# VERTEX OPERATORS FOR IIB STRINGS ON $AdS_3 \times S^3$ WITH RAMOND BACKGROUND

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# CFT / AdS STRING THEORY DUALITY

- COLLECTION OF P-BRANES WHOSE NEAR HORIZON GEOMETRY LOOKS LIKE $AdS_{p+2} \times S^{D-p-2}$ , $D = 10, 11$

BRANES IN STRING OR M-THEORY

$p = 2, 3, 5$

$\Rightarrow$ IIB SUPERSTRING ON $AdS_5$ IS DUAL TO $SU(N)$ gauge $CFT_4$

$$(g^2_{ym} N)^{\frac{1}{2}} (4\pi)^{\frac{1}{2}} = \frac{R^2_{sph}}{\alpha'}$$

't Hooft COUPLING $x = g^2_{ym} N$

$\Rightarrow$ (large $x$) $\equiv$ ($\alpha' \to 0$ SG LIMIT OF AdS STRING)

For 1.

$\left(\text{Large } N \to \infty\right) \equiv$ (AdS) string tree amplitude

and

$\left(\text{Large } x \equiv g_{YM}^2 N\right) \equiv$ $\alpha' \to 0$ lim of string tree amplitude

(strong coupling)

For 2.

$\left(\text{large } N \to \infty\right) \equiv$ string tree amplitude

and

$\left(\text{small } x \equiv g_{YM}^2 N\right) \equiv$ Large $\alpha'$ lim of string tree amplitude

weak coupling
$\Rightarrow$ perturbative YM, (QCD)

# EXAMPLES :

MALDACENA, KLEBANOV

CFT

D=6
N=(2,0)
(Tensor multiplet) CFT

5-BRANES

M-THEORY ON
$AdS_7 \times S^4$

$M^7 \times T^3$ $(\underline{14}, 5\,\underline{10}, \boxed{10}\,\underline{5})$ + fermions

D=4
N=4
SU(N) gauge CFT

3-BRANES

IIB SUPERSTRING ON $AdS_5 \times S^5$

$M^5 \times T^5$ $(\underline{5}, \boxed{27}\,\underline{3}, 42\,\underline{1})$ + f.

D=3 CFT

2-BRANES

M-THEORY ON
$AdS_4 \times S^7$

$M^4 \times T^6$ $(\pm 2, \boxed{28}(\pm 1), 70(0))$

D=2 CFT
N=(4,4)

1-BRANE
D1-D5 SYSTEM

IIB STRING ON
$AdS_3 \times S^3 \times M_4$

((3,3), 5(3,1))
((1,3), 5(1,1))

BVW 2D-SIGMA MODEL

# DIFFICULTIES IN FORMULATING STRINGS ON AdS:

- P-BRANE SOLUTIONS INCLUDE NON-ZERO FLUX OF RAMOND-RAMOND BOSON FIELDS

- IN RNS FORMALISM, R-R FIELDS COUPLE TO WORLDSHEET SPIN FIELDS

$\Rightarrow$ SUPERCONFORMAL WORLDSHEET SYMMETRY IS VIOLATED (AS WORLDSHEET SUPERCURRENTS ARE NOT LOCAL WITH RESPECT TO SPIN FIELDS).

1. RNS $\quad \partial x^\mu, \psi^\mu, b, c, \beta, \gamma$

- LORENTZ COVARIANT
- NOT SUPERSYMMETRY INVARIANT

2. GREEN-SCHWARZ $\quad \partial x^i, S_\alpha$

- NOT LORENTZ COVARIANT $\quad SO(8) \to U(4)$
- SUPER SYMMETRIC

3. BERKOVITS-VAFA

- LORENTZ COVARIANT FOR $D \leq 6$
- SUPERSYMMETRIC

$D=6$: $\quad \partial x^m, \theta^a, p^a, \rho, \sigma$

- RR VERTEX OPERATORS ARE ORDINARY (SUPER)CONFORMAL W.S. FIELDS
- FOR RR BACKGROUND, INTEGRATE OUT $p^a, \bar{p}^{\bar{a}}$ (BVW)

BERKOVITS - VAFA - WITTEN

$\sigma$ - model for IIB on $AdS_3 \times S^3 \times M_4$ :

- TARGET SPACE IS SUPERGROUP MANIFOLD $PSU(2|2)$

  COUPLED TO GHOSTS $\rho, \sigma$

- $x^p, \theta^a, \bar{\theta}^{\bar{a}}$ COORDINATES ON MANIFOLD

  $1 \le p \le 6$

  $1 \le a, \bar{a} \le 4$

- BERKOVITS - VAFA - WITTEN W.S. VARIABLES

- N=4 W.S. CONFORMAL GENERATORS

* • CONSTRAINT EQUATIONS FOR VERTEX OPERATORS $PSU(2|2)^2$ INVARIANT

* • SOLUTION OF CONSTRAINT EQ.'S, IDENTIFICATION WITH LINEARIZED AdS SUPERGRAVITY EQ.'S OF MOTION

* → VERTEX OPERATORS ARE EXACT IN $\alpha'$

[ COULD BE USED TO GO BEYOND SG APPROXIMATION AND COMPUTE $\alpha'$-CORRECTIONS OF CORRELATION FUNCTIONS ].

# FLAT CASE:

## N=4 SUPERVIRASORO ALGEBRA IN B-V-W VARIABLES:

$$T = -\tfrac{1}{2}\partial X^m \partial X_m - p_a \Theta^a - \tfrac{1}{2}\partial\rho\partial\rho - \tfrac{1}{2}\partial\sigma\partial\sigma + \partial^2(\rho+i\sigma) \quad \Big| \; c=0$$
$$+ T_c \quad \Big| \; c=6$$

$$G^+ = -e^{-2\rho-i\sigma} p^4 + \tfrac{i}{2} e^{-\rho} p_a p_b \partial x^{ab}$$
$$+ e^{i\sigma}\left(-\tfrac{1}{2}\partial X^m \partial X_m - p_a \partial\Theta^a - \tfrac{1}{2}\partial(\rho+i\sigma)\partial(\rho+i\sigma) + \tfrac{1}{2}\partial^2(\rho+i\sigma)\right)$$
$$+ G_c^+$$

$$G^- = e^{-i\sigma} + G_c^-$$

$$J = \partial(\rho+i\sigma) + J_c$$

$$\tilde{G}^+ = e^{iH_c+\rho} + e^{\rho+i\sigma}\tilde{G}_c^+$$

$$\tilde{G}^- = e^{-iH_c}\left(-e^{-3\rho-2i\sigma} p^4 + \tfrac{i}{2} e^{-2\rho-i\sigma} p_a p_b \partial x^{ab}\right)$$
$$+ e^{-\rho}\left(-\tfrac{1}{2}\partial X^m \partial X_m - p_a \partial\Theta^a - \tfrac{1}{2}\partial(\rho+i\sigma)\partial(\rho+i\sigma) + \tfrac{1}{2}\partial^2(\rho+i\sigma)\right)$$
$$+ e^{-\rho-i\sigma}\tilde{G}_c^-$$

$$J^+ = e^{\rho+i\sigma} J_c^+$$

$$J^- = e^{-\rho-i\sigma} J_c^-$$

WHERE $\quad i\partial H_c \equiv J_c$

$$e^{\pm iH_c} \equiv J_c^\pm$$

IN BVW VARIABLES:

$$\{ q_a^+, q_b^- \} = \tfrac{1}{2} \varepsilon_{abcd} P^{cd}$$

$$[ P_{ab}, P_{cd} ] = 0$$

$$[ P_{ab}, q_c^\pm ] = 0 \quad , \quad \{ q_a^+, q_b^+ \} = 0$$
$$\{ q_a^-, q_b^- \} = 0$$

WHERE

$$q_a^- \equiv \oint F_a(z)$$
$$q_a^+ \equiv \oint \left( e^{-\rho - i\sigma(z)} F_a(z) + i E_a(z) \right)$$
$$P^{ab} \equiv \oint \partial X_m(z) \, \sigma^{m\,ab}$$

$1 \le a \le 4$
$1 \le m \le 6$

8 odd gen.
6 even
(SUPERCHARGES + TRANSLATIONS)

$$F_a \equiv p_a$$
$$E_a \equiv \tfrac{1}{2} \varepsilon_{abcd} \, \theta^b \, \partial X_m \, \sigma^{mcd}$$

↑

IIB ON $(\text{MINKOWSKI})_6 \times M_4$
$D=6$, $N=1$ SUPERSYMMETRY ALGEBRA (LEFT + RIGHT)

↘

$$[ P_{ab}, P_{cd} ] = 0 \quad , \quad [ E_a, F_b \} = \tfrac{1}{2} \varepsilon_{abcd} P^{cd}$$
$$[ P_{ab}, F_c ] = 0 \quad \quad \{ E_a, E_b \} = [ F_a, F_b \} = 0$$
$$[ P_{ab}, E_c ] = 0$$

$$\left( \text{WHERE } P_{ab} \equiv \delta_{ac} \delta_{bd} P^{cd} \right)$$

• $V(z,\bar z) = \sum_{n,m} e^{m(i\sigma+\rho)+n(i\bar\sigma+\bar\rho)} V_{m,n}(x,\theta,\bar\theta)$

$(\sigma, \rho, x^p, \theta^a, \bar\theta^{\bar a})$ : WORLSHEET FIELDS

• N=4 CONSTRAINTS (LEFT)

$G_0^- V = \tilde G_0^- V = \bar G_0^- V = \bar{\tilde G}_0^- V = T_0 V = \bar T_0 V = 0$

$J_0 V = \bar J_0 V = 0$ , $G_0^+ \tilde G_0^+ V = \bar G_0^+ \bar{\tilde G}_0^+ V = 0$

• PHYSICAL DEGREES OF FREEDOM :

$V_{n,m}$ (for $n \le -2$ or $m \ge 2$) ~ GAUGED TO 0.

$V_{n,m}$ (for $-1 \le m,n \le 1$) SATISFY :

• $\nabla^4 V_{1,n} = \nabla_a \nabla_b \sigma^{ab\,p} \frac{\partial}{\partial x^p} V_{1,n} = 0$

$\frac{1}{6} \varepsilon^{abcd} \nabla_b \nabla_c \nabla_d V_{1,n} = -i \nabla_b \partial^{ab} V_{0,n}$

$\nabla_a \nabla_b V_{0,n} - \frac{i}{2} \varepsilon_{abcd} \partial^{cd} V_{-1,n} = 0$ ; $\nabla_a V_{-1,n} = 0$

$\partial^p \partial_p V_{m,n} = 0$ $\left(\nabla_a = \frac{d}{d\theta^a}\right)$

FLAT SPACE CONSTRAINT EQUATIONS

6 EVEN GENERATORS ~ $SU(2)\times SU(2) \sim SO(4)$
8 ODD "

$$[K_{ab}, K_{cd}] = \delta_{ac} K_{bd} - \delta_{ad} K_{bc} - \delta_{bc} K_{ad} + \delta_{bd} K_{ac}$$
$$[K_{ab}, E_c] = \delta_{ac} E_b - \delta_{bc} E_a$$
$$[K_{ab}, F_c] = \delta_{ac} F_b - \delta_{bc} F_a$$
$$\{E_a, F_b\} = \tfrac{1}{2} \varepsilon_{abcd} K^{cd}$$

$1 \le a \le 4$

$\{E_a, E_b\} = \{F_a, F_b\} = 0$ $K_{ab} = -K_{ba}$

SYMMETRY OF IIB ON $\underline{AdS_3 \times S^3} \times M_4$

~ $SO(4)$ GROUP MANIFOLD

$\Rightarrow$ ISOMETRIES : $K_{ab}, \bar{K}_{\bar{a}\bar{b}}$

$$q_a^- = \oint F_a(z)$$
$$q_a^+ = \oint \left(e^{-\rho - i\sigma(z)} F_a(z) + i E_a(z)\right)$$
$$K^{ab} = \oint K^{ab}(z)$$

$$\{q_a^+, q_b^-\} = \tfrac{1}{2} \varepsilon_{abcd} K^{cd}$$
$$[K_{ab}, K_{cd}] = \delta_{ac} K_{bd} - \delta_{ad} K_{bc} - \delta_{bc} K_{ad} + \delta_{bd} K_{ac}$$
$$[K_{ab}, q_c^+] = \delta_{ac} q_b^+ - \delta_{bc} q_a^+$$
$$[K_{ab}, q_c^-] = \delta_{ac} q_b^- - \delta_{bc} q_c^-$$

$\{q_a^+, q_b^+\} = 0$
$\{q_a^-, q_b^-\} = 0$

SPACETIME SUPERSYMMETRY GROUP IS $PSU(2|2) \times PSU(2|2)$

- ACTS BY LEFT AND RIGHT MULTIPLICATION ON THE $PSU(2|2)$ MANIFOLD:

$$g \to a\, g\, b^{-1}$$

WHERE $a, b \in PSU(2|2)$ ARE SYMMETRY GROUP ELEMENTS

AND $g$ IS A $PSU(2|2)$-VALUED FIELD:

$$g = e^{\theta^a f_a} \underbrace{e^{\frac{1}{2}\chi^{ab} t_{ab}}}_{"h} e^{\bar{\theta}^a e_a}.$$

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$$F_a\, g = \frac{d}{d\theta^a}\, g \qquad = f_a g$$

$$E_a\, g = \left[\tfrac{1}{2}\varepsilon_{abcd}\,\theta^b\left(t_L{}^{cd} - \theta^c \frac{d}{d\theta_d}\right) + h_{a\bar{b}}\frac{d}{d\bar{\theta}_{\bar{b}}}\right] g \qquad = g\, e_a g$$

$$K_{ab}\, g = \left(-\theta_a \frac{d}{d\theta_b} + \theta_b \frac{d}{d\theta_a} + t_{L\,ab}\right) g \qquad = -t_{ab} g$$

$$t_{ab\,L}\, g = e^{\theta^a f_a}\left(-t_{ab}\, e^{\frac{1}{2}\chi^{ab} t_{ab}}\right) e^{\bar{\theta}^a e_a}$$

↑ INVARIANT DERIVATIVE ON GROUP MANIFOLD

$$\bar{E}_{\bar{a}}\, g = \frac{d}{d\bar{\theta}^{\bar{a}}} g \qquad = g\, e_{\bar{a}}$$

$$\bar{F}_{\bar{a}}\, g = \left( \tfrac{1}{2} \varepsilon_{\bar{a}\bar{b}\bar{c}\bar{d}}\, \bar{\theta}^{\bar{b}} \left( -t_R^{\bar{c}\bar{d}} + \bar{\theta}^{\bar{c}} \frac{d}{d\bar{\theta}_{\bar{d}}} \right) + h^{-1}_{\bar{a}b} \frac{d}{d\theta_b} \right) g$$

$$= g\, f_{\bar{a}}$$

$$\bar{K}_{\bar{a}\bar{b}}\, g = \left( -\bar{\theta}_{\bar{a}} \frac{d}{d\bar{\theta}^{\bar{b}}} + \bar{\theta}_{\bar{b}} \frac{d}{d\bar{\theta}^{\bar{a}}} + t_{\bar{a}\bar{b}\,R} \right) g = g\, t_{\bar{a}\bar{b}}$$

QUADRATIC CASIMIR OPERATOR OF THE SUPERGROUP PSU(2/2) IS

$$F_a E^a + \tfrac{1}{8} \varepsilon_{abcd} K^{ab} K^{cd}$$

$$= \bar{F}_{\bar{a}} \bar{E}^{\bar{a}} + \tfrac{1}{8} \varepsilon_{\bar{a}\bar{b}\bar{c}\bar{d}} \bar{K}^{\bar{a}\bar{b}} K^{\bar{c}\bar{d}}$$

$$= h_{a\bar{b}} \frac{d}{d\theta_a} \frac{d}{d\bar{\theta}_{\bar{b}}} + \tfrac{1}{8} \varepsilon_{abcd}\, t_L^{ab} t_L^{cd}$$

$$= h^{-1}_{\bar{b}a} \frac{d}{d\theta_a} \frac{d}{d\bar{\theta}_{\bar{b}}} + \tfrac{1}{8} \varepsilon_{\bar{a}\bar{b}\bar{c}\bar{d}}\, t_R^{\bar{a}\bar{b}} t_R^{\bar{c}\bar{d}}$$

$\doteq$ SUPER LAPLACIAN

1. GENERALIZE FLAT SPACE CONSTRAINT EQ.'S TO BE INVARIANT UNDER $V \to V + \Delta V$ :

$$[q_a^{\pm}, V_{m,n}] = \Delta_a^{\pm} V_{m,n}$$
$$[K_{ab}, V_{m,n}] = \Delta_{ab} V_{m,n},$$

FOR $q_a^{\pm}(E_a, F_a)$, $K_{ab} \in$ SU(2|2)

INSTEAD OF D=6 SUPER POINCARE

2. $$F^4 V_{1,n} = F_a F_b K^{ab} V_{1,n} = 0$$
$$\tfrac{1}{6} \varepsilon_{abcd} F_b F_c F_d V_{1,n} = \left( -i F_b K^{ab} V_{0,n} + 2i F^a V_{0,n} - E^a V_{-1,n} \right)$$
$$F_a F_b V_{0,n} - \tfrac{i}{2} \varepsilon_{abcd} K^{cd} V_{-1,n} = 0 \ ; \ F_a V_{-1,n} = 0$$

ZERO LAPLACIAN CONDITION:

$$\left( F_a E_a + \tfrac{1}{2} \varepsilon_{abcd} K^{ab} K^{cd} \right) V_{n,m} = 0$$

$\Rightarrow F^4 F_a V_{1,n} = 0$, ETC. $\qquad \Delta_a^- V_{m,n} = F_a V_{m,n}$

INFINITESIMAL TRANSFORMATIONS :

$$\Delta^-_a V_{m,n} = F_a V_{m,n} \quad ; \quad \Delta_{ab} V_{m,n} = K_{ab} V_{m,n}$$

$$\Delta^+_a V_{1,n} = E_a V_{1,n}$$

$$\Delta^+_a V_{0,n} = E_a V_{0,n} + i F_a V_{1,n}$$

$$\Delta^+_a V_{-1,n} = E_a V_{-1,n} - i F_a V_{0,n}$$

OF GROUP MANIFOLD:

TO EVALUATE CONSTRAINTS: $\underline{\underline{t_{ab\,L}}}$

CONVERT INVARIANT DERIVATIVE
ON GROUP MANIFOLD $t_{ab\,L}$

INTO COVARIANT DERIVATIVE:

LET $\tau_L^{ab} = -\sigma^{p\,ab} D_p$

$$t_L^{ab}\varphi = -\sigma^{p\,ab} D_p \varphi = \mathcal{J}_L^{ab}\varphi$$

$$t_L^{ab} V_c = \mathcal{J}_L^{ab} V_c + \tfrac{1}{2}\delta^a{}_c\,\delta^{bd} V_d - \tfrac{1}{2}\delta^b{}_c\,\delta^{ad} V_d$$

# LEFT AND RIGHT-INVARIANT VIELBEINS

FOR SO(4):

$$e^{mab} = -\sigma^{mab} \quad ; \quad e^{m\bar{a}\bar{b}} = \sigma^{m\bar{a}\bar{b}}$$

WHERE $\quad e^{mab} e_m{}^{cd} = \varepsilon^{abcd}$

$$e^{mab} e^{ncd} \varepsilon_{abcd} = 4\bar{g}^{mn}$$

COVARIANT DERIVATIVE:

$$D_m V_{gh} = \partial_m V_{gh} - \frac{1}{4} \underset{\substack{\uparrow \\ SO(4) \\ \text{s.c.}}}{f_{gh}}{}^{cd}{}_{ef}\, e_m{}^{ef} V_{cd}$$

$$t_L^{ab} V_{gh} = -\sigma^{mab} \partial_m V_{gh} = e^{mab} \partial_m V_{gh}$$

$$J_L^{ab} V_{gh} = -\sigma^{mab} D_m V_{gh} = e^{mab} D_m V_{gh}$$

$$\Rightarrow t_L^{ab} V_{gh} = J_L^{ab} V_{gh} + \frac{1}{2} f_{gh}{}^{cdab} V_{cd}$$

# PHYSICAL DEGREES OF FREEDOM:

$$V_{1,1}(x, \theta, \bar\theta)$$

$$\left.\begin{array}{l} V_{10}, V_{1,-1} \\ V_{01}, V_{00}, V_{0,-1} \\ V_{-11}, V_{-10}, V_{-1-1} \end{array}\right\} \text{GAUGED TO ZERO}$$

$$\Rightarrow V_{11}(x^p, \theta^a, \bar\theta^{\bar a})$$

$$\begin{aligned}
&= \theta^a \bar\theta^{\bar a} V^{--}_{a\bar a}(x) \\
&+ \theta^a \theta^b \bar\theta^{\bar a} \sigma^m_{ab} \bar\xi^-_{m\bar a}(x) \\
&+ \theta^a \bar\theta^{\bar a} \bar\theta^{\bar b} \sigma^m_{\bar a\bar b} \xi^-_{ma}(x) \\
&+ \theta^a \theta^b \bar\theta^{\bar a} \bar\theta^{\bar b} \sigma^m_{ab} \sigma^n_{\bar a\bar b} (g_{mn}(x) + b_{mn}(x) + \bar g_{mn} \varphi(x)) \\
&+ (\theta^a \bar\theta^3_{\bar a}) A_a^{-+\bar a}(x) \\
&+ (\theta^3_a \bar\theta^{\bar a}) A_{\bar a}^{+-a}(x) \\
&+ \theta^a \theta^b \bar\theta^3_{\bar a} \sigma^m_{ab} \bar\chi_m^{+\bar a}(x) \\
&+ \theta^{3a} \bar\theta^{\bar a} \bar\theta^{\bar b} \sigma^m_{\bar a\bar b} \chi_m^{+a}(x) \\
&+ \theta^{3a} \bar\theta^{3\bar a} F^{++a\bar a}(x)
\end{aligned}$$

# MASSLESS VERTEX OPERATOR $V_{11}$
# (COMPACTIFICATION INDEPENDENT FIELDS)

$D=6$, $N=(2,0)$

SG: $(3,3)+5(3,1)+4(3,2)$

1 TENSOR MULTIPLET: $(1,3)+5(1,1)+4(1,2)$

- $\bar{\xi}^{--}_{ma}(x)$, $\xi^{-}_{ma}(x)$, $\bar{\chi}^{+a}_{m}(x)$, $\chi^{+a}_{m}(x)$ — 4 GRAVITINOS, 4 SPINORS

- $V^{--}_{a\bar{a}}(x)$, $F^{++a\bar{a}}(x)$, $A^{-+\bar{a}}{}_{a}(x)$, $A^{+-a}{}_{\bar{a}}(x)$

  4 SELF-DUAL TENSORS
  +4 SCALARS

- $g_{mn}(x)$, $b_{mn}(x)$, $\varphi(x)$ — GRAVITON, 1 S-D TENSOR, 1 ANTI S-D TENSOR, 1 SCALAR

FLAT SPACE:

CONSTRAINT EQ'S FOR VERTEX OPERATORS IMPLY

$$\partial^m g_{mn} = -\partial_n \varphi \quad , \quad \partial^m b_{mn} = 0$$

$$\partial^m \chi_m^{\pm b} = 0 = \partial^m \bar{\chi}_m^{\pm b}$$

$$\partial_{ab} \chi_m^{\pm b} = 0 = \partial_{\bar a \bar b} \bar{\chi}_m^{\pm \bar b}$$

$$\partial_{cb} F^{\pm\pm\, b\bar a} = 0 = \partial_{\bar c \bar b} F^{\pm\pm\, \bar b \bar a} = 0$$

MASSLESS $\partial^p \partial_p$ (ALL FIELDS) $= 0$.

EQUIVALENT TO

D=6, N=(2,0) SUPERGRAVITY
WITH 1 SG + 1 TENSOR MULTIPLET
EXPANDED AROUND $\bar{\eta}_{mn}$
(L. ROMANS 1986)

CONSTRAINT EQ'S FOR VERTEX OPERATORS IMPLY

$$D^m g_{mn} = -D_n \varphi - \frac{1}{2} (\sigma^m \sigma_n \sigma^r)_{ab} \delta^{ab} b_{mr}$$

$$D^m b_{mn} = 0$$

$$\begin{aligned}
\frac{1}{2} D^p D_p G_{rs} = & -\frac{1}{2} (\sigma_r \sigma^p \sigma^q)_{ab} \delta^{ab} D_p G_{qs} \\
& + \frac{1}{2} (\sigma_s \sigma^p \sigma^q)_{ab} \delta^{ab} D_p G_{rq} \\
& - \bar{R}_{\tau r s \lambda} G^{\tau \lambda} \\
& - \frac{1}{2} \bar{R}_r{}^\tau G_{\tau s} - \frac{1}{2} \bar{R}_s{}^\tau G_{r \tau} \\
& + \frac{1}{4} F^{++gh}_{asy} (\sigma_r{}^{ab} \sigma_s{}^{ef} \delta_{ah} \delta_{be} \delta_{gf}) \\
& + \frac{1}{4} F^{++gh}_{sym} (\sigma_{rga} \sigma_{shb} \delta^{ab})
\end{aligned}$$

$$G_{rs} \equiv g_{rs} + b_{rs} + \bar{g}_{rs} \varphi$$

$\equiv$ IDENTIFY AS LINEARIZED L. ROMANS $D=6$, $N=(2,0)$ SG + 1 TENSOR EXPANDED AROUND $\bar{g}_{mn}$ = METRIC OF $AdS_3 \times S^3$.

# SUMMARY

- BVW $\sigma$-model with supergroup manifold target space FOR TYPE IIB on $AdS_3 \times S_3 \times K_3$

* GENERALIZED THE VERTEX OP'S CONSTRAINT EQUATIONS FOR $AdS_3 \times S_3$ FROM $R_6$

* SOLVED THE CONSTRAINT EQ'S TO GET THE VERTEX OP'S WHICH ARE EXACT IN $\alpha'$.

* POLARIZATION VECTORS SATISFIED $D=6$, $N=(2,0)$ supergravity (+ tensor) LINEARIZED AROUND $AdS_3 \times S^3$.

# OPEN PROBLEM

A DEEPER UNDERSTANDING OF RAMOND-RAMOND FIELDS

- DIRAC QUANTIZATION IS HANDLED DIFFERENTLY
  K-THEORY

- SELF-DUAL NATURE
  NON-LAGRANGIAN FIELDS
  PARTITION FUNCTIONS DEPEND ON SPIN STRUCTURE

- STRING EXTENSIONS OF GAUGED SUPERGRAVITIES INVOLVE BACKGROUND R-R FIELDS