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Generalized Conformal Symmetry and Supergravity - Matrix Theory Correspondence

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Main purpose:

Investigate the large N behavior of Matrix theory on the basis of the generalized AdS-CFT correspondence

- Brief review of the BFSS conjecture
- Generalized conformal symmetry and AdS-CFT correspondence for Matrix th.
- Predictions for 2-point correlators and its implications on the BFSS

Based mainly on

T. Sekino + T. Y. hep-th/9907029

also on some of earlier works

T. Y. hep-th/9902200

A. Jevicki, T. Kazama + T. Y. hep-th/9808...

hep-th/9810...

A. Jevicki + T. Y. hep-th/9804...

Units : l_s fixed

1. Brief review of the BFSS conjecture

M-theory

- $R = g_s l_s$
- $l_p = g_s^{1/3} l_s$

is described by the matrix model

$$H = R \,\mathrm{Tr}\, h = \frac{N}{P_-} \mathrm{Tr}\, h \quad (= -2P^-)$$

$$h = \frac{1}{2}\Pi^2 - \frac{1}{4 l_p^6}[X^i, X^j]^2 + \cdots$$

in the IMF

$$P_- = \frac{N}{R} \to \infty$$

- $\boxed{\text{For finite } \tfrac{\not{h}}{R} \Rightarrow N \to \infty}$

  - N = the number of D0 partons

- A different interpretation of IMF is possible for finite N — DLCQ

but $R \to 0$ or $g_s \to 0$ — Susskind, Seiberg, Sen

Assumption for DLCQ :

"kinematical" boost invariance

$$X^{\pm} \to e^{\pm\rho} X^{\pm} \;;\; R' \to e^{-\rho} R'$$

space-like compactification $R \to 0$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ ||

light-like compactification

$$x^- \to x^- + 2\pi R'$$

- higher string modes are decoupled if $E \ll l_s^{-1}$ or $r \ll l_s$

- Does this guarantee that the matrix model is consistent with classical SUGRA (IIA) at long-distance regime?

Not necessarily!

- characteristic length $\sim l_p = g_s^{1/3} l_s \ll l_s$
- classical SUGRA is good only for $r \gg l_s$

naively, gravity from closed string is decoupled.

However,

Perturbative computations in Matrix Th. tell us (good for $r \gg l_p$)

- 1-graviton exchange processes for general 2-body scattering
- self interaction of gravitons for 3-body scatering

are accurately described.

At present,

it is not clear:

- To what extent, is SUSY responsible for the agreement of Matrix th. with SUGRA?
- Is there any hidden symmetry in Matrix th.?

In the present formulations of matrix models of D-branes,

no manifest symmetry related to space-time general covariance

BFSS : large N scaling with fixed R

("dynamical" boost)

$$H = -2P^- = R\,\mathrm{Tr}\,h = \frac{N}{P_-}\,\mathrm{Tr}\,h$$

$$\Downarrow$$

$$P_+ P_- = N\,\mathrm{Tr}\,h$$: boost invariant

$$\boxed{\mathrm{Tr}\,h \sim O\left(\frac{1}{N}\right),\ H \sim O\left(\frac{R}{N}\right)}$$

or $t \sim O(N)$

- It is important to study the large time scaling property

$$t \to N t$$

- Holography $\Rightarrow$ the tansverse size of the system expands indefinitely as $N \to \infty$

  - Mean field approx.: $L \sim N^{1/9}\,l_p$
  - Matrix black hole : $r_s \sim N^{1/9}\,l_p$
  - eigenvalue distribution : $L \gtrsim N^{1/3}\,l_p$

Polchinski

- How can this be reconcilcled with Lorentz invariance ?

2. Generalized Conformal Symmetry and AdS-CFT correspondence

- Can we extrat any nontrivial information on the $N \to \infty$ behavior of Matrix th. from the AdS-CFT correspondence?

Basic assumptions for AdS-CFT correspondence (D3-brane)

- near horizon limit

$$1 + \frac{g_s N l_s^4}{r^4} \to \frac{g_s N l_s^4}{r^4} \gg 1$$

conformally non-invariant condition

- small curvature condition

$$(g_s N)^{1/4} \gg 1$$

$\Leftarrow N \to \infty$

conformally invariant condition

- small string coupling

$$g_s \ll 1$$

conformally invariant

- strong-coupling large N limit : $g_s N \gg 1$
- Single characteristic scale : $(g_s N)^{1/4} l_s$
- The correspondence is extended to the whole "conformal" region

$$r < (g_s N)^{1/4} l_s$$

  - crucial for extracting correlators for boundary theory by following the prescription of Gubser-Klebanov-Polyakov Witten

◇ Extension of these conditions to Dp-branes has been discussed

(Itzhaki - Maldacena - Sonnenshein - Yankielowicz
...)

but let us reconsider the problem from the viewpoint of generalized conformal symmetry.

Generalized Conformal Symmetry

- D0 background

$$ds^2 = -e^{-\frac{2}{3}\tilde{\Phi}}dt^2 + e^{\frac{2}{3}\tilde{\Phi}}dx_i^2$$

$$e^{\Phi} = g_s e^{\tilde{\Phi}}, \quad e^{\tilde{\Phi}} = \left(1 + \frac{q}{r^7}\right)^{3/4}$$

$$q = g_s N l_s^7$$

$$A_0 = -\frac{1}{g_s}\left(\frac{1}{1+\frac{q}{r^7}} - 1\right)$$

- near-horizon region : $1 + \frac{q}{r^7} \to \frac{q}{r^7}$

"quasi"- $AdS_2 \times S^8$

$$ds^2 = -\frac{r^2}{\rho^2}dt^2 + \frac{\rho^2}{r^2}(dr^2 + r^2 d\Omega_8^2)$$

$$\rho = \rho(r) = \left(\frac{q}{r^3}\right)^{1/4}$$: r-dependent radius

characterized by the "generalized" conformal symmetry" $SO(2.1)$

- scale :

$$r \to \lambda r, \quad t \to \lambda^{-1} t, \quad g_s \to \lambda^3 g_s$$

- special conf :

$$\delta r = 2\epsilon t r, \quad \delta t = -\epsilon\left(t^2 + \frac{2q}{5r^5}\right)$$

$$\delta g_s = 6\epsilon t g_s$$

- $\rho(r)$ is invariant!
- Matrix theory enjoys the same symmetry

Symmetry in what sense?

- generalized symmetry:

  $$\begin{array}{c} \delta(\text{dynamical variables}) \\ \parallel \\ \text{compensated by} \\ \parallel \\ \delta g_s \end{array}$$

- original (and deeper) motivation

  stringy uncertainty relation in space-time

  $$\Delta T \Delta X \gtrsim \ell_s^2$$

- contains 11D (kinematical) boost

  changing the engineering units of length

  $\Rightarrow$ keep X_i and ℓ_P $(\ell_s \to \lambda^{-1} \ell_s)$

  $t \to \lambda^{-2} t$, $R \to \lambda^2 R$

  kinematical boost

  $\Rightarrow$ Keep t $(\ell_s \to \lambda \ell_s)$

  $X_i \to \lambda^2 X_i$, $\ell_P \to \lambda^2 \ell_P$, $R \to \lambda^4 R$

  Seiberg-Sen's "~" transformation

Conditions for generalized AdS-CFT

- near horizon $r < (g_s N)^{1/7} l_s$

- non conformally invariant

- small curvature $r < (g_s N)^{1/3} l_s$

- conformally invariant

- small string coupling

$$g_s^{1/3} N^{1/7} l_s < r$$
$$\parallel$$
$$(g_s N)^{1/3} N^{-\frac{4}{21}} l_s \to 0 \quad N \to \infty$$

$g_s N$: fixed

- conformally invariant

By requiring that the whole near-horizon region is allowed for the correspondence

$$g_s \ll 1, \quad g_s N > 1$$

$$N \to \infty$$

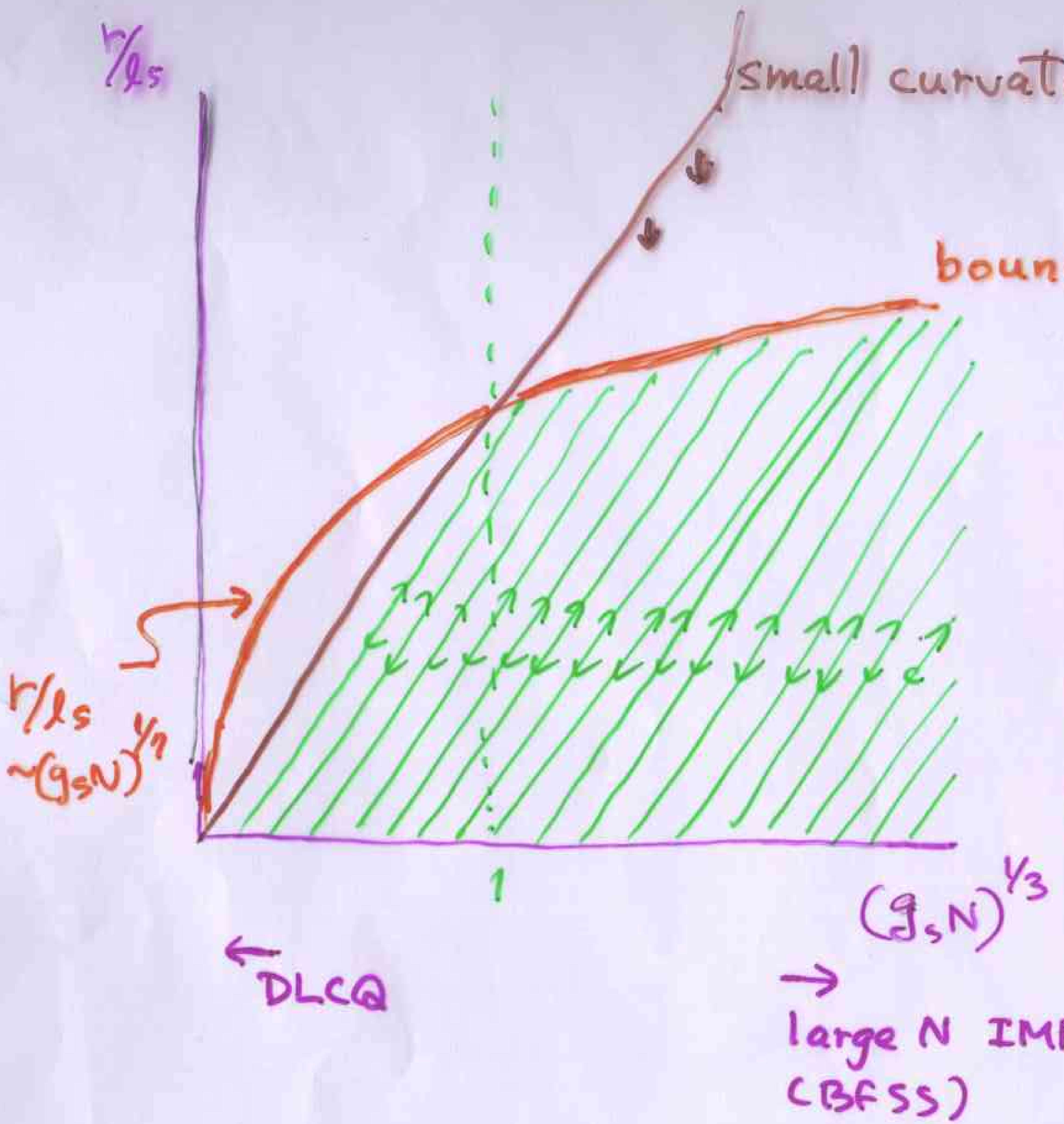
- $g_s N \to 1$ ⇒ connected with DLCQ region

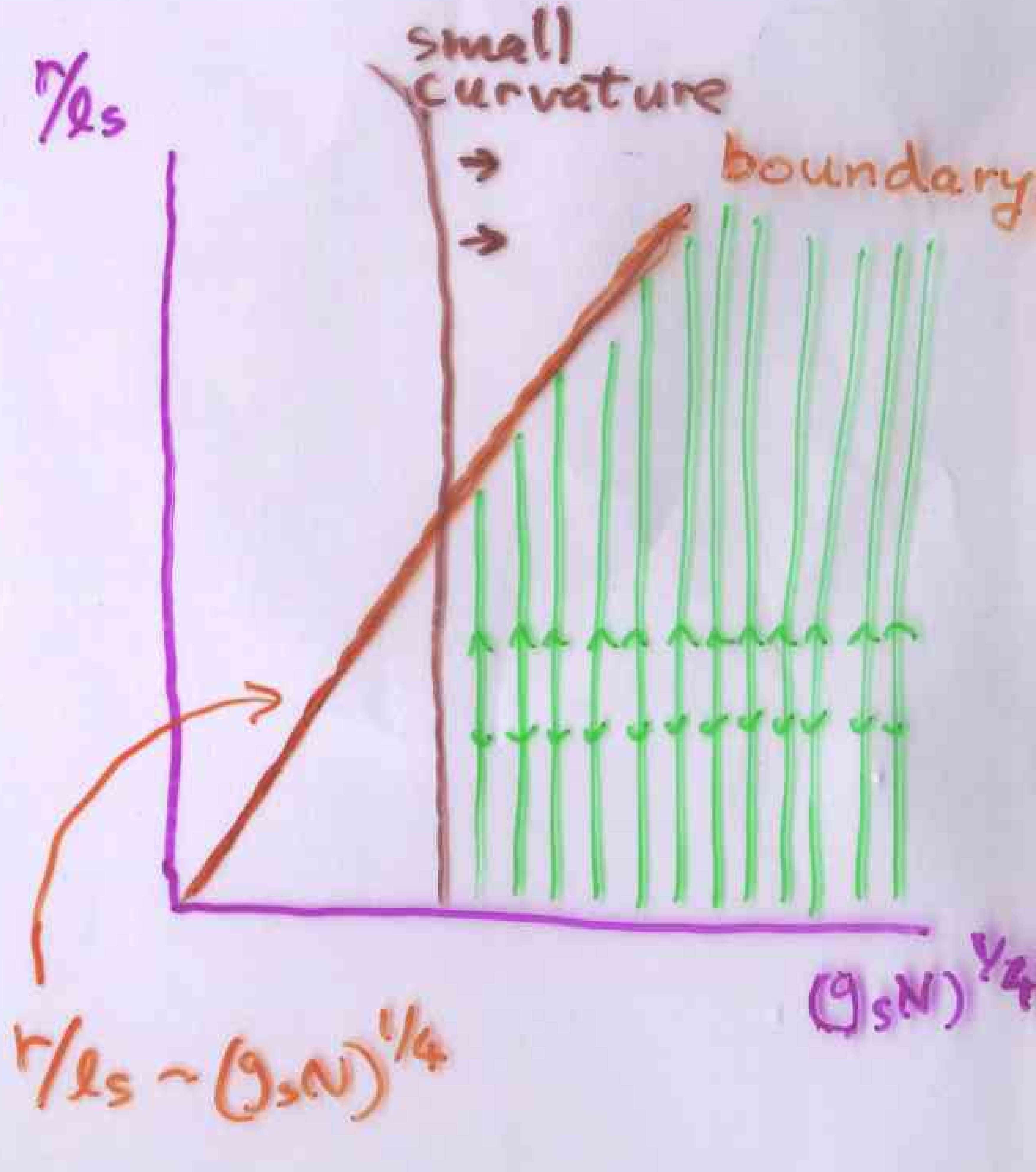
- $g_s N \gg 1$ ⇒ the near-horizon region allows to go beyond DLCQ

$$(g_s N)^{1/7} l_s > r > l_s$$

F-1

Ordinary AdS/CFT correspondence (D3-brane)





Generalized AdS/CFT correspondence for D0

or "Oblique" AdS/CFT correspondence

3. Prediction for 2-body correlators

- Strategy

  - establish the correspondence:
    bulk fields $\leftrightarrow$ Matrix operators
    using generalized conf. symmetry
    ($\cup$ kinematical boost invariance)

    - well defined conf. dimensions on both sides

      (M. Quantum Mechanics requires no ultraviolet regularizations)

  - extend the correspondence to the region

    $$\boxed{\begin{array}{l} g_s N \gg 1 \\ \quad \Downarrow \\ \text{BFSS} \quad g_s : \text{fixed} \quad N \to \infty \end{array}}$$

  - use the GKP prescription

$$e^{-S_{SUGRA}[\phi_0]} = \left\langle e^{\int d\tau \sum_I \phi_0^I \mathcal{O}^I} \right\rangle$$

$$\phi^I(x)\Big|_{x = x_b} = \phi_0^I$$

- The singular nature of the classical D0-background does not cause any harm

$$\text{e.g.}\quad \partial_\mu \sqrt{g}\, e^{-2\phi} g^{\mu\nu} \partial_\nu \sim r^8 \left(\partial_r^2 + 8 \frac{1}{r} \partial_r - \frac{q_0}{r^7} \partial_t^2 \right)$$

- General form of 2-point correlators: fixed by the generalized conf. symmetry

$$\boxed{\langle \mathcal{O}(\tau) \mathcal{O}(\tau') \rangle \sim \frac{1}{g_s^2 l_s^8} \frac{q^{(\Delta + \Delta_e + 7)/5}}{|\tau - \tau'|^{(7\Delta - 3\Delta_e + 9)/5}}}$$

- Δ : generalized conformal dimension of $\mathcal{O}$
- Δ_e: engineering dimension of $\mathcal{O}$
- $q = (g_s N) l_s^7$

It is convenient to normalise $\mathcal{O}$ s.t. $\Delta_e = -1$

- SUGRA side

bosonic

We have performed a complete harmonic analysis of the linearized perturbations around the D0 background

- the explicit diagonlization of the whole KK excitations
- the confirmation of the above formula for 2-point correlators
- the natural correspondence with the Matrix operators

◆ Notable features of the result

- fractional dependence on g_s and N

  never be reproduced by perturbation th. even for $g_s N \ll 1$.

- the angular-momentum independent part of the coupling and N dependence

$$\parallel$$

$$S \sim N^2 (g_s N)^{-\frac{3}{5}} (T_H l_s)^{\frac{9}{5}}$$

entropy of non extremal D0 solution

Correspondence:

SUGRA FIELDS $\leftrightarrow$ Matrix Operators

SUGRA FIELDS	Δ	MATRIX OPERATORS
$h^i{}_j$	$1+\frac{4}{7}l \ (l\geq 2)$	T_l^{ij}
h^0_i, h^z_i, A_i	$3+\frac{4}{7}l \ (l\geq 1)$	T_l^{-i}
	$-1+\frac{4}{7}l \ (l\geq 2)$	T_l^{+i}
ϕ, h^0_0, h^0_z, h^z_z, h^i_i, A_0, A_z	$5+\frac{4}{7}l \ (l\geq 0)$	T_l^{--}
	$1+\frac{4}{7}l \ (l\geq 1)$	T_l^{+-}
	$-3+\frac{4}{7}l \ (l\geq 2)$	T_l^{++}
B_{0i}, B_{zi}	$1+\frac{4}{7}l \ (l\geq 1)$	J_l^{+-i}
B_{ij}, A_{0ij}, A_{zij}	$3+\frac{4}{7}l \ (l\geq 1)$	J_l^{-ij}
	$-1+\frac{4}{7}l \ (l\geq 1)$	J_l^{+ij}
A_{ijk}	$1+\frac{4}{7}l \ (l\geq 1)$	J_l^{ijk}

MATRIX-MODEL OPERATORS

Identified by Kabat-Taylor using perturbation th.

$$T_\ell^{++} = \frac{1}{R} STr(\tilde{X}_{i_1} \tilde{X}_{i_2} \cdots \tilde{X}_{i_\ell} + \cdots)$$

$$T_\ell^{+i} = \frac{1}{R} STr(\dot{X}_i \tilde{X}_{i_1} \tilde{X}_{i_2} \cdots \tilde{X}_{i_\ell} + \cdots)$$

$$T_\ell^{+-} = \frac{1}{R} STr\left(\tfrac{1}{2}(\dot{X})^2 \tilde{X}_{i_1} \tilde{X}_{i_2} \cdots \tilde{X}_{i_\ell} + \cdots\right)$$

$$T_\ell^{ij} = \frac{1}{R} STr(\dot{X}_i \dot{X}_j \tilde{X}_{i_1} \tilde{X}_{i_2} \cdots \tilde{X}_{i_\ell} + \cdots)$$

$$\vdots$$

$$J_\ell^{+ij} = \frac{1}{6R} STr(F_{ij} \tilde{X}_{i_1} \tilde{X}_{i_2} \cdots \tilde{X}_{i_\ell} + \cdots)$$

$$J_\ell^{+-i} = \frac{1}{6R} STr(F_{ij} \dot{X}_j \tilde{X}_{i_1} \tilde{X}_{i_2} \cdots \tilde{X}_{i_\ell} + \cdots)$$

$$\vdots$$

$$\tilde{X}_i = \frac{X_i}{z} \qquad , \; F_{ij} = \frac{1}{\ell_s^2}[X_i, X_j]$$

Implications for $N\to\infty$ IMF ($\Rightarrow$ BFSS conjecture)

study the scaling behavior under

$$\tau-\tau' \to N(\tau-\tau')$$

$$\Downarrow \quad N\to\infty$$

- $$\langle \mathcal{O}(\tau)\mathcal{O}(\tau')\rangle \underset{N\to\infty}{\to} N^{2d_{IMF}} \langle \mathcal{O}(\tau)\mathcal{O}(\tau')\rangle$$

$$d_{IMF} = \underline{\left(1+\frac{1}{5}\right)}(n_+ - n_- - 1) - \underline{\frac{1}{5}\ell} - \underbrace{\frac{1}{7}\ell}_{\text{normalization}}$$

- scaling behavior is determined by the external 11D Lorentz indices,

  however

  anomalous!

  - transverse indices $\Rightarrow N^{-\frac{1}{5}}$
  - $\pm$ light-cone $\Rightarrow N^{\pm\frac{1}{5}}$

- In particular, a shrinking behavior with respect to the transverse directions

Does this contradict holography?

Not necessarily ?!

- large-time correlation with respect to angular momentum

  $\nparallel$

  direct measure of the extension of wave functions.

- It is very hard to believe that the dynamics at IMF could be consistent with SUGRA, unless the expansion caused by Holography is somehow "screened" or unobservable.

  → Might be a good news for the BFSS conjecture

- Here, we have to recall that the generalized AdS/CFT correspondence has the infrared cut off

$$r < r_B \sim q^{\frac{1}{7}} \propto \underline{\underline{N^{\frac{1}{7}}}}$$

$\Rightarrow$ Necessary to renormalize the system to larger sizes at fixed N.

We can estimate the real size ~~of~~ of the system in the BFSS limit by requiring the consistency with large N boost invariance for correlators

$$N^{-\frac{1}{5}} \times \frac{L}{r_B} = 1$$

$$\Downarrow$$

$$\boxed{L \sim N^{\frac{1}{3}+\epsilon}, \qquad \epsilon = \frac{1}{105}}$$

- very close to the bound proposed by Polchinski: $L \gtrsim N^{\frac{1}{3}}$

- To achieve the full consistency with boost invariance, the anomalous dimensions $\pm 1/5$ with respect to the light-cone indices must also be cancelled by the size-renormalization procedure.

Conclusions

- Generalized conformal symmetry

  ⇓

  "Oblique" AdS/CFT correspondence
  for Matrix Theory

- large N behavior of Matrix Theory
  (with special cut off)

  ⇓

  suggests a screening effect
  with respect to the transverse directions
  (in correlators)

  holography ⟺ Lorentz invariance in 11D

renaming problems:

- representation th. of generalized superconformal algebra
- computations of higher-point correlators
- computation of S-matrix

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