FIELD THEORY ANOMALIES
FROM THE
AdS/CFT CORRESPONDENCE

work done in collaboration with

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MOTIVATION: test correspondence at subleading order in N

here we will consider

SUSY
- R - current anomaly
- trace anomaly

- for $\mathcal{N}=4 \;\hat{=}\; AdS_5 \times S^5$

anomalies have been successfully reproduced at $O(N^2)$

Witten

Henningson, Skenderis

there: no $O(N)$ contribution

$O(1) \rightarrow$ string loops (Bilal, Chu).

- $\mathcal{N}=2 \;\hat{=}\; AdS_5 \times X^5$

↳ Einstein, ... see below

Strategy: on AdS_5 side of correspondence

need $d=5$ low energy effective SUGRA action

from here: (in principle) straightforward

- complete for $U(1)_R$ anomaly ①
- incomplete for trace-anomaly ②

THE $\mathcal{N}=2$ MODEL

look for $O(N)$ effect

- D-branes (disc)
- O-planes ($\mathbb{RP}_2$)

the model we consider

Sen

$T_{8,9}$ - dual to type I $\hat{=}$ IIB on $M^{7,1} \times T^2 / (I_{8,9} \cdot (-1)^{F_L} \cdot \Omega)$

$$\begin{matrix} 32\ D9 \\ 1\ O9 \end{matrix} \xrightarrow{T_{8,9}} \begin{matrix} 32\ D7 \\ 4\ O7 \end{matrix}$$

$$4 \times (\underbrace{8\ D7 + 1\ O7})$$

all coincident

$$+\ 2N\ D3 \xleftarrow{T_{8,9}} D1$$

inside D7

note: $\mu_7^{I} = \frac{1}{2} \mu_7^{II}$ D7

$\mu_7^{\prime I} = -8 \mu_7^{I}$ O7

Field content

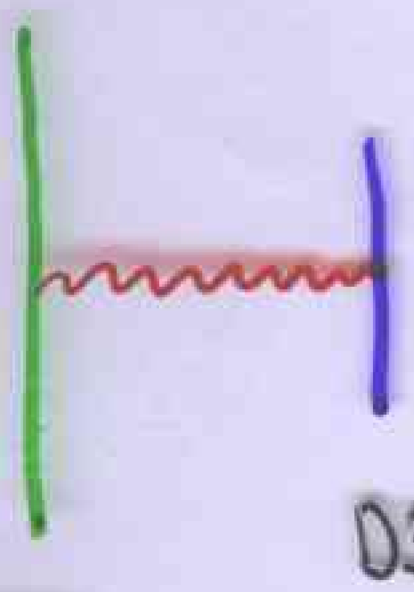
Aharony, Sonnenschein, Yankielowicz, S.T.
Douglas, Lowe, Schwarz

$G = USp(2N)$

		b f
vector multiplets :	$\underline{N(2N+1)}$	$(2,1)$
hypermultiplets :	$4 \times \underline{2N}$ *.)	$(0,-1)$
	$\underline{N(2N-1)-1}$	$(0,-1)$

$\uparrow$ $U(1)_R$-charges

*.)



D3

D7

$\beta = 0$

global symmetries

$$\underbrace{SU(2)_L \times \overbrace{SU(2)_R \times U(1)_R}^{\mathcal{N}=2\ R\text{-symmetry}}}_{\subset\, SU(4)_R} \times \underbrace{SO(8)}_{\text{flavour} \leftarrow 8\ D7\text{'s}}$$

Anomalies (field theory) to $\Theta(N)$

couple to external $g_{\mu\nu}$

A_μ^{flavour}

$$\Rightarrow \langle \underbrace{\partial_\mu(\sqrt{g}\, R^\mu)}_{\text{U(1) R-current}} \rangle = -\frac{N}{32\pi^2}\left[(\tilde{R}R) - 4(\tilde{F}F)\right]$$

$$(\tilde{F}F) = \frac{1}{2}\,\mathrm{tr}(\epsilon F F)$$

$$(\tilde{R}R) = \frac{1}{2}\,(\epsilon R R)$$

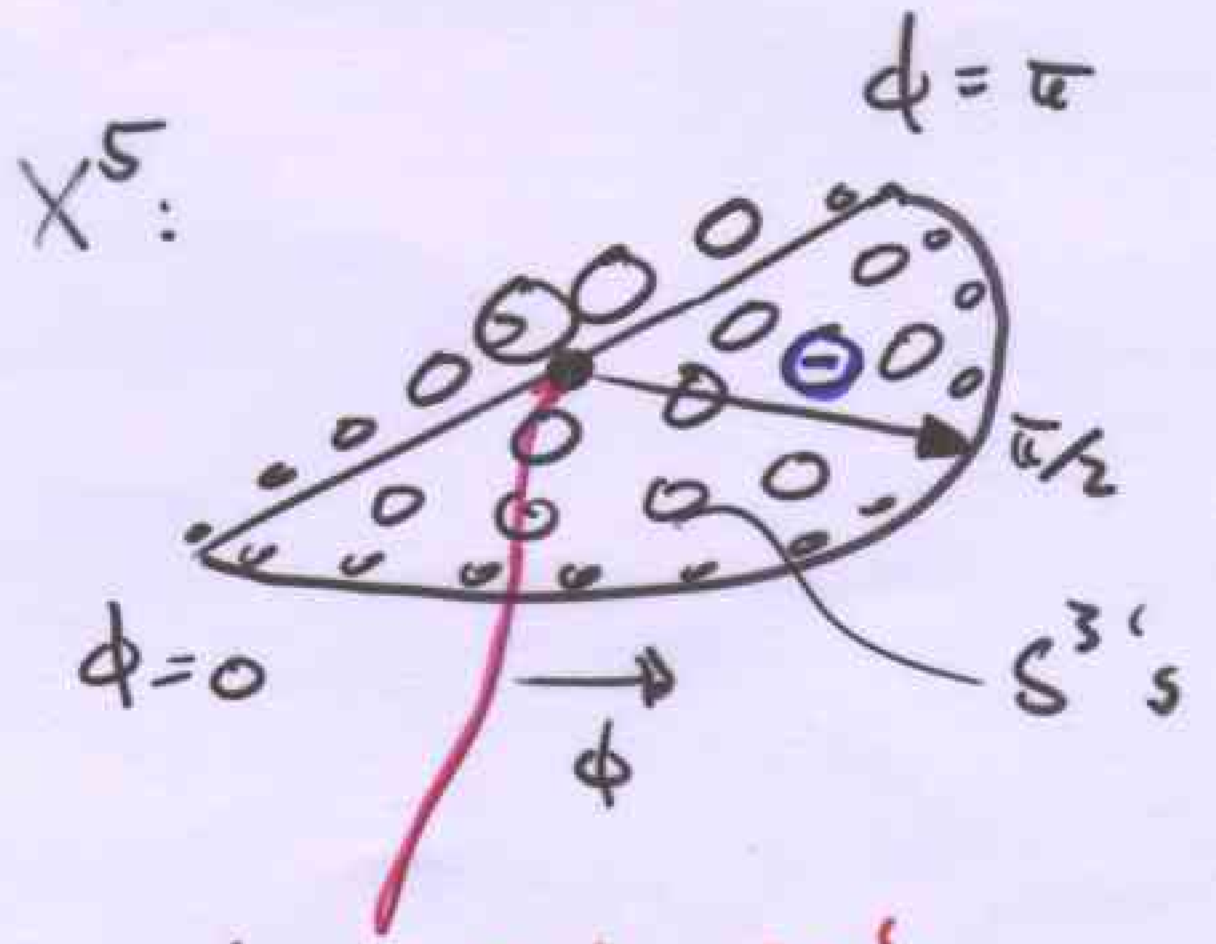
$$\langle T^\mu{}_\mu \rangle = \frac{N^2}{32\pi^2}\left\{\left(1+\frac{3}{2}\frac{1}{N}\right)C^2 - \left(1+\frac{1}{N}\right)\mathrm{G.B.}\right\}$$

Near horizon limit of this brane configuration

Spalinski, Fayyazuddin
Aharony, Fayyazuddin, Maldacena

- $ds^2 = \alpha' \sqrt{8\pi g_s N} \; (ds^2_{AdS_5} + ds^2_{X^5})$

$$ds^2_{X^5} = d\theta^2 + \sin^2\theta \, d\phi^2 + \cos^2\theta \, \underbrace{d\Omega_3^2}_{S^3}$$



$X^5 = S^5/\mathbb{Z}_2$

here sit the D7's, wrapped around S^3
D4 singularity at $\theta = 0$

- F_5 normalized s.t. $\int_{X^5} F_{(5)} = N$

$\mathrm{Vol}(X^5) = \frac{1}{2} \mathrm{Vol}(S^5)$

- $\Phi = \mathrm{const.}$

Anomalies (string theory)

need: CS terms of SUGRA action

- of the form $\int A^R \wedge \mathrm{tr}(R \wedge R) + \int A^R \wedge \mathrm{tr}(F \wedge F)$

  $U(1)_R$ gauge field — $SO(8)$ field strength

- gauged $(SU(2) \times U(1))$ $\mathcal{N}=4$, $d=5$ SUGRA

get them from CP-odd terms of world-volume action of D7 and O7

They are

D7 $\quad \mu_7 \int C \wedge \sqrt{\hat{A}(4\pi^2\alpha' R)}\, \mathrm{tr}_f\, e^{2\pi\alpha' F} \Big|_8$ — Green, Harvey, Moore

O7 $\quad + \mu_7' \int C \wedge \sqrt{\hat{L}(\pi^2\alpha' R)} \Big|_8$ — Morales, Scrucca, Serone, Stefanski

$$= \frac{N}{16\pi^4} \int C_{(4)} \wedge \mathrm{tr}(R \wedge R) + \frac{N}{4\pi^2} \int C_{(4)} \wedge \mathrm{tr}(F \wedge F)$$

here:
- kept only relevant terms
- rescaled $C_{(4)} \to 32\pi\alpha'^2 N\, C_{(4)}$ to SUGRA convent.

Now: compactify on $S^3 \to$ CS-term of 5-dim SUGRA

for this note:

5-dim vectors from 2 sources:

metric $\quad g_{ma} = B_m(x)\, Y_a(y)$ ← vector harmonic $\frac{1}{2}\sin^2\Theta\ \hat{\phi}$

$C_{(4)} \quad C_{mabc} \equiv \phi_m(x)\, \epsilon_{abc}{}^{de}\, D_e Y_e(y)$

from Kim, Romans, van N. (note: $U(1) \subset SU(4)$)

$$A_m \equiv B_m - 16\,\phi_m \qquad \text{massless}$$

$$V_m \equiv B_m + 8\,\phi_m \qquad \text{massive}$$

$$\Rightarrow \quad C_{mabc} = \eta\, A_m^R\, \omega_{abc}$$

volume element on unit S^3

η: normalization: to be fixed (c.f. below)

$$\phi_m = -\frac{1}{24} A_m \equiv \eta\, A_m^R$$

A_m^R: couples canonically to R_μ current

$$\int_{S^3} \Longrightarrow \text{ 5-dim C.S. terms}$$

$$\eta \frac{N}{8\pi^2} \int_{AdS_5} \left[A^R \wedge \mathrm{tr}(R\wedge R) + 4\, A^R \wedge \mathrm{tr}(F\wedge F) \right]$$

$U(1)$ R-transf. $A^R \to A^R + d\Lambda$

$$\Longrightarrow \quad \langle \partial_\mu (\sqrt{g}\, R^\mu) \rangle = -\eta \frac{N}{16\pi^2} \left[(\tilde{R}R) - 4 (F\wedge F) \right]$$

$$F = F^a T^a$$

SUGRA

c.f. field theory result:

$$\langle \partial_\mu (\sqrt{g}\, R^\mu) \rangle = -\frac{N}{32\pi^2} \left[(\tilde{R}R) - 4 (F\wedge F) \right]$$

$$F = F^a t^a$$

FIELD THEORY

$\Longrightarrow$ need to check two normalizations

(i) T^a vs t^a

(ii) $\eta = ?$

both checks use AdS/CFT correspondence

(i)

- Compute for SO(8) flavour current J^a_μ in F.T.

$$\langle J^a_\mu(x) J^b_\nu(0)\rangle = 2N \operatorname{tr}(t^a t^b) \frac{1}{(2\pi)^4} (\eta_{\mu\nu}\Box - \partial_\mu\partial_\nu) \frac{1}{x^4}$$

- same in AdS/CFT

Freedman, Mathur, Matusis, Rastelli

$$\langle J^a_\mu(x) J^b_\nu(0)\rangle = \delta^{ab} \frac{1}{2\pi^2 g^2_{SO(8)}} (\eta_{\mu\nu}\Box - \partial_\mu\partial_\nu) \frac{1}{x^4}$$

- $g^2_{SO(8)}$ from BI-action of D7's

$$S = -\mu_7 \int d^8x\, e^{-\phi} \operatorname{tr} \sqrt{-\det(G_{ab} + 2\pi\alpha' F_{ab})}$$

$$\underset{\substack{\uparrow \\ \text{integrate over } S^3}}{=} \lambda \frac{N}{16\pi^2} \int d^5x \sqrt{-G}\, F^a_{\mu\nu} F^{a\,\mu\nu}$$

$\operatorname{tr}(T^a T^b) = \lambda \delta^{ab}$ $\Longrightarrow g^2_{SO(8)} = \frac{1}{\lambda} \frac{4\pi^2}{N}$

Comparison $\Longrightarrow$ $\operatorname{tr}(T^a T^b) = \operatorname{tr}(t^a t^b)$

(ii)

- From quadratic action of massless gauge fields

Arutyunov, Frolov

$$S = \frac{4(2N)^2}{(2\pi)^5} \int_{AdS_5} d^5x \sqrt{g}\, \frac{\pi^3}{24}\, \frac{1}{3}\left(-\frac{1}{4} F(A)^2\right)$$

$$\equiv -\frac{1}{4 g^2_{SUGRA}} \int d^5x \sqrt{g}\, F(A^R)^2$$

recall: $\therefore A = -24\eta A^R$

- compute $\langle R_\mu(x) R_\mu(0) \rangle$

  - in field theory
  - using AdS/CFT Freedman et al.

$$\Rightarrow g^2_{SUGRA} = \frac{\pi}{N}$$

$$\Rightarrow \eta = \frac{1}{2}$$

Now: Trace anomaly to $\mathcal{O}(N)$ w\ A. Schwimmer, S. Yankielowicz

related work by

Nojiri, Odintsov

Blau, Gava, Narain

based on Henningson, Skenderis

preliminaries

- field theory

$$\langle T^\mu{}_\mu \rangle = a\, E_4 - c\, C^2$$

- $\mathcal{N}=4$: $a = c$

$$\rightsquigarrow \langle T^\mu{}_\mu \rangle \sim N^2 \left(R^2, R_{\mu\nu}^2, \cancel{R_{\mu\nu\rho\sigma}^2} \right)$$

- $\mathcal{N}=2$: $a \neq c$

$$\rightsquigarrow \langle T^\mu{}_\mu \rangle \sim N^2 \left(R^2, R_{\mu\nu}^2, \frac{1}{N} R_{\mu\nu\rho\sigma}^2 \right)$$

note: $U(1)_R$ anomaly is prop. to $(a-c)$ $\leftarrow \mathcal{N}=2$ SUSY

$$\langle \partial_\mu(\sqrt{g}\, R^\mu) \rangle \quad \longleftarrow \quad \int d^5x \text{ "}(A R R)\text{"} \qquad \text{CP odd}$$

$$\Big\updownarrow \mathcal{N}=2 \text{ susy} \quad \Longrightarrow \quad \Big\updownarrow \mathcal{N}=2 \text{ SUSY}$$

$$\langle T^\mu{}_\mu \rangle \quad \longleftarrow \quad \int d^5x \text{ "}(R + R^2)\text{"} \qquad \text{CP even}$$

e.g. D7 branes

Note: the CP even terms of gauged $\mathcal{N}=4$ $d=5$ SUGRA to $O(\partial^4)$ are not completely known

- coefficients of R^2, R_{ij}^2, R_{ijkl}^2
- Cosmological constant Λ

⇝ matching SUGRA ↔ CFT has not been achieved yet

GENERAL APPROACH

$(d+1)$ - dim action

$$S = \int d^{d+1}x \sqrt{G}\, f(R) \quad (+ \text{ boundary terms}$$

(arbitrary) local function of R, ∇R, ..
s.t. AdS_{d+1} is a solution of eqs. of motion

Ansatz for metric (Fefferman, Graham)

$$ds^2 = G_{ij}\, dx^i dx^j = \frac{\ell^2}{4}\left(\frac{d\rho}{\rho}\right)^2 + \frac{1}{\rho}\, g_{ij}(x,\rho)\, dx^i dx^j$$

$\rho = 0 \;\hat{=}\;$ boundary

$g_{ij}(x, \rho = 0)$: boundary metric

look for those $(d+1)$-dim diffeos which

- leave form of metric invariant
- induce Weyl rescalings of boundary metric

ansatz: $$\rho = \rho' e^{-2\sigma(x')}$$

$$x^i = x'^i + a^i(x', \rho')$$

to $\mathcal{O}(\sigma)$

$$\rightarrow \quad a^i(x,\rho) = \frac{\ell^2}{2} \int_0^\rho d\rho' \, g^{ij}(x,\rho') \partial_j \sigma(x)$$

(*)

$$\delta g_{ij}(x,\rho) = 2\sigma(1-\rho\partial_\rho) g_{ij}(x,\rho)$$
$$+ \nabla_i a_j(x,\rho) + \nabla_j a_i(x,\rho)$$

ansatz

$$a^i(x,\rho) = \sum_1^\infty a^i_{(n)}(x)\rho^n$$

$$g_{ij}(x,\rho) = \sum_0^\infty g^{(n)}_{ij}(x)\rho^n \qquad \text{(+ log terms}$$

Can solve (*)

e.g. $\delta \overset{(0)}{g}_{ij} = 2\sigma \overset{(0)}{g}_{ij}$ Weyl rescaling of boundary metric

$$\delta \overset{(1)}{g}_{ij} = \overset{(0)}{\nabla}_i a_{(1)j} + (i \leftrightarrow j)$$

$$\text{w/} \quad a^i_{(1)} = \frac{\ell^2}{2} \overset{(0)}{g}{}^{ij} \partial_j \sigma$$

$$\Longrightarrow \overset{(1)}{g}_{ij} = \frac{\ell^2}{d-2} \left[\overset{(0)}{R}_{ij} - \frac{1}{2(d-1)} \overset{(0)}{R} \overset{(0)}{g}_{ij} \right]$$

$$\delta \overset{(2)}{g}_{ij} = \dots$$

$$\Longrightarrow \overset{(2)}{g}_{ij} = \underset{\underset{(\text{Weyl})^2}{\ni}}{(\text{hom})} + (\text{inhom})$$

$\vdots$

So far, the action has not yet been used

Now

$$S = \int d^{d+1}x \sqrt{G}\, f(R)$$
$$= \int d\rho\, d^d x\, \rho^{-\frac{d}{2}-1} \sqrt{\overset{(0)}{g}(x)}\, b(x,\rho)$$

requiring

$$S = \int (x,\rho) = \int (x',\rho')$$

gives

$$\delta b(x,\rho) = -2\sigma(x)\rho\, \partial_\rho b(x,\rho) + \overset{(0)}{\nabla}_i \left(b(x,\rho)\, a^i(x,\rho) \right)$$

Note that b satisfies WZ consistency:

$$\int d^d x \sqrt{\overset{(0)}{g}} \left(\sigma_1 \delta_{\sigma_2} - \sigma_2 \delta_{\sigma_1} \right) b(x,\rho) = 0$$

Expand

$$b(x,\rho) = \sum_0^\infty b_n(x)\, \rho^n$$

One finds, e.g.

- $\delta b_0 = 0$ $\quad \rightsquigarrow b_0(x) = \text{const}$

in fact: $b_0 = \ell f \; (AdS_{d+1})$

$$R_{\mu\nu\rho\sigma} = \frac{1}{\ell^2}\left(G_{\mu\rho}G_{\nu\sigma} - G_{\mu\sigma}G_{\nu\rho}\right)$$

- $\delta b_1 = -2\sigma b_1 + \frac{\ell^2}{2} b_0 \overset{(0)}{\Box}{}^2 \sigma$

$$\rightsquigarrow b_1 = b_0 \frac{\ell^2}{4(d-1)} \overset{(0)}{R}$$

- $\delta b_2 = -4\sigma b_2 - \frac{b_0 \ell^2}{4(d-2)} \left[\overset{(0)ij}{R} \overset{(0)}{\nabla}_i \overset{(0)}{\nabla}_j \sigma - \frac{1}{2} \overset{(0)}{R} \overset{(0)}{\Box}{}^2 \sigma \right]$

$$\rightsquigarrow b_2 = \underbrace{\frac{\ell^4 b_0}{32(d-2)(d-3)} E_4}_{\text{"universal"}} + \underbrace{c\, C^2}_{\text{solution to hom. eq.}}$$

-
-
-

Significance of the b_n: Henningson/Skenderis

$$\langle T^i{}_i \rangle_{d=2n} = b_n(x)$$

since

$$S = \int_\varepsilon \rho^{-n-1} d^{2n}x \sqrt{g^{(0)}} \left(\cdots + b_n(x) \rho^n + \cdots \right)$$

$$\sim \ln \varepsilon \int d^{2n}x \sqrt{g^{(0)}}\, b_n(x) \quad + \cdots \qquad (*)$$

note: the detailed form of action (i.e. $f(R)$) enters through c's

coeff's of hom. solutions to $\delta b_n = \cdots$

as in $(*)$ the action is evaluated on-shell ($(d+1)$-dim'l)

to get c's: need to solve eqs. of motion to appropriate power of ρ.

For $O(N)$ terms of trace anomaly for above $\mathcal{N}=2$ SCFT this leads to

$$S = \frac{1}{2k_5^2}\int d^5x\sqrt{G}\left\{R - 2\Lambda + \underbrace{\alpha R^2 + \beta R_{\mu\nu}^2 + \gamma R_{\mu\nu\rho\sigma}^2}\right\}$$

$\mathcal{N}=2$ SUSY

CS term $A \wedge R \wedge R$

$$\alpha, \beta, \gamma \sim 1/N$$

$$\frac{1}{2k_5^2} \sim N^2 (1 + O(1/N))$$

from $l^4 = l_0^4(1+O(1/N))$, $l_0^4 = 8\pi\alpha'^2 N g_s$

$$\Lambda \sim 1 + O(1/N)$$

above procedure

$$\langle T^i{}_i\rangle = \frac{N^2}{32\pi^2}\left\{-\left(1+\frac{c_1}{N}\right)E_4 + \left(1+\frac{c_2}{N}\right)C^2\right\}$$

c_1, c_2 can be determined once α, β, γ and Λ are known

note that $\Lambda = \frac{6}{l^2} + \frac{1}{l^4}(40\alpha + 8\beta + 4\gamma)$

solution of eqs of motion at $O(\delta^{-1})$
$\Rightarrow$ AdS_5 is a solution

To compare with field theory result need all $O(1/N)$ terms in 5-dim action : $\underbrace{\alpha, \beta, \gamma, \Lambda}$

$O(\partial^4)$ terms

$\{\alpha, \beta, \gamma, \Lambda\}$ are however not completely known.