M THEORY DUALITY

AND

BPS - EXTENDED SUPERGRAVITY

Strings 2000

B. de Wit

based on

Abou-Zeid, dW, Lüst, Nicolai, hep-th/9908169

dW, Lüst, hep-th/9912225

in progress.

TOROIDAL COMPACTIFICATIONS* OF M-THEORY

$E_{d(d)}(\mathbb{Z})$ duality

$\supset SL(2,\mathbb{Z})$ of IIB

toroidal compactifications of 11-D SUGRA

$\rightarrow$ ONLY KK STATES

CENTRAL CHARGE $\longleftrightarrow$ EXTRA DIMENSIONS

$\uparrow$

INCOMPLETE

HENCE SUGRA IS <u>INCOMPLETE</u>

MOREOVER central charges <u>NOT</u> always in U-duality representations

INCLUDE MORE CENTRAL CHARGES

$\rightarrow$ ADDITIONAL DIMENSIONS

$\rightarrow$ $D>11$ SUGRA'S OF A PARTICULAR KIND

<u>BPS</u> <u>EXTENDED</u> <u>SUPERGRAVITY</u>

$\rightarrow$ 9 spacetime dimensions

* <u>note</u> $E_{d(d)}$ relevant for full D11 SUGRA?

(dW, Nicolai)

IIA/B T-DUALITY & IIA/M S-DUALITY

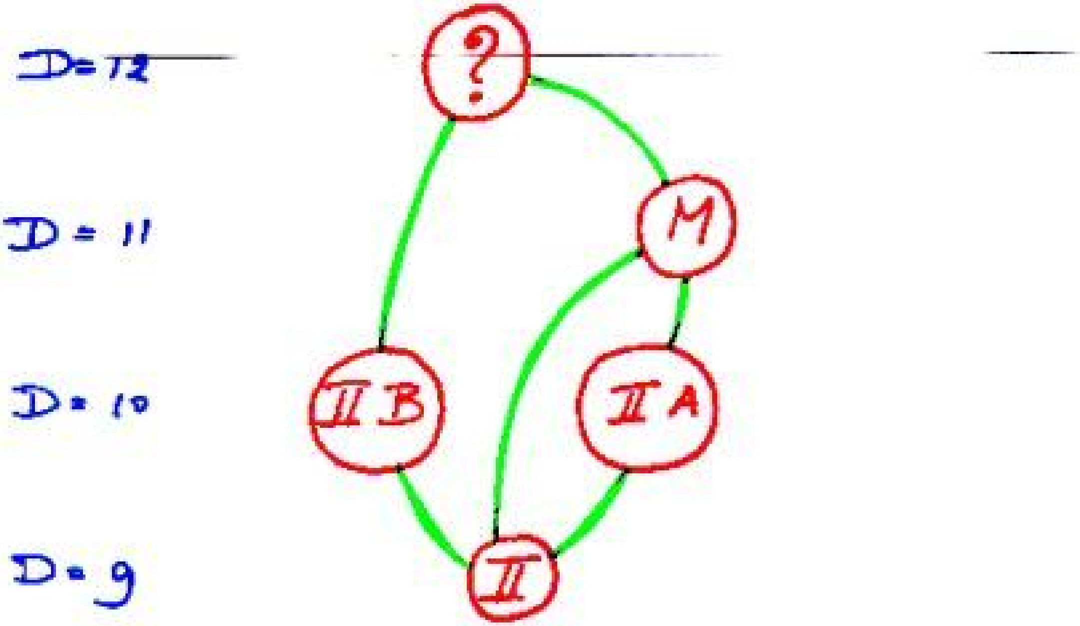
combines as a duality between

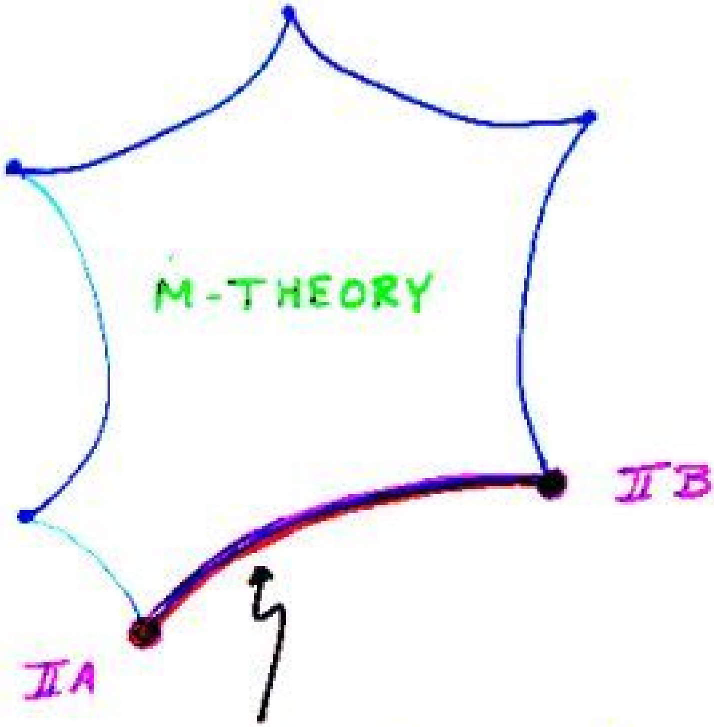
IIB $\mathbb{R}^9 \times S^1$

M-THEORY $\mathbb{R}^9 \times T^2$

11-D (SUPER) MEMBRANE PERSPECTIVE

(Schwarz)





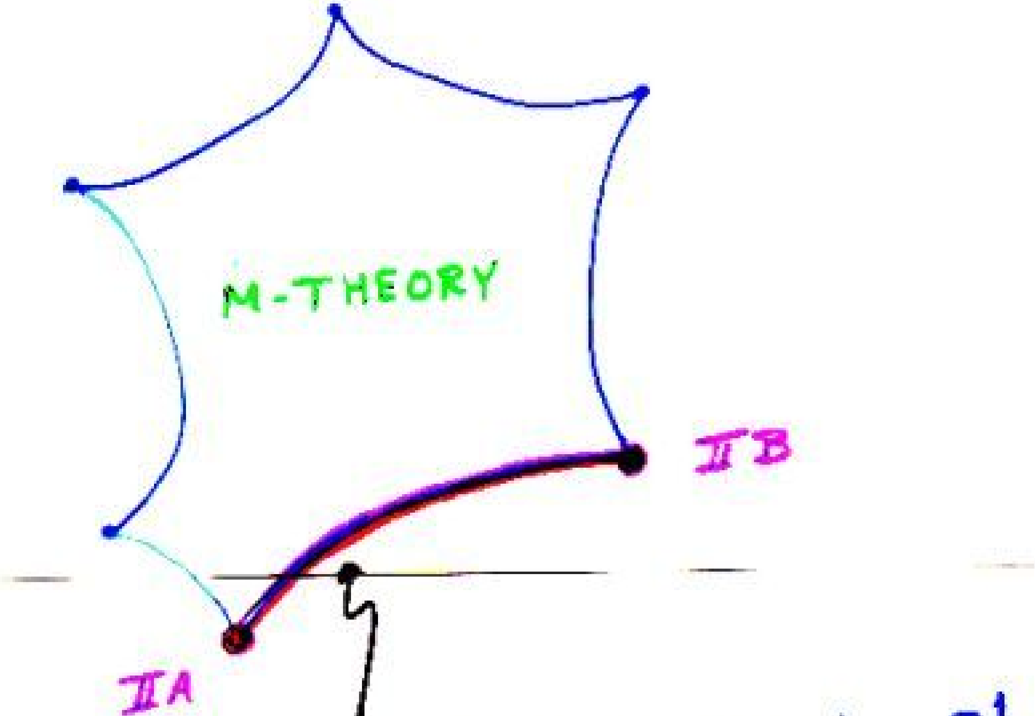
parametrized by the S^1 compactification radius

$$R \to \begin{cases} 0 & \text{decompactification to A/B} \\ \infty & \text{decompactification to B/A} \end{cases}$$

two *inequivalent* decompactification limits

Dine, Huet, Seiberg

Dai, Leigh, Polchinski



parametrized by the S^1
compactification radi

$R \to \begin{cases} 0 & \text{decompactification to A/B} \\ \infty & \text{decompactification to B/A} \end{cases}$

two inequivalent decompactification limit

Dine, Huet, Seiberg
Dai, Leigh, Polchinski

II A/B STRING THEORY

massive supermultiplets coincide

massless supermultiplets inequivalent

→ different SO(8) representations

IN D=9 S^1 compactification

massless states equivalent

(unique N=2 sugra)

BPS states are inequivalent

SO(8) helicity in D=10

↓ SO(7) helicity in D=9

↘ SO(8) spin in D=9

consequences for T duality ?

D = 9 N=2 SUSY ALGEBRA

$$\{Q^i_\alpha, Q^{+j}_\beta\} = (\not{P}\gamma^0)_{\alpha\beta}\,\delta^{ij} + Z^{ij}\,(i\gamma^0)_{\alpha\beta}$$

$i,j = 1,2 \qquad \alpha,\beta = 1,\dots,16$

SYMMETRIC REAL

(C = 1)

$$Z^{ij} \propto b\,\delta^{ij} + a\,(\cos\theta\,\sigma_3^{ij} + \sin\theta\,\sigma_1^{ij})$$

SO(2) SINGLET — SO(2) DOUBLET

THREE TYPES OF BPS MULTIPLETS

KKA : $M = |a| \quad b = 0$ — ½ BPS

KKB : $M = |b| \quad a = 0$ — ½ BPS

INTERMEDIATE : $a, b \neq 0$ — ¼ BPS

BPS mass formula

$$M_{BPS} = |a| + |b|$$

(linear!)

IIA/B STRING

$$Z^{ij} = \begin{cases} \frac{1}{2}(p_L + p_R)\,\delta^{ij} + \frac{1}{2}(p_L - p_R)\,\sigma_3^{ij} & \text{IIB} \\ \frac{1}{2}(p_L - p_R)\,\delta^{ij} + \frac{1}{2}(p_L + p_R)\,\sigma_3^{ij} & \text{IIA} \end{cases}$$

KKA STATES $\begin{cases} \text{IIA momentum} \\ \text{IIB winding} \end{cases}$

INEQUIVALENT

KKB STATES $\begin{cases} \text{IIB momentum} \\ \text{IIA winding} \end{cases}$

CONFIRMS T DUALITY

BPS Mass formula

$$M_{BPS} = \frac{1}{2}|p_L + p_R| + \frac{1}{2}|p_L - p_R|$$

$\frac{1}{2}$ BPS : $p_L = \pm p_R$

INTERMEDIATE : $M = |p_L|$ or $|p_R|$

i.e. right-moving BPS
left-moving oscillator

or vv

MEMBRANE INTERPRETATION

11-D SUSY ALGEBRA

$$\{Q, \bar{Q}\} = -i P_M \Gamma^M + \frac{1}{2} i Z_{MN} \Gamma^{MN}$$

→ 9-DIMENSIONS

$$Z^{ij} = Z_{9\,10}\, \delta^{ij} - (P_9\, \sigma_3{}^{ij} - P_{10}\, \sigma_1{}^{ij})$$

$$M_{BPS} = \sqrt{P_9^2 + P_{10}^2} + |Z_{9\,10}|$$

(linear!)

$$= \frac{1}{\sqrt{A\tau_2}} |q_1 + \tau q_2| + T_m A |p|$$

momentum #   wrapping #

KKA   KKB

A: area
T_m: membrane tension
τ : T^2 modular parameter

(Schwarz)

M-THEORY/STRING INTERPRETATION

$$M_{q_1 q_2 p} = \left| \frac{q_1}{R^A} + \frac{q_2}{g^A} \right| + |p| R^A$$

KK mom   D0 brane   winding

IIB: likewise

D=9 BPS EXTENDED SUPERGRAVITY

N=2 SUGRA COUPLED TO KKA + KKB

$$
\begin{tikzcd}
\boxed{\text{11 D SUGRA}} \arrow[dr, "T^2"] & & \boxed{\text{IIB SUGRA}} \arrow[dl, "S^1"] \\
& \boxed{\begin{array}{l}\text{D9 N=2 SUGRA} \\ \text{+KKA + KKB}\end{array}} &
\end{tikzcd}
$$

12-DIM. FIELD THEORY

NOT D=10 or 11 LORENTZ INVARIANCE

MASSLESS SECTOR $SO(1,1) \times SL(2,\mathbb{R})$

+ nonlinear SIGMA MODEL

KKA CHARGES $q_1 \; q_2$

KKB CHARGES p

break invariance to $SL(2,\mathbb{Z})$

perturbative phenomena

T duality incorporated

contains : D=9 N=2 SUGRA

but is not IIA IIB SUGRA

identical to : D=11 SUGRA

	D=11 SG	IIA	D=9 SG	IIB	
	$\hat{G}_{\mu\nu}$	$G_{\mu\nu}$	$g_{\mu\nu}$	$G_{\mu\nu}$	0
B	$\hat{A}_{\mu 9 10}$	$C_{\mu 9}$	B_{μ}	$G_{\mu 9}$	−4
A	$\hat{G}_{\mu 9}, \hat{G}_{\mu 10}$	$G_{\mu 9}, C_{\mu}$	A_{μ}^{α}	$A^{\alpha}_{\mu 9}$	3
	$\hat{A}_{\mu\nu 9}, \hat{A}_{\mu\nu 10}$	$C_{\mu\nu 9}, C_{\mu\nu}$	$A^{\alpha}_{\mu\nu}$	$A^{\alpha}_{\mu\nu}$	−1
	$\hat{A}_{\mu\nu\rho}$	$C_{\mu\nu\rho}$	$A_{\mu\nu\rho}$	$A_{\mu\nu\rho 9}$	2
	$\hat{G}_{9 10}, \hat{G}_{99}, \hat{G}_{10 10}$	ϕ, G_{99}, C_{9}	ϕ^{α} e^{σ}	ϕ^{α} G_{99}	0 7

SO(1,1)

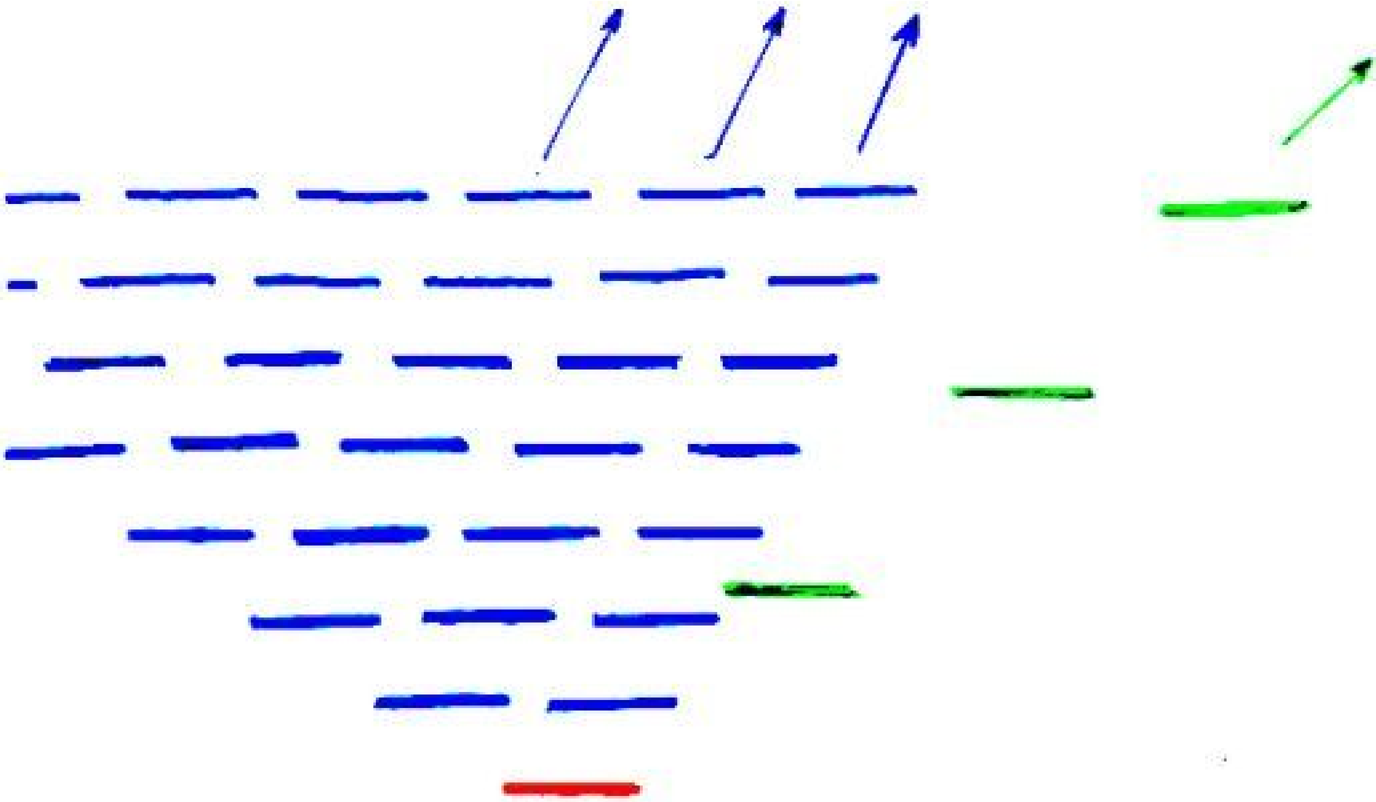
$$G_{99} = e^{\sigma} \qquad \det \hat{G}_{ij} = e^{-4\sigma/3}$$

A : graviphotons coupling to KKA charges

B : graviphoton coupling to KKB charge

Note : no truncation so far !!

$\langle \phi^{\alpha} \rangle \sim$ modular parameter of T^2



Mass spectrum dichotomic field theory

9 spacetime dimensions

$2+1 \neq 3$ extra dimensions

associated with
3 central charges

2 mass scales

BPS mass formula

$$M^{BPS}_{q_1 q_2 p} = m_{KKA}\, e^{3\sigma/7} |q_\alpha \phi^\alpha| + m_{KKB}\, e^{-4\sigma/7} |p|$$

→ 11D membrane formula

$$m^2_{KKA}\, m_{KKB} = T_m \qquad \text{(tension)}$$

BPS fields can transform linearly under $SL(2,\mathbb{Z})$, provided one keeps the local $SO(2)$ of the $SL(2,\mathbb{R})/SO(2)$ nonlinear sigma model

$SL(2,\mathbb{Z})$ does act (only) on the charges and T^2 indices.

(minimal coupling) inconsistencies ?

Geometric structure ?

application ?

encompasses a common sector of sugra / string / membrane theory

SUPERTRACES AND R^4-TERMS

INTEGRATE OVER KKA & KKB MULTIPLETS

$\rightarrow$ R^4 terms (Green, Gutperle, Vanhove)

$$A_4^{KKA+KKB} = \frac{1}{(2\pi)^9} \int d^4k \sum_{q_1 q_2 p} \frac{1}{\left(k^2 + M^2_{q_1, q_2 p}\right)^4}$$

$$= \frac{2}{3} \frac{A^{-1/2}}{(4\pi)^6} \sum_{q_1' q_2'} \frac{\tau_2^{3/2}}{|q_1' + \tau q_2'|^3}$$

$$+ \frac{4}{3} \frac{T_m A}{(4\pi)^6} \sum_{p'} \frac{1}{p'^2}$$

uv divergences $q_1' = q_2' = 0$ and $p' = 0$

drop $\rightarrow$ T-duality consistent result

(questions) (dW, Lüst)

proportionality constant

super-helicity traces B_8

KKA, KKB $B_n = 0$ for $n < 8$

INTERMEDIATE $B_n = 0$ for $n < 12$

LONG $B_n = 0$ for $n < 16$