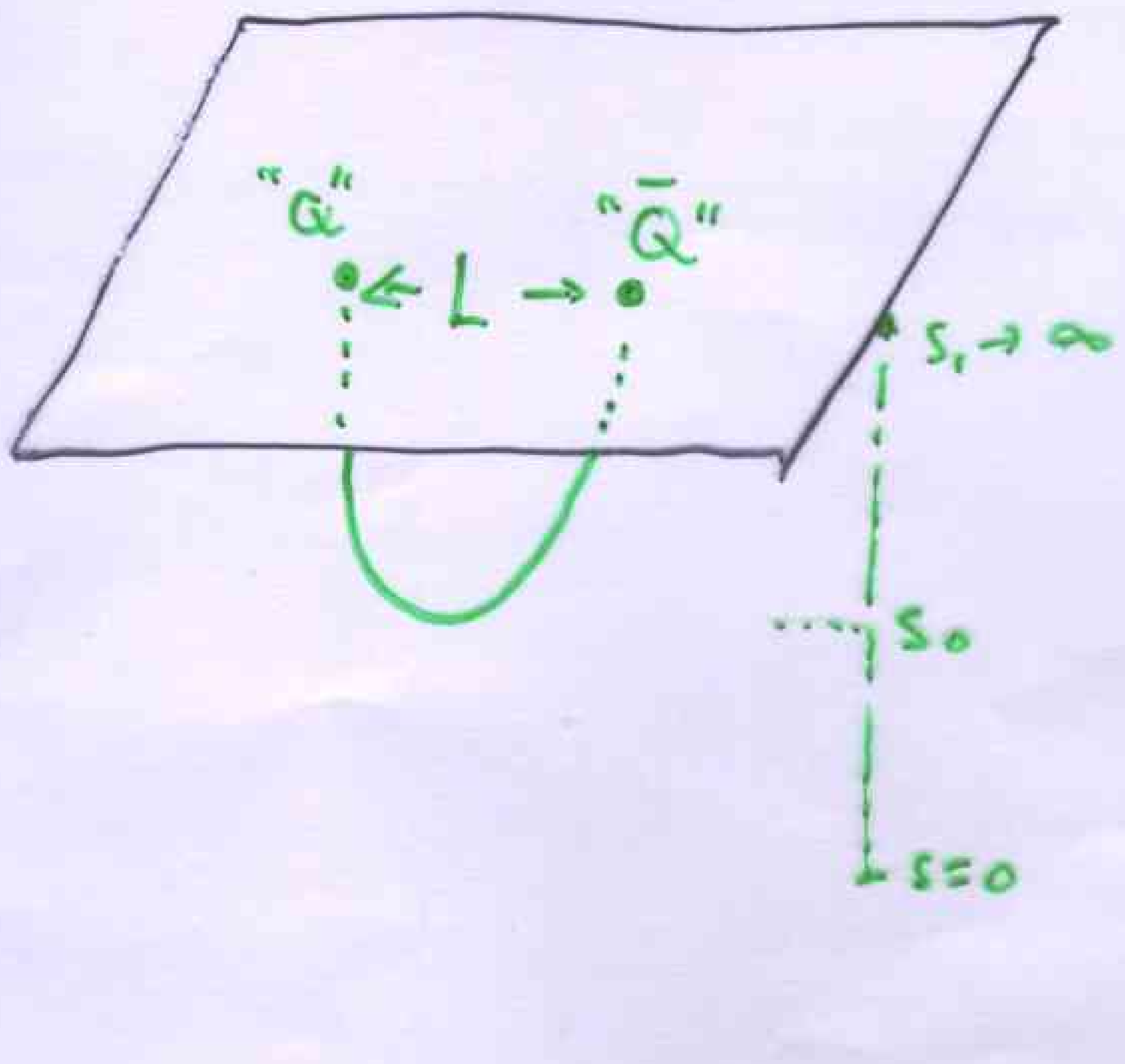
The action is renormalized by subtracting the quark masses. For $AdS_5 \times S_5$ it is equivalent to the Legendre transform of Drukker, Gross, Ooguri

$$\boxed{m_q \equiv \int_0^{s_1} g(s)ds} \qquad (2)$$

So that the renormalized quark anti-quark potential is

$$\boxed{\begin{aligned} E &= f(s_0) \cdot L \\ &+ 2 \int_{s_0}^{s_1} \frac{g(s)}{f(s)} \left(\sqrt{f^2(s) - f^2(s_0)} - f(s)\right) ds - 2 \int_0^{s_0} g(s)ds \ . \end{aligned}}$$

The behavior of the potential is determined by the following theorem



Theorem 1. *Let S_{NG} be the NG action defined above, with functions $f(s), g(s)$ such that:*

1. *$f(s)$ is analytic for $0 < s < \infty$. At $s = 0$, (we take here that the minimum of f is at $s = 0$) its expansion is:*

$$f(s) = f(0) + a_k s^k + O(s^{k+1})$$

   with $k > 0$, $a_k > 0$.

2. *$g(s)$ is smooth for $0 < s < \infty$. At $s = 0$, its expansion is:*

$$g(s) = b_j s^j + O(s^{j+1})$$

   with $j > -1$, $b_j > 0$.

3. *$f(s), g(s) \geq 0$ for $0 \leq s < \infty$.*

4. *$f'(s) > 0$ for $0 < s < \infty$.*

5. *$\int^{\infty} g(s)/f^2(s)ds < \infty$.*

Then for (large enough) L there will be an even geodesic line asymptoting from both sides to $s = \infty$, and $x = \pm L/2$. The associated potential is

1. if $f(0) > 0$, then

   (a) if $k = 2(j+1)$,

   $$\boxed{E = f(0) \cdot L - 2\kappa + O((\log L)^{\beta} e^{-\alpha L})}$$

   (b) if $k > 2(j+1)$,

   $$\boxed{E = f(0) \cdot L - 2\kappa - d \cdot L^{-\frac{k+2(j+1)}{k-2(j+1)}} + O(L^{\gamma}).}$$

   where $\gamma = -\frac{k+2(j+1)}{k-2(j+1)} - \frac{1}{k/2-j}$ and β and κ, α d and $C_{n,m}$ are positive constants determined by the string configuration.

   In particular, there is

   $\boxed{\text{linear confinement}}$

2. *if* $f(0) = 0$, *then if* $k > j+1$,

$$\boxed{E = -d' \cdot L^{-\frac{j+1}{k-j-1}}} + O(L^{\gamma'})$$

where $\gamma' = -\frac{j+1}{k-j-1} - \frac{2k-j-1}{(2k-j)(k-j-1)}$ *and* d' *is a coefficient determined by the classical configuration.*

In particular,

$$\boxed{\textit{there is no confinement}}$$

A general statement about confining scenarios

As a consequence of this theorem

A sufficient condition for confinement is

f *has a minimum at* s_{min}	g *diverges at* s_{div}
$f(s_{min}) \not> 0$	$f(s_{div}) \not> 0$

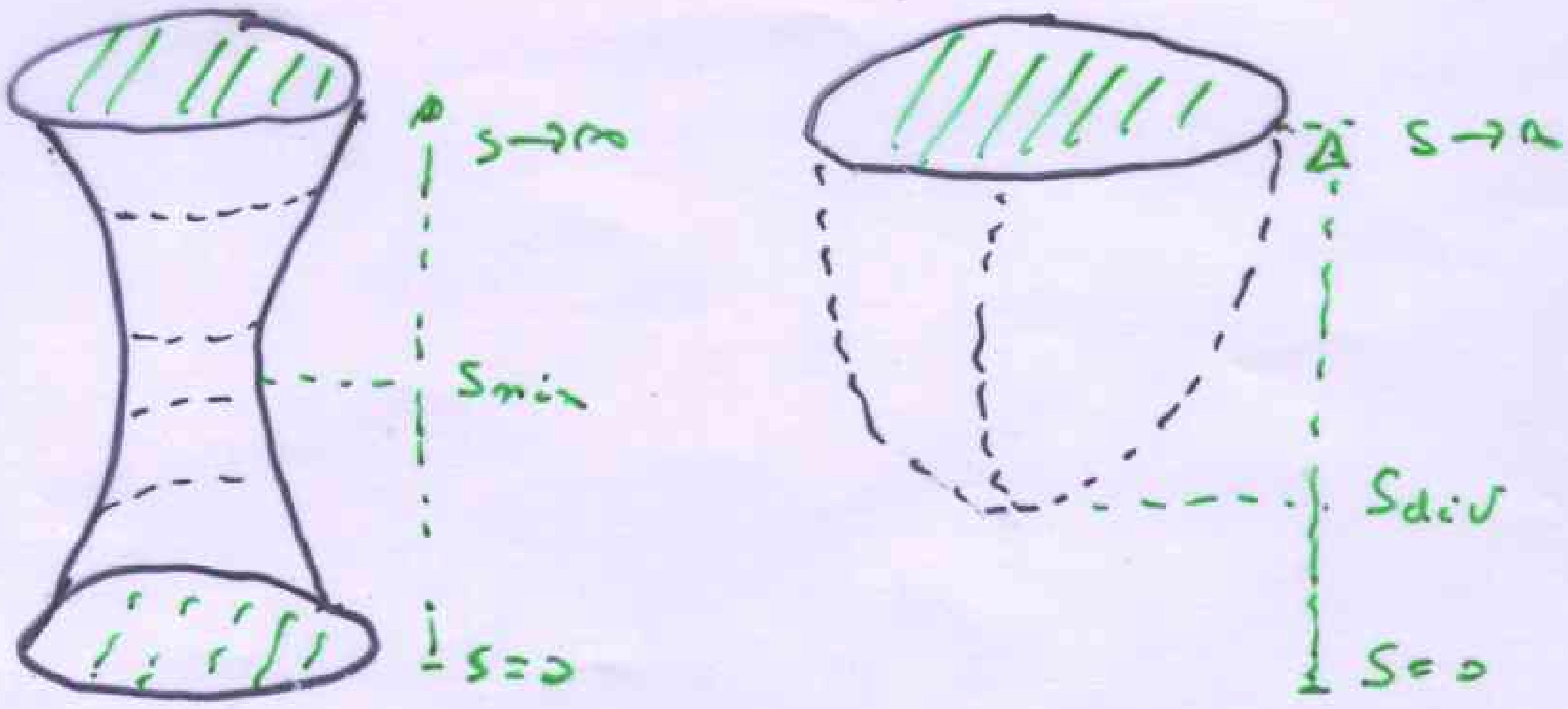
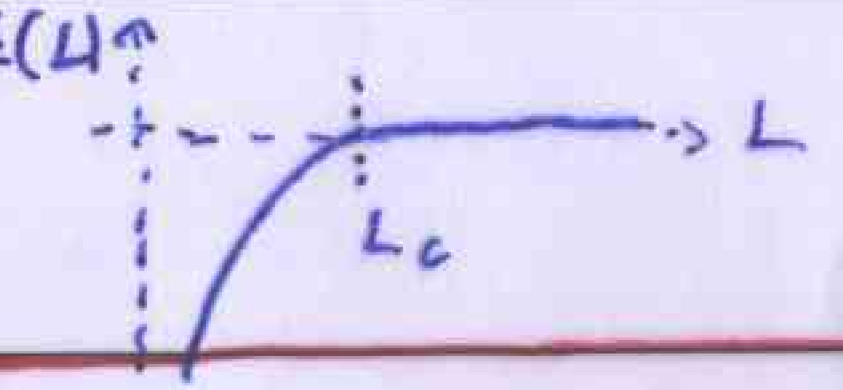


Figure 1: Wilson loops in various scenarios

	Model	Nambu-Goto Lagrangian	Energy	remarks	refs.
Non Confining	$AdS_5 \times S^5$	$\frac{1}{2\pi}\sqrt{U^4/R^4 + (U')^2}$	$-\frac{2\sqrt{2}\pi^{3/2}R^2}{\Gamma(\frac{1}{4})^4} D_{4,4} \cdot \underline{L^{-1}} + O(L^{-7/4})$	4d $\mathcal{N}=4$ SYM	Maldacena, Gross Ooguri, Ooguri
	Itzhaki, maldacena, J.S, Yankielowicz; non-conformal D_p brane (16 supersymmetries)	$\frac{1}{2\pi}\sqrt{(U/R)^{7-p} + (U')^2}$	$-d' \cdot \underline{L^{-2/(5-p)}} + O(l^{-2/(5-p)-2(6-p)/(5-p)(7-p)})$		BISY
	Pure YM in 4d at finite temperature	$\frac{1}{2\pi}\sqrt{(U/R)^4(1-(U_T/U)^4) + (U')^2}$		full screening above L_c	Rey, Theisen, Yee; BISY
Confining; $f(S_{div}) \neq 0$	Witten; Dual model of Pure YM in 3d; D_3	$\frac{1}{2\pi}\sqrt{(U/R)^4 + (U')^2(1-(U_T/U)^4)^{-1}}$	$\frac{U_T^2}{2\pi R^2} \cdot \underline{L} - 2\kappa + O(\log l\, e^{-\alpha L})$	• time on S_1 • anti-periodic b.c	Otto Dorn, Brandhuber, Itzhaki
	Dual model of Pure YM in 4d; D_4	$\frac{1}{2\pi}\sqrt{(U/R)^3 + (U')^2(1-(U_T/U)^3)^{-1}}$	$\frac{U_T^{3/2}}{2\pi R^{3/2}} \cdot \underline{L} - 2\kappa + O(\log L\, e^{-\alpha L})$	• $U_T \to a$ $(R_T \to 0)$	J.S, Yankielowicz
	Russo; Rotating D_3 brane	$\sqrt{C}\sqrt{\frac{U^6}{U_0^4}\Delta + (U')^2\frac{U^2\Delta}{1-a^4/U^4-U_0^6/U^6}}$	$\frac{4}{3}\frac{U_T^2}{R^2}\cdot C\, \underline{L} + \ldots$		KSS
$f(S_{min} \neq 0 = S_{div})$	$D_3 + D_{-1}$ system	$\sqrt{\left(\frac{U^4}{R^4}+q\right) + (U')^2\left(1+\frac{qR^4}{U^4}\right)}$	$q\,\underline{L} + \ldots O(\log L\, e^{-\alpha L})$	q-instanton density	Liu, Tseytlin
$f(S_{min}) \neq 0$	$MQCD$ system	$2\sqrt{2\zeta}\sqrt{\cosh(s/R_{11})}\sqrt{1+s'^2}$	$E = 2\sqrt{2\zeta} \cdot \underline{L} - 2\kappa + O(\log L\, e^{-1/\sqrt{2}R_{11}L})$	d_s, D_4, d_s'	Kinar, Schreiber, JS
	't Hooft loop	$\sqrt{\left(\frac{U^3}{R_4^3}\right)\left(1-\frac{U_T^3}{U^3}\right) + (U')^2} \Big/ g^2_{YM}$	full screening of monopole anti-monopole pair	D_2 action	Gross Ooguri, BISY

Wilson loops in type 0 string theory

- **What is type 0 string**

  Klebanov Tseytlin

  Type 0 string is supersymmetric on the world sheet but not in space-time due to a non-chiral GSO projection. The type 0_A and type 0_B differ from the type II_A and type II_B

  (i) No space-time fermions

  (ii) Doubling of the RR fields

  (iii) Tachyons

- A type 0 model can be made consistent only provided

  (i) The Tachyon m^2_{tach} can be shifted to $m^2_{tach} > -\frac{c}{R^2}$

  (ii) No dilaton (and possible other massless fields) tadpoles

  (iii) The low energy effective theory is reliable if

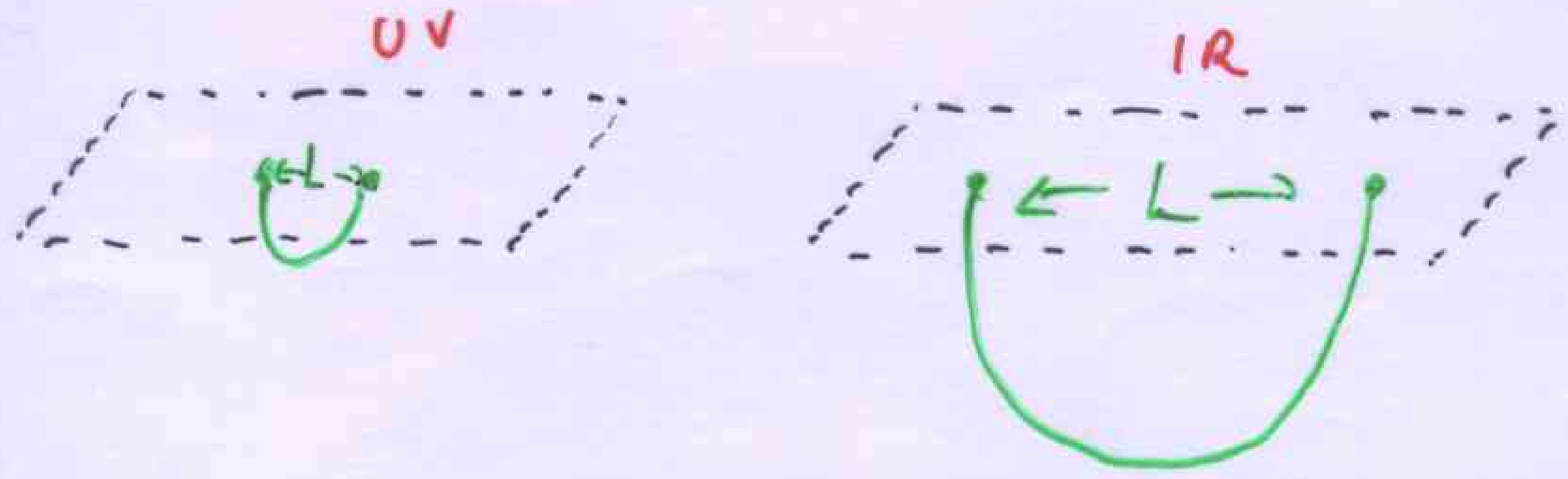
  $g_{st} << 1 \qquad \mathcal{R} << 1$ where $\mathcal{R}$ is the scalar curvature in the string frame.

- The Wilson loops were discussed both in the critical string and in Polyakov's non-critical string model. Armoni, Fuchs, JS

- The equations of motion of the low energy effective theory guarantee that

$$\boxed{\partial_s^2 f(s) \geq 0}$$

- The interpretation of an IR and UV domains may be in terms of the structure of the Wilson line is as follows



so that the large u_s regime corresponds to the gauge theory UV regime and the small u regime to the IR.

- the IR the generic solution has

$$\boxed{\partial_s f(s)_{min} = 0 \qquad with \qquad f(s_{min}) \neq 0}$$

So that generically the solution in the IR admits a linear confinement behavior.

- This can also be verified from arguments based on the 5d bulk theory and in particular also from the screening nature of the 't Hooft loop.

- In the UV a fixed point in the form of $AdS_5 \times S^5$ was observed. Moreover around the fixed point $f \sim logL$ so that it was argued by Minaham that

$$\boxed{\Delta V_1 \sim \frac{1}{log\frac{L_0}{L}}\,\frac{1}{L}}$$

Klebanov and Tseytlin found the higher order correction produces a WIlson line

$$\boxed{\Delta V_2 \sim \frac{1}{(log\frac{L_0}{L} - clog\ log\frac{L_0}{L})}\,\frac{1}{L}}$$

which resembles the 2 loop correction in the gauge theory picture. Note however that in the UV generically the curvature in the string frame is not negligible and thus the assertions have to be made with a grain of salt.

Quantum fluctuations

Introduce quantum fluctuations around the classical configuration

$$x^\mu(\sigma,\tau) = x^\mu_{cl}(\sigma,\tau) + \xi^\mu(\sigma,\tau)$$

The quantum corrections to the Wilson line is
(to quadratic order)

$$\langle W \rangle = \mathrm{e}^{-E_{cl}(L)T} \int \prod_a d\xi_a \exp\left(- \int d^2\sigma \sum_a \xi^a \mathcal{O}^a \xi^a \right)$$

where ξ^a are the fluctuations left after gauge fixing.

The corresponding correction to the free energy is

$$\boxed{F_B = -\log \mathcal{Z}_{(2)} = -\sum_a \frac{1}{2} \log\det \mathcal{O}_a}$$

gauge fixing

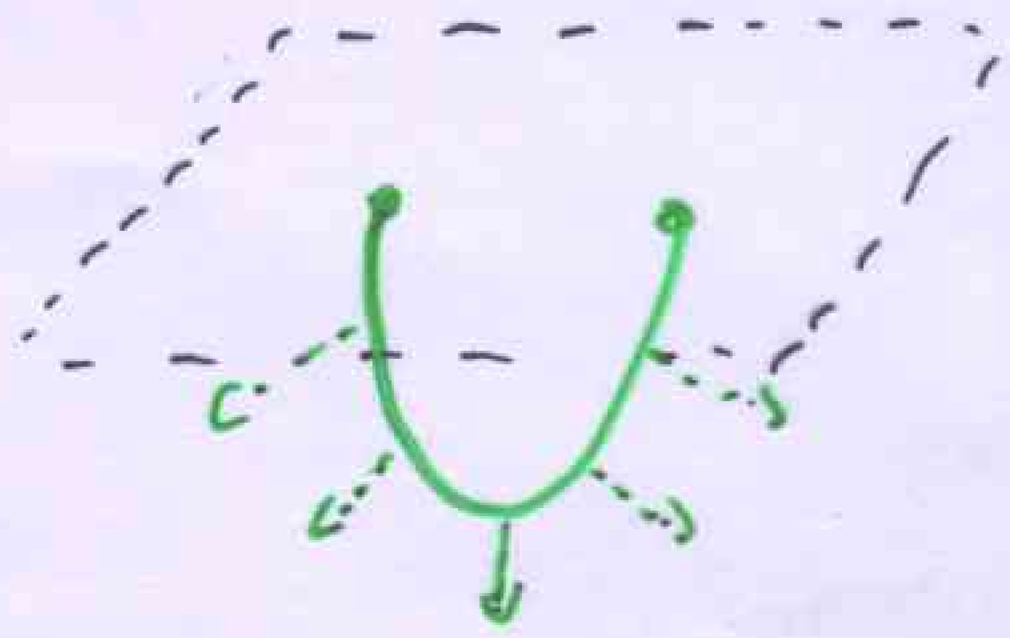
It is convenient to choose $\tau = x^0$

In addition one can further fix (we denote here s by u)

- $\sigma = u \rightarrow \xi_u = 0$ - No fluctuations from the metric
- $\sigma = x \rightarrow \xi_x = 0$
- "normal coordinate gauge"

  $\sigma = u_{cl}$

  and the fluctuation

  in x, u plane

  is in the coordinate normal to u_{cl}

  The normal gauge is safer to use than the other two gauges (that suffer from singularities).

general form of the bosonic determinant

In the $\sigma = u$ gauge (after a change of variables) the free energy is given by

$$
\begin{aligned}
F_B &= -\frac{1}{2}\log\det\mathcal{O}_x - \frac{(p-1)}{2}\log\det\mathcal{O}_{x_{II}} \\
&- \frac{(8-p)}{2}\log\det\mathcal{O}_{x_T}
\end{aligned}
\tag{3}
$$

where

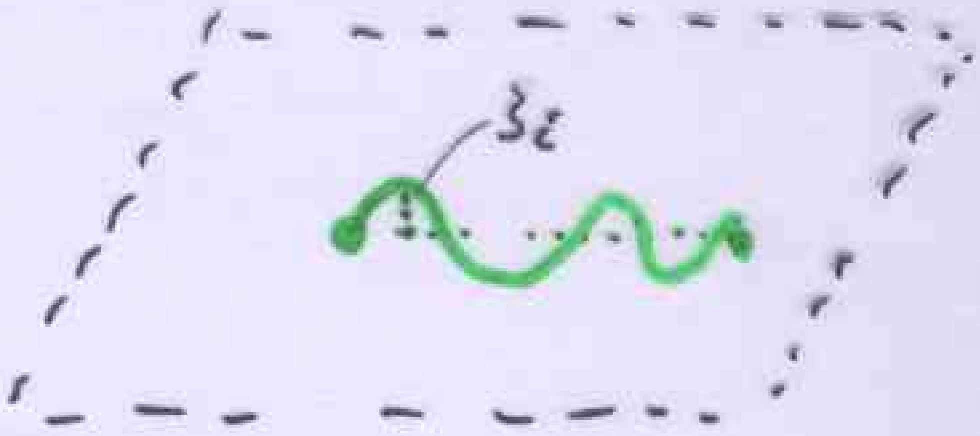
$$
\boxed{
\begin{aligned}
\hat{\mathcal{O}}_x &= \left[\partial_x\left((1-\frac{f^2(u_0)}{f^2(u_{cl})})\partial_x\right) + \frac{G_{xx}(u_{cl})}{G_{tt}(u_{cl})}(\frac{f^2(u_{cl})}{f^2(u_0)}-1)\partial_t^2\right. \\
\hat{\mathcal{O}}_{x_{II}} &= \left[\partial_x\left(\frac{G_{y_iy_i}(u_{cl})}{G_{xx}(u_{cl})}\partial_x\right) + \frac{G_{y_iy_i}(u_{cl})}{G_{tt}(u_{cl})}\frac{f^2(u_{cl})}{f^2(u_0)}\partial_t^2\right] \\
\hat{\mathcal{O}}_{x_T} &= \ldots
\end{aligned}
}
$$

where $\hat{\mathcal{O}} = \frac{2}{f(u_0)}\mathcal{O}$ the boundary conditions are

$$\hat{\xi}(-L/2,t) = \hat{\xi}(L/2,t) = 0$$

Bosonic fluctuations in flat space-time Let us recall first the determinant in flat space-time. The fluctuations in this case are determined by the follwoing action

$$S_{(2)} = \frac{1}{2}\int d\sigma d\tau \sum_{i=1}^{D-2}\left[(\partial_\sigma \xi_i)^2 + (\partial_\tau \xi_i)^2\right]$$



The corresponding eigenvalues are

$$E_{n,m} = (\frac{n\pi}{L})^2 + (\frac{m\pi}{T})^2$$

and the free energy is given by

$$-\frac{2}{D-2}F_B = \log \prod_{nm} E_{n,m} = T\frac{\pi}{2L}\sum_n n + O(L)$$

Regulating this result using Riemann ζ function we find that the quantum correction to the linear quark anti-quark potential is

$$\boxed{\Delta V(L) = -\tfrac{1}{T}F_B = -(D-2)\tfrac{\pi}{24}\cdot\tfrac{1}{L}}$$

which is the so-called **Lüscher term**.

What can be said about the L dependence of ΔV in cur-

Consider an operator of the form -ved space-time

$$\mathcal{O}[A,B] = A^2 F_t(v)\partial_t^2 + B^2\partial_v(F_v(v)\partial_v)$$

The correction to the potential $V[A,B]$ due to fluctuations determined by such an operator is

$$\boxed{V[A,B] = (B/A)\cdot V[1,1]}$$

For the operators that describe the fluctuations associated with metrics such that

$$f(u) = au^k \quad (5)$$

$$g(u) = bu^j \quad (6)$$

(For instance the D_p brane solution in the near horizon limit) Then

$$A^2 \qquad B^2 = \frac{a^2}{b}u_0^{2k-j-2} \rightarrow B/A = \frac{a}{b}u_0^{k-j-1}$$

Therefore, the potential is proportional to

$$\boxed{B/A = \frac{a}{b} u_0^{k-j-1} \rightarrow \Delta V \propto L^{-1}}$$

The quantum correction of the quark anti-quark potential is of Luscher type for models of D_p brnaes with 16 supersymmetries, in particular also the $AdS_5 \times S^5$ model.

The fermionic fluctuations

- The NSR action of the type II superstring with a RR fields like on $AdS_5 \times S^5$ is not known.

- On the other hand the manifestly space-time supersymmetric Green Schwarz action was written down for the $AdS_5 \times S^5$ case.

- To demonstrate the use of the GS action we start with the

  fermionic determinant in flat space-time

- The target space is the coset

  super-Poincarégroup $/SO(9,1)$.

- The fermioic part of the κ gauged fixed GS-action is

$$\boxed{S_F^{flat} = 2i \int d\sigma d\tau \bar{\psi} \Gamma^i \partial_i \psi}$$

  where ψ is a Weyl-Majorana spinor, Γ^i are the SO(1,9) gamma matrices, $i, j = 1, 2$ and we explicitly considered a flat classical string.

Thus the fermionic operator is

$$\hat{\mathcal{O}}_F \equiv D_F \equiv \Gamma^i \partial_i \qquad (7)$$

and squaring it we get

$$(\hat{\mathcal{O}}_F)^2 = \Delta = \partial_x^2 - \partial_t^2$$

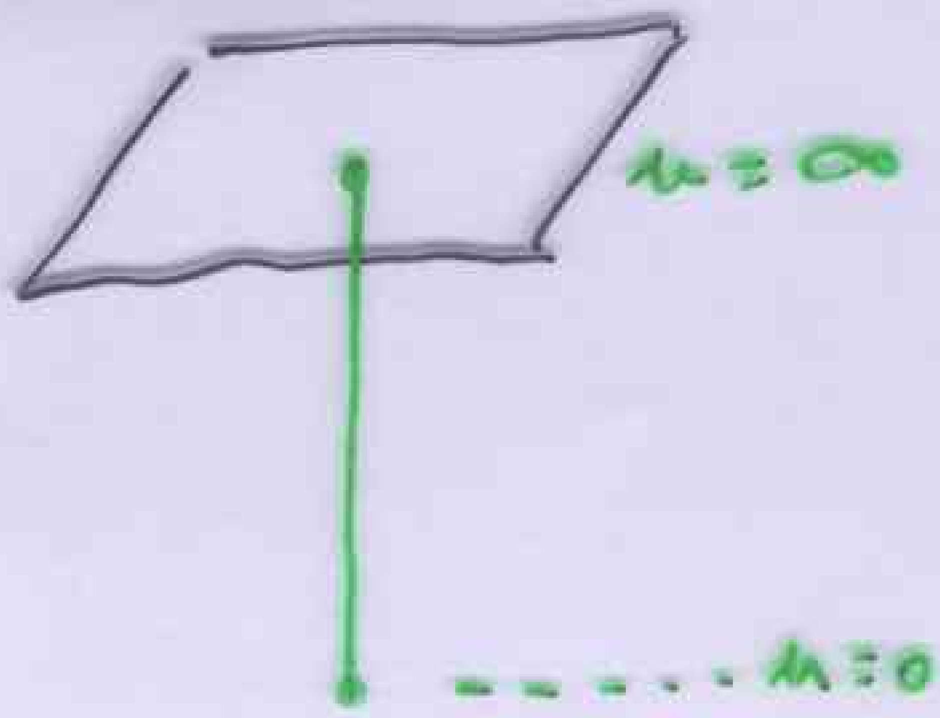
The total free energy is

$$\boxed{F = 8 \times \left(-\frac{1}{2} \log \det \Delta + \log \det D_F \right) = 0}$$

since for D=10, we have 8 transverse coordinates and 8 components of the unfixed Weyl-Majorana spinor.

- In flat space-time the energy associated with supersymmetric string is not corrected by quantum fluctuations. (BPS)

The determinant for a free BPS quark of $AdS_5 \times S^5$



- Masaev and Tseytlin wrote a κ fixed GS action by treating the target space as the coset

$$SU(2,2|4)/(SO(1,4) \times SO(5))$$

Their action incorporates the coupling to the RR field.

- The square of the operator associated with the fermionic fluctuations is

$$8 \times \qquad \mathcal{O}_\psi = \partial_\sigma^2 + \partial_\tau^2 - \frac{3}{4\sigma^2}$$

- The bosonic operators are of the form

$$\begin{aligned} 3 \times \qquad \mathcal{O}_x &= \partial_\sigma^2 + \partial_\tau^2 - \frac{2}{\sigma^2} \\ 5 \times \qquad \mathcal{O}_\theta &= \partial_\sigma^2 + \partial_\tau^2 \end{aligned} \tag{8}$$

where $\{x^0, x, u, \theta\} \equiv \{\tau, \sigma\xi_x, \frac{1}{\sigma}, \xi_\theta\}$ and θ is the coordinate on the S^5

- According to a theorem of McKean and Singer the divergences of Laplacian type operator

$$\Delta = \nabla^2 + X = -\frac{1}{\sqrt{g}} D_a(g^{ab}\sqrt{g} D_b) + X$$

vanish if there is a match between the fermionic and bosonic coefficients of

(i) ∇^2

$\rightarrow \quad 8 \times (B) - 8 \times (F) = 0 \rightarrow$ No Quadratic divercence

(ii) the "mass term" X

$3 \times 2(B) - 8 \times 3/4(F) = 0 \rightarrow$ No Logrithmic divergence

it is thus clear that the divergent parts of the det
erminant vanish

This problem is related to the general issue of quantum correction to soliton in supersymmetric theories

Van Nieuwenhausen et.al
Shifman Weinstein
Jaffe et al

The determinant for a Wilson line of $AdS_5 \times S^5$

Kalosh and Tseytlin simplified the κ fixed GS action

$$
\begin{aligned}
S_{GS} &= \int d^2\sigma[\sqrt{g}g^{\alpha\beta}(y^2[\partial_\alpha x^p - 2i\bar{\psi}\Gamma^p\partial_\alpha\psi][\partial_\beta x^p - \\
&+ \frac{1}{y^2}\partial_\alpha y^t \partial_\beta y^t) + 4\epsilon^{\alpha\beta}\partial_\alpha y^t \bar{\psi}\Gamma^t\partial_\beta\psi]
\end{aligned}
$$

where ψ is a Majorana-Weyl spinor and the $Ads_5 \times S^5$ metric is written in terms of the $4+6$ coordinates

$$ds^2 = y^2 dx^p_{II} dx^p_{II} + \frac{1}{y^2} dy^i_T dy^i_T$$

The bosonic operators in the normal gauge now read

$$
\begin{aligned}
2\times \quad \mathcal{O}_{x_{II}} &= \partial_x^2 - \frac{u^4}{u_0^4}\partial_t^2 \\
5\times \quad \mathcal{O}_\theta &= \partial_x^2 - \frac{u^4}{u_0^4}\partial_t^2 + 2\frac{u^6}{u_0^4}
\end{aligned}
$$

$$1\times \qquad \mathcal{O}_{normal} = \partial_x^2 - \frac{u^4}{u_0^4}\partial_t^2 + 5u^2 - 3\frac{u^4}{u_0^2} \tag{10}$$

The fermionic part of the action for the classical solution leads to the operator

$$\hat{\mathcal{O}}_\psi = \frac{u_0^2}{R^2}\Gamma^1\partial_x + \left(\frac{u_{cl}^4}{u_0^2R^2}\Gamma^0 + \frac{u_{cl}^4}{R^4}\cdot\frac{\sqrt{u_{cl}^4 - u_0^4}}{u_0^2}\Gamma^2\right)\partial_t$$

where we use Γ matrices of SO(1,4) - the AdS_5 tangent space. Squaring this operator, we find

$$\left(\frac{R^2}{u_0^2}\hat{\mathcal{O}}_F\right)^2 = \partial_x^2 - \frac{u_{cl}^4}{u_0^4}\partial_t^2 = \frac{R^2}{u_0^2}\hat{\mathcal{O}}_{x\parallel}$$

Thus the transverse fluctuations $\mathcal{O}_{x\parallel}$ are cancelled by the fermionic fluctuations. We are left with 6 fermionic degree of freedom and the normal bosonic fluctuations $+\ 5\times\mathcal{O}_\theta$

Using our general result we know that the quamtum correction of the potential is of a Luscher type. The universal coefficient and in particular its sign has not yet been determined.

Ghoshal, Forste and Theisen found using a different fermionic action that the final result has a logarithmic divergence

The determinant for "confining scenarios"

- Let us consider first the the setup which is dual to the pure YM thoery in 3d. For that case Greensite Olesen

$$
\begin{aligned}
f(u) &= u^2/R^2 && (11)\\
g(u) &= (1-(\frac{u_T}{u})^4)^{-1/2} && (12)
\end{aligned}
$$

In the large L limit

$$
\boxed{
\begin{aligned}
\hat{\mathcal{O}}_y &\longrightarrow \frac{u_T^2}{2}\left[\partial_x^2+\partial_t^2\right] && (13)\\
\hat{\mathcal{O}}_{x_T} &\longrightarrow 2u_T^2 e^{-2u_T L}\left[\partial_x^2+\partial_t^2\right] && (14)\\
\hat{\mathcal{O}}_n &\longrightarrow \left[\frac{4u_T^2}{2R^4}+\frac{1}{2}\partial_x^2+\frac{1}{2}\partial_t^2\right] && (15)
\end{aligned}
}
$$

- We see that the operators for transverse fluctuations, $\hat{\mathcal{O}}_y$, $\hat{\mathcal{O}}_{x_T}$, turn out to be simply the Laplacian in flat spacetime, multiplied by overall factors, which are irrelevant.

- Therefore, the transverse fluctuations yield the standard Lüscher term proportional to $1/L$.

- The longitudinal normal fluctuations give rise to an operator $\hat{\mathcal{O}}_n$ corresponding to a scalar field with mass $2u_T/R^2 = \alpha$. Such a field contributes a Yukawa like term

$$\boxed{\approx -\frac{\sqrt{\alpha}e^{-\alpha L}}{\sqrt{L}}}$$

to the potential.

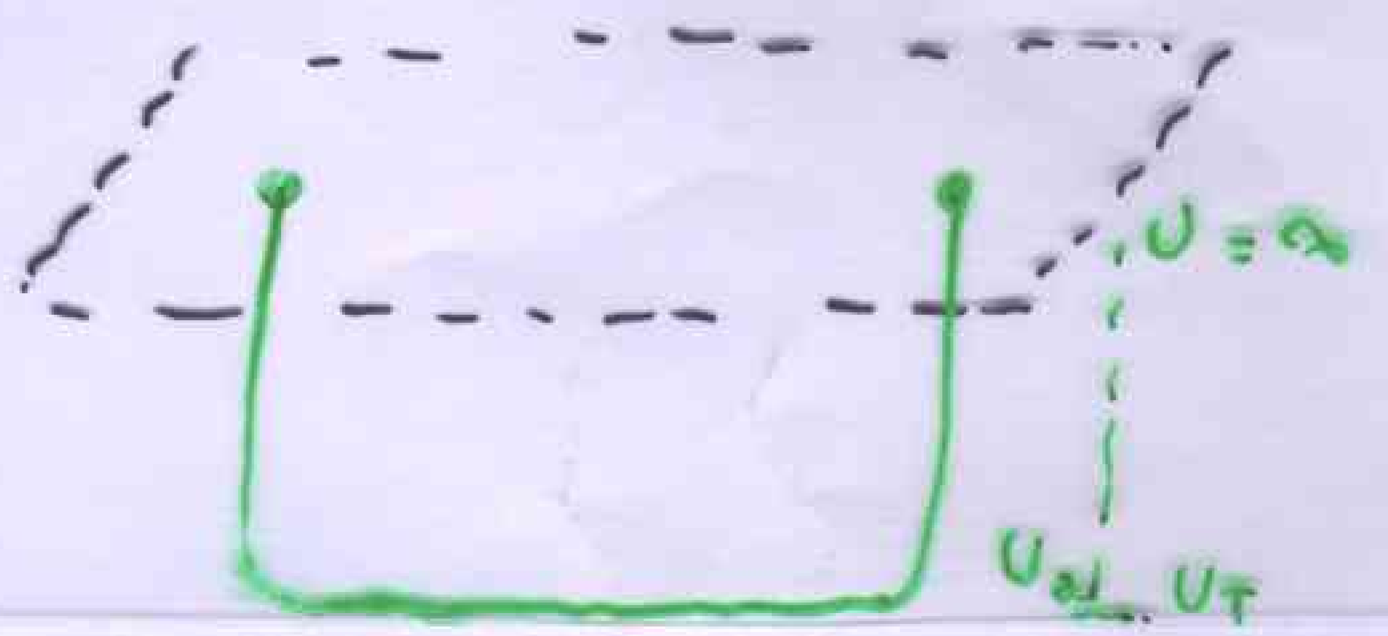
- The square of the fermionic operator

$$\hat{\mathcal{O}}_\psi = \frac{u_T^2}{2}\left[\partial_x^2 + \partial_t^2\right] + \ldots \text{ mass corr.}$$

- In the general setup that corresponds to confinement, namely,

$$f(s_{min}) \ngtr 0 \qquad or \qquad f(s_{div}) \ngtr 0$$

- One can prove that most of the string is flat along u_0. In the large L limit the non-flat part behaves like L^{-a} $(a > 0)$ for the first type and as $log(L)$ for the second type. ~~type~~



* Since most of the string is along U_0 again like the "pure YM case" there are 7 Luscher type modes and one massive mode.

Greensite Olesen

* Had the fermionic modes been those of flat space-time then the total coefficient infront of the Luscher term would have been $+8 -7$ $\rightarrow$ a repulsive Culomb like potential. This contradicts gauge dynamics. Bachas

* However the point is that due to the RR flux the corresponding GS action cannot be that of a flat space-time. Moreover, (some of) the fermionic fluctuations also become massive.

* Thus it is plausible that total inteaction is attractive after all.

* This is in accordance with a general statement about the nature of the potential made recently by Dorn and Pershin

Can the Wilson line be evaluated exactly?

- Consider the bosonic string in flat space-time Arvis with the boundary conditions

$$X^i(\sigma = 0) = 0 \qquad X^i(\sigma = \pi) = L^i \qquad with\, L^iL_i =$$

The solution of the equations of motion are

$$\begin{aligned} X^i(\sigma\tau) &= \frac{L^i}{\pi}\sigma + \sum_{n=1}^{\infty}\frac{\alpha_n^i}{n} sin(n\sigma)e^{-in\tau} \\ X^0(\sigma\tau) &= 2\pi\alpha' E\tau + i\sum_{n=1}^{\infty}\frac{\alpha_n^0}{n} cos(n\sigma)e^{-in\tau} \end{aligned} \tag{16}$$

- The energy

$$E = \frac{1}{2\pi\alpha'}\int_0^\pi d\sigma \partial_\tau X^0(\sigma)$$

for the lowest **tachyonic** state is given by

$$E^2 = P^iP_i + {m_{tach}}^2 = (\frac{L^i}{2\pi\alpha})^2 - \frac{(D-2)}{24}\frac{1}{\alpha'}$$

so that

$$\boxed{E = T_s L\sqrt{1 - \frac{\pi(D-2)}{12}\frac{1}{T_s L^2}}}$$

which can be expanded

$$\sim T_{st}L - \pi\frac{(D-2)}{24}\frac{1}{L} + \frac{1}{T_{st}L^2} + \ldots$$

where the string tension $T_{st} = 2\pi\alpha'$. Thus this expansion yields the Luscher quadratic fluctuation term.

- (Recall also that in the limit $L \to \infty$ the Virasoro anomaly ~~sim~~ $\frac{T_{st}^2}{L^2} \to 0$)

- Moreover for a bosonic string in Flat space-time it was shown that in the large D limit O. Alvarez

$$D \to \infty \qquad \frac{\pi}{24T_{st}L^2} \to 0 \qquad \frac{D\pi}{24T_{st}L^2} \to finite$$

$$\boxed{S_{NG} = T_{st}L\sqrt{1 - \frac{(D-2)}{24\;T_s L^2}} = E}$$

- Recall also that for a static classical configurations

$$E_{Poly} = S_{NG}$$

with A. Tseytlin

Can one find such "exact" solution for non-flat m

metric

- A naive conjecture is that for the $Ads_5 \times S^5$ the result is $\sim \frac{\sqrt{g^2N}}{L}\sqrt{1-\frac{c}{\sqrt{g^2N}}}$. But this does not yield the perturbative result $\sim \frac{g^2 N}{L}$
- Exact results are known for group manifolds and coset spaces.
- The sigma model associated with such target spaces is equipped with a WZ term The bosonic action is therefore

$$S_B = S_{NG} + \int d^2\sigma e^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu B_{\mu\nu}$$

For the case that the only non-trivial component of $B_{\mu\nu}$ is $B_{01} = B$ one finds that for $B \neq f$

$$S_{NG+WZ} = \int_{u_0}^{\infty} du \frac{g}{f}\frac{f^2 - B(f_0 + B - B_0)}{\sqrt{f^2 - (f_0 + B - B_0)^2}}$$

and

$$S_{NG+WZ} = 0 \qquad for\ B \equiv f$$

In the string models

- The $SL(2,R)$ group manifold
- The $\frac{SL(2,R)}{R} \times R$ (The F and K models of Tseytlin and Horowitz

  The B term match the f, namely $f = B$ so that the Wilson line is a straight line and the energy $E = 0$.

  $\boxed{E_{cl} = 0 \quad ; \quad E_{quan} = 0}$

- **are there models with non-trivial Wilson line that can be determined exactly?** To be explored.

Summary

– The Luscher term may serve as a "precision" tool to compare between string models belonging to the confining universality class and real life QCD dynamics.

– Several obstacles in this program have been overcome, however there is still a long way to go.

For the former case

$$S = (f_0 + B_0)L + 2\int_{u_0}^{\infty} du \frac{g}{f}\sqrt{f^2 - (f_0 + B - B_0)^2}$$

For sting theories where $f_0 + B_0$ does not vanish the WIlson loop admits an area law behavior with a string tension equal to $f_0 + B_0$.