SUPERCONFORMAL INTERPRETATION OF BPS STATES IN AdS GEOMETRIES

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Summary of the talk

1) Motivation of the study

2) Non supersymmetric conformal fields in "d" dimensions (or states in AdS_{d+1}) ($d \geq 3$) and unitarity bounds. (reducible Harish-Chandra modules)

3) Superconformal fields in "d" dimensions and "all" short (unitary) representations of superconformal algebras (generalization of "chiral" superfields of $N=1$, $d=4$ supersymmetry ($N=2$ sugra in AdS_5)

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1) Motivation:

1) Study of conformal invariant quantum field theories. OPE expansions. n-point functions

2) AdS_{d+1}/CFT_d correspondence:

Short multiplets $\to$ BPS states in AdS

Non renormalization theorems are a consequence of the reduction phenomenon

K-K states of AdS compactifications with maximal supersymmetry (32 spinor charges) are necessarily $\frac{1}{2}$ BPS.

~~Not true for compactification with~~ lower supersymmetry (example: **IIB on $AdS_5 \times T_{11}$**)

Some multiparticle states can be $< \frac{1}{2}$ BPS (in $AdS_5 \times S_5$) implying non renormalization of certain (multi trace) conformal primary operators

Example:

double trace

$$\mathrm{Tr}(\phi^2)_{20} \times \mathrm{Tr}(\phi^2)_{20} \supset (\mathrm{Tr}\phi^2\,\mathrm{Tr}\phi^2)_{105} +$$

$\frac{1}{2}$ BPS $\qquad\qquad$ $\frac{1}{2}$ BPS (max spin 2)

$$+ (\mathrm{Tr}\phi^2\,\mathrm{Tr}\phi^2)_{84} + (\mathrm{Tr}\phi^2\,\mathrm{Tr}\phi^2)_{20} + (\mathrm{Tr}\phi^2\,\mathrm{Tr}\phi^2)_{1}$$

$\frac{1}{4}$ BPS (max spin 3) $\quad$ non-BPS $\qquad$ non-BPS

From the study of 4-point functions of ($\frac{1}{2}$ BPS) single trace operators (dual to K-K states)

$$\gamma_{105} = \gamma_{84} = 0 \qquad \gamma_{20}\,,\ \gamma_{1} \neq 0$$

in perturbation theory (Bianchi, Kovacs, Rossi, Stanev; D'Hoker, Freedman, Mathur, Matusis, Rastelli; Chalmers, Schalm; Smith)

$$\mathrm{Tr}(\phi^2)\,\mathrm{Tr}(\phi^2)\,\mathrm{Tr}(\phi^2) \supset (\mathrm{Tr}\phi^2\,\mathrm{Tr}\phi^2\,\mathrm{Tr}\phi^2)_{35} + \text{etc}$$

$\frac{1}{8}$ BPS (max spin $7/2$)

$$+ \tfrac{1}{2}\ \text{BPS} + \tfrac{1}{4}\ \text{BPS} + \text{non BPS}$$

Classification of BPS AdS black-holes in AdS_4 and AdS_5 preserving different fraction of supersymmetry

(AdS₄ black-holes with different BPS fraction of susy studied by Duff, Liu)

One expects the same fraction of supersymmetries as in asymptotically flat black-holes but with quantum numbers dictated by the appropriate reps. of the superconformal algebra

(This is indeed what happens)

Curious facts about higher dimensions:

1) Unlike $D=4$ (where every massless UIR (Poincaré) can be extended to the conformal group, in higher D this is not true.
(Enright, Howe, Wallach; Siegel, Minwalla Angelopoulos, Laoues; Fronsdal, S.F.)
This fact is closely related to the fact that $SO(D-2)$ becomes non-abelian

Result: $D=2n$ $(SO(2,2n))$
weights of $SO(2n)$: $(SO(2)\times SO(2n))$

$$w_1 = w_2 = \ldots\ldots = |w_n| \qquad E_0 = n-1+w_1$$

(infinitely many massless conformal fields labeled by a "single" quantum number),

$D=2n+1$ $(SO(2,2n+1)$

$$\vec{w}=0 \quad E_0 = n-\tfrac{1}{2} \;, \quad w_1 = \ldots w_n = \tfrac{1}{2} \quad E_0 = n$$

singletons of $AdS_{2n+1\,(2n+2)}$
(even AdS Di and Rac of Flato, Fronsdal)

$SO(2,d) \supset SO(2) \times SO(d)$

$\downarrow$

(rank $n+1$) E_0 $d = 2n$

$d = 2n+1$ (rank n)

$\mathcal{L}ie\ SO(2,d) \rightarrow L^+ + L^0 + L^- = \mathcal{L}_{SO(2,d)}$

three grading

Harish-Chandra module:

UIR of L^0, $|\Omega\rangle$, $L^-|\Omega\rangle = 0$

$|\Omega\rangle, L^+|\Omega\rangle, \dots L^+ \dots L^+|\Omega\rangle$

$|\Omega\rangle \Longrightarrow D(E_0, m_1, \dots m_n)$

weights of a UIR of $SO(d)$

For E_0 large enough the module is unitary.

Find all values of E_0 for which the module becomes reducible (null vectors)

Solved by mathematicians for a certain class of non compact groups having a 3-grading decomp.

$SO(2,d)$, $SO^*(2n)$, $Sp(2n,R)$, $SU(p,q)$ etc.

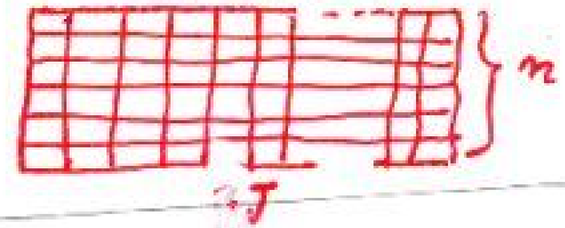
For $D=2n$ the next bosonic massless conformal field (other than the scalar) is a (self-dual n odd) antisymmetric tensor field (Fronsdal, SF) (Theories of tensionless $n-2$ self-dual branes!)

(i.e. IIB supergravity in $D=10$ contains a massless $SO(10,2)$ field, the self-dual five form field strength!) Seiberg Witten

(Bosonic singletons of AdS_{2n+1}):

$E_0 = n + J - 1$

Young-Tableaux



The classes of degenerate Harish-Chandra modules is essentially equal to n (including the above) ($n+1$ = Rank of conformal algebra)

(Enright-Howe-Wallach, Fronsdal SF)

For special cases (bosons only):

Metsaev

Brink, Metsaev, Vasiliev

Unitary bounds for $SO(2,d)$

(degenerate Harish-Chandra modules)

$d=2n$

$E_0 \geq d-1+w_1-p \qquad 1 \leq p \leq n \left(\frac{d}{2}\right)$

$(\vec{w}=0,\ E_0=0$ or $E_0=n-1)$

$w_1 \geq w_2 \geq \dots \geq |w_n|$

$\rightarrow w_1 = w_2 = \dots w_p > w_{p+1}$

Singletons: $p=n \qquad \frac{d}{2}-1+w_1$

diff forms: $w_1=1$ ($p-1$ branes) $d-p$ (zero center modules)

tensor currents: $p=1 \qquad d-2+w_1$

$d=2n+1$ as before $+ (\vec{w}=0;\ E_0=0,\ E_0=n-\frac{1}{2};\ (\vec{w})_i=\frac{1}{2},\ E_0=n)$

(Enright, Howe, Wallach; Fronsdal, S.F)

(Siegel, Minwalla, Angelopoulos, Laoues)

Example: $SO^*(8) \simeq SO(2,6)$

$2w_1 = a_1+a_3+2a_2\ ,\ 2w_2 = a_1+a_3\ ,\ 2w_3 = a_1-a_3$

$p=1 \quad E_0 \geq 4+w_1 = 4+a_2+(a_1+a_3)/2 \qquad a_2 \neq 0$

$p=2 \quad E_0 \geq 3+w_1 = 3+(a_1+a_3)/2 \qquad a_2=0$

$p=3 \quad E_0 \geq 2+w_1 = 2+w_1/2 \qquad$ (less) $a_2 a_3=0$

Massless:

$a_1 = 2J$

Conformal fields:

fields on $\mathcal{M}_c = \dfrac{SO(2,d)}{ISO(1,d-1)\times R^+} = S \times S$

(light rays in 2+d dimensions)

conformal primary: $K_\mu \varphi(x=0) = 0$

(this is a H.W. rep. in the non-compact basis: (Gunaydin, Minic, Zagermann)

CFT/AdS $\quad \ell \leftrightarrow E_0$

$w_1 \ldots w_n \; (SO(d-1,1) \leftrightarrow SO(d))$

Degenerate Harish-Chandra modules correspond to "differential constraints" on $\varphi(x)$.

Example $d=4 \qquad \mathcal{D}(\ell, J_1, J_2)$

$\ell = 1+J \qquad \partial^{\dot\alpha_1 \alpha_1} \varphi_{\alpha_1 \ldots \alpha_{2J}}(x) = 0$ (see Dyson)

$\ell = 2+J_1+J_2 \qquad \partial^{\alpha_1 \dot\alpha_1} \varphi_{\alpha_1 \ldots \alpha_{2J_1}, \dot\alpha_1 \ldots \dot\alpha_{2J_2}} = 0$

Superalgebras (Bars, Günaydin; Günaydin, Minic, Zagermann; Dobrev, Petrova, Binegar, Minwalla, ...)

Five grading: $\mathcal{L} = \underset{B}{L^{++}} + \underset{F}{L^{+}} + \underset{B}{L^{0}} + \underset{F}{L^{-}} + \underset{B}{L^{--}}$

For highest-weight reps.

$$\mathcal{L} = P + Q + \underline{L^0 + S + K}$$

$$(SO(2)\times SO(2d) \longrightarrow SO(1,1)\times SO(1,d-1))$$

$$L^0 = SO(1,1)\times SO(1,d-1)\times \text{T symmetry} \to G$$

Superfields: fields on $\mathcal{G}(\mathcal{L}) / \mathcal{G}(L^0+S+K)$

(Salam, Strathdee)

$\mathcal{G}(\mathcal{L}) \longrightarrow$ supergroup

Harmonic superfields:

superfield whose bosonic coordinates are

$\mathcal{M}_c \times \frac{G}{H}$ $\quad \frac{G}{H}$ = "flag manifolds"

Infinitely many superalgebras for $d=4,6$, finite for $d=3,5$ [essentially only one (two) supermultiplet]

(For $d=6$ not all in the short eqs.)

Physical interest:

BPS massive particle states in AdS →

K-K states, AdS black-hole,

multiparticle AdS states

The conformal dimensions (AdS masses) are "protected" by non-renormalization theorems

They cannot be deformed from their value at the conformal point.

Generally ℓ is quantized in terms of the R-charges (Dynkin labels).

(the quantization is milder in $d=4$ ($N \neq 4$) because of a non-semisimple factor in the R-symmetry)

(Dobrev-Petkova, Bianchi, SF, Howe, West, Heslop)

The most powerful approach to study reps. of superconformal algebras is the superfield method, with superfields defined in "Harmonic superspaces" with bosonic part:

$\mathcal{M}_{d-1,1} \times G/(H)$ $\quad$ G: R-symmetry

x, θ $\quad$ H: subgroup of G

H is generally chosen to be the maximal torus of G, however for short (BPS) multiplets it is more convenient to choose H larger depending on the particular BPS condition. (In this case G/H is smaller and H is maximal for $\frac{1}{2}$ BPS states)

Harmonic superspace: Galperin, Ivanov, Kalitzin, Ogievetsky, Sokatchev; Howe, West; Hartwell, Howe

Highest weight UIR's $\quad SO(2,d) \times$ Int. $SO(d)$

$$\mathcal{D}(E_0, J_1 .. J_{[\frac{d}{2}]}; a_1 .. a_R) \Longleftrightarrow$$

$(E_0, J_1 .. J_{[\frac{d}{2}]}) \rightarrow SO(2,d) = SO(2) \times SO(d)$

$(a_1 .. a_R) \rightarrow$ Dinkin labels of R-symmetry

Superfields:

$$\mathcal{D} \rightarrow \Phi_\ell(x=0, \theta=0, u_i^J) \quad \ell = E_0$$

coset representative of G/H.

$$\Phi_{\ell, J_1 .. J_{[\frac{d}{2}]}}^{a_1 .. a_R}$$

Algebra of Covariant derivatives

$$\{\mathcal{D}_\alpha^I, \mathcal{D}_\beta^J\} = d^{IJ} \partial_{\alpha\beta}$$

$G = U(N) \quad (d=4) \qquad s-t = 6, 2, \bmod 8$

$= SO(N) \quad (d=3) \qquad s-t = 1, 7, 8, \bmod 8$

$= USp(2N) \quad (d=5,6) \qquad s-t = 3,4,5 \bmod 8$

d^{IJ} invariant tensor

The most interesting (and maximal) cases are:

(Maldacena)

M_2 (M theory branes) $AdS_4 \times S_7$ $OSp(8,4,R)$

M_5 $AdS_7 \times S_4$ $OSp(8^*,4)$

D_3 (II B) $AdS_5 \times S_5$ $SU(2,2/4)$

D_4 (II A) $AdS_6 \times S_4$ $F(4)$

(Kehagias, Ferrara, Zaffaroni, S.F.) (Brandhuber, Oz) (Cvetic, Lü, Pope)

Unlike D_4 all other branes are controlled by "classical" superalgebras of the (infinite series)

$OSp(N,4,R) \supset SO(N) \times SO(3,2)$

$OSp(8^*,2N) \supset SO(2,6) \times USp(2N)$

$SU(2,2/N) \supset SO(2,4) \times SU(N) \times U(1)$

$F(4) \supset SO(2,5) \times SU(2)$

$N=1$: $SU(2,2/1)$ (Flato-Fronsdal, Dobrev-Petkova $SU(2,2/N)$)

a) Ultrashorts $\bar{D}_{\dot{\alpha}} W_{\alpha_1 \ldots \alpha_{2J}} = 0$ $(J,0)$

$\ell = -r = 1+J$

Space-time constraint: $\begin{cases} D^{\alpha_1} W_{\alpha_1 \ldots \alpha_{2J}} = 0 \\ J=0 \quad D^2 W = 0 \end{cases}$

(supersingleton in AdS_5)

b) short "chiral superfield"

$\ell = -r \geq 1+J$

c) intermediate shorts $\ell = 2 + 2J_1 + r$

$$D^{\alpha_1} W_{\alpha_1 \ldots \alpha_{2J_1}, \dot{\alpha}_1 \ldots \dot{\alpha}_{2J_2}} = 0$$

$J_1 = J_2 = \frac{1}{2}$ massive AdS graviton

$$(J_1 = 0 \quad D^2 W_{\dot{\alpha}_1 \ldots \dot{\alpha}_{2J_2}} = 0)$$

$J_2 = 0$ massive AdS vector

if also satisfies the $\bar{J}^{\dot{\alpha}}$ conditions then it is a "conserved current multiplet" (space-time constraint) $\rightarrow$ massless AdS_5-state

$(r = J_2 - J_1)$

b) c) occur in IIB on $AdS_5 \times T_{11}$

(Klebanov, Witten; Gubser; Ceresole, Dall'Agata, D'Auria, S.F.)

Harmonic Superfields:

(Galperin, Ivanov, Kalitzin, Ogievetsky
(Sokatchev, Hartwell Howe)

Superspace: $A^{4/2N-2p,\ 2N-2q}$:

$$\frac{SU(2,2/N)}{(M, D, H, R, K, S, \bar{S}, Q^1 .. Q^p\ \bar{Q}_{N-q+1} .. \bar{Q}_N)}$$

Bosonic superspace: $M_c \times G/H$

$$G/H = SU(N)/H \longrightarrow u_i^I \in SU(N)$$

$I \to H$, $i \to SU(N)$

$$H = SU(p) \times SU(q) \times U(1)^{N-p-q-1}$$

Chiral Superfields in Extended Superspace

Superspace: $\dfrac{SU(2,2/N)}{(\bar{L}_\mu^{0}, \bar{Q}_i, S, K)}$ $i = 1 \ldots N$

$(J_{max} = J + N/2)$

$\{D_\alpha^i, \bar{D}_{\dot\alpha j}\} = \partial_{\alpha\dot\alpha}\,\delta^i_j$ $\{D_\alpha^i, D_\beta^j\} = 0$

Superconformal symmetry requires that they have only left spin and $\ell = -2$ and Rep $SU(N)$ trivial (from the $\{Q_\alpha, S_\beta\}$ commutator)

G-Analytic superfields $(u_i^J$ $SU(N)$ matrix$)$
(Howe, West...) (Ogievetsky et al.)

$I = 1 \ldots p$, $J = N-q+1, \ldots N$ $1 \le q \le N-p$

$\{D_\alpha^I, \bar{D}_{\dot\alpha J}\} = 0$ $D_\alpha^I = u_i^I D_\alpha^i$

$D_\alpha^I \Phi(x, \theta_I, \bar\theta^J, u) = 0$ $\theta_I = u_I^i \theta_i$

$\bar{D}_{\dot\alpha J} \Phi(x, \theta_J, \bar\theta^J, u) = 0$

$D^I{}_J = \partial^I{}_J + 2i\,\theta_J \sigma^\mu \bar\theta^I \partial_\mu - \theta_J \partial^J + \bar\theta^I \partial_J$

$1 \le I < J \le N$ (raising operator)

Superconformal invariance implies:

$$D^{I}{}_{I+1}\,\Phi^{[a_1\ldots a_{N-1}]} = D_{I}{}^{I+1}\,\Phi^{[a_1\ldots a_{N-1}]} = 0$$

$$1 \le I \le p-1$$

$$N-q+1 \le I \le N-1$$

Moreover $\left\{\begin{matrix} a_1 = \ldots a_{p-1} = 0 \\ a_{N-q+1} = \ldots a_{N-1} = 0 \end{matrix}\right\}$

$$\phi^{[0\ldots a_p\ldots a_{N-q}\ldots 0]}(x_A,\theta_{p+1}\ldots\theta_N,$$
$$\left[x_A = x^\mu - (\theta_I\sigma^\mu\bar\theta^I - \theta_J\sigma^\mu\bar\theta^J)\right]\bar\theta^1\ldots\bar\theta^{N-q}, u)$$

Bosonic part of Superspace $\dfrac{SU(N)}{SU(p)\times SU(q)\times U(1)^{N-p-q+1}} \times M_4$

(Flag-manifold)

$$\left\{\begin{matrix} \ell = m_1 & J_1 = J_2 = 0 \\ z = \dfrac{2m}{N} - m_1 & J_{max} = \left(\dfrac{N-p}{2}, \dfrac{N-q}{2}\right) \end{matrix}\right.$$

$$\left(\ell_0 = -\frac{1}{2},\ z_0 = \frac{1}{2}\left(1 - \frac{4}{N}\right)\right)$$

BPS states of $SU(2,2/N)$

(p,q) BPS states:

$1 \leq p, q \leq N-1$
$p+q \leq N$

$\frac{p+q}{2N}$ BPS $\qquad \frac{1}{N} \leq \frac{p+q}{2N} \leq \frac{1}{2}$

$(0,q)$ BPS $\qquad 1 \leq q \leq N-1$ $\qquad$ ceu cally left spin

$\frac{q}{2N}$ BPS $\qquad \frac{1}{2N} \leq \frac{q}{2N} \leq \frac{N-1}{2N}$

chiral: $\frac{1}{2}$ BPS $\qquad$ left spin.

D=4 SUPERSINGLETONS

G-Analytic: $m_1 = \ell$ $\quad \ell = \frac{2k}{N} - 1$

$(a_1 \dots a_{N-1}) = (0 \dots 0 \ \underset{k}{1} \dots 0)$  k

Chiral: $SU(N)$ singlets, $\ell \mathrel{\hat{=}} 1 + J_1 = -2$

$AdS_5 \times S_5$

$PSU(2,2|4)$ $\quad R=0$

$K = \frac{N}{2}$ $\quad R=0$. Y.h. multiplet

Analytic superfields: $m_1 = \frac{2m}{N}$

$(0, p, 0)$ $\quad \frac{1}{2}$ BPS $\quad (2,2)$

(q, p, q) $\quad \frac{1}{4}$ BPS $\quad (1,1)$

$(q+2s, p, q)$ $\quad \frac{1}{8}$ BPS $\quad (0,1)$

$\ell = p + 2q + 3s$

$$W^{12} = W^{12}(x_A, \theta_3 \theta_4 \bar{\theta}^1 \bar{\theta}^2)$$
$$W^{13} = W^{13}(x_A, \theta_2 \theta_4 \bar{\theta}^1 \bar{\theta}^3)$$
$$W^{23} = W^{23}(x_A\ \theta_1 \theta_4\ \bar{\theta}^2 \bar{\theta}^3)$$

$$(W^{12})^{p+q+s}(W^{13})^{q+s}(W^{23})^{s}$$

I will not consider here the D_3 brane algebras which have been widely discussed (SU(2,2|N) is[etc]

special in two respect: SO(6) is semisimple and G has a nonsemisimple U(1) factor

This gives families of BPS states which do not exist in the other cases (Osp and F(4))

I'll consider rather

$Osp(8/4,R)$ $\qquad SO(N)/H$

$Osp(8^*/2N)$ $\qquad USp(2N)/H$

$F(4)$ $\qquad SU(2)/U(1) = S^2$

Harmonic spaces: Depend on the shortening condition (degeneration of the representation)

For each BPS different situations may be possible if certain symmetries of the Young-Tableaux occurs.

$d=4$ is special because $SO(4) = SU(2) \times SU(2)$

Existence of chiral superfields:

Consider a set of $\{I'\} \subset \{I\}$ such that

$$\{D_\alpha^{I'}, D_\beta^{J'}\} = 0$$

Then a constraint of the type

$$D_\alpha^{I'} \phi(x, \theta_I, u) = 0$$

is called Grassmann-analytic constraint

An irreducible (H.W.) Grassmann superfield is such that

$$D^{\alpha} \Phi = 0$$

when D^{α} are the generators corresponding to the simple-roots of the Lie algebra of G (subset of positive roots)

Some conventions:

$SO(8) \to (a_1\ a_2\ a_3\ a_4)$

$(0,0,1,0)$ 8_s

$(0,0,0,1)$ 8_c

$(1,0,0,0)$ 8_v

$(0,1,0,0)$ 28

$$\underline{D_\alpha^i \phi_a = \tfrac{1}{8}(\gamma^i \gamma \cdot D\, \phi)_a}$$

SO(8) Triality: For one choice of the spinor charge in 8_v $(8_s)(8_c)$ the two superunification multiplets have spin $(0,\tfrac{1}{2})$ states in (Sezgin, Nicolai / Blencowe, Duff)

I $(8_s, 8_c)$ II $(8_c, 8_s)$

I $(8_v, 8_c)$ II $(8_c, 8_v)$

I $(8_v, 8_s)$ II $(8_s, 8_v)$

Three choices are equivalent (give isomorphic algebras and multiplets)

$SO(8) \to SO(2)^4 \qquad \pm \, (\pm) \, [\pm] \, \{\pm\}$

$8_v \to Q_\alpha^{++}, Q_\alpha^{(++)} \; Q_\alpha^{[+]\{+\}} \; Q_\alpha^{[+]\{-\}}$

$d_{MAX} = 4$

$\frac{1}{8}$ BPS $\quad 7/2 \quad D_\alpha^{++} = 0$

$\frac{1}{4}$ BPS $\quad 3 \quad D_\alpha^{++} = D_\alpha^{(++)} = 0$

$\frac{3}{8}$ BPS $\quad \frac{5}{2} \quad D_\alpha^{++} = D_\alpha^{(++)} = D_\alpha^{[+]\{+\}} = 0$

$\frac{1}{2}$ BPS $\quad 2 \quad D_\alpha^{++} = D_\alpha^{(++)} = D_\alpha^{[+]\{+\}} = D_\alpha^{[+]\{-\}} = 0$

$7/2 \; \frac{1}{8}$ BPS $\quad \mathcal{D}(E_0 = a_1 + a_2 + \frac{1}{2}(a_3 + a_4) \,,\, a_1, a_2, a_3, a_4)$

$3 \; \frac{1}{4}$ BPS $\quad a_1 = 0$

$\frac{3}{8}$ BPS $\quad a_1 = a_2 = 0$

$2 \; \frac{1}{2}$ BPS $\quad a_1 = a_2 = a_{\cancel{3}4} = 0 \quad (a_1 = a_2 = a_3 = 0)$

$\frac{1}{8}$ BPS : $\dfrac{Spin(8)}{U(1)^4}$

(if $a_2=a_3=a_4=0$) $\to \dfrac{Spin(8)}{U(4)}$

$\frac{1}{4}$ BPS : $\dfrac{Spin(8)}{U(1)^2 \times U(2)}$ (if $a_3=a_4=0$) $\to \dfrac{Spin(8)}{U(2)\times SU(2)}$

$\frac{1}{2}$ BPS : $\dfrac{Spin(8)}{U(4)}$

$\frac{3}{8}$ BPS : $\dfrac{Spin(8)}{U(1)\times U(3)}$

If we multiply only one type of supercharge we only get certain cases:

$\frac{1}{8}$ BPS $(a_3 - a_4 = 2s)$

$\frac{1}{4}$ BPS $a_1 = a_4 = 0$

$\frac{3}{8}$ BPS none

$\frac{1}{2}$ BPS $(0, 0, a_3, 0)$ (all)

$Osp(8^*/2N)$ superalgebra:

infinitely many supersingleton representations of $(2N,0)$ superconformal algebra.

Some analogies with $SU(2,2/N)$.

Supersingletons live on conformal boundary =

superspace = θ's are purely right-handed

Q_s (D_s) purely left-handed

Harmonic spaces: $\dfrac{Osp(2N)}{H}$

Analytic superfields: $W^{\{i_1 \ldots i_n\}}$ $1 \le n \le N$

$D_\alpha^{(k\{i_1)} W^{i_2 \ldots i_n\}} = 0 \Rightarrow$

$D(2;0,0,0;0 \ldots 0, a_n = 1, 0, \ldots 0)$

higher spin $(2N-n;0,0)$

$SU^*(4)$

(non analytic)

lowest spin $(2N,0,0)$

$$D^{(i}_{[\alpha} D^{j)}_{\beta]} W = 0 \Rightarrow \mathcal{D}(2,0,0,0;00\ldots0)$$

$$D^{i}_{[\beta} W_{(\alpha_1]\ldots\alpha_n)} = 0 \quad \mathcal{D}(2+\tfrac{n}{2};n,0,0;0\ldots0)$$

All compound fields are in $(k,0,0)$ UIR's of $SU^*(4)$ and $(\frac{k}{2},0)$ UIR's of $SU(2)_L \times SU(2)_R$

Because of this observation these massless UIR's are identical to massless Poincaré UIR's in $d=5$ dimensions! (Hull)

$$D^1_\alpha W^{12\ldots n} = D^2_\alpha W^{12\ldots n} = D^n_\alpha W^{1\ldots n} = 0$$

$$W^{12\ldots n}(x, \theta^1, \theta^2 \ldots \theta^{2N-n}, u)$$

We have seen that in $d=6$ $(2,0)$ there are only two types of BPS states $(1/4, 1/2)$ –

For $d=6$ $(1,0)$ there is therefore only one type of BPS state $(1/2)$ since there is only one type of harmonic space: $SU(2)/U(1) = S^2$

The only G-analytic superfield is the hypermultiplet

$$D^{(i}W^{j)}(x,\theta)=0 \qquad \mathcal{D}(2,0,0,0;I=1)$$

which implies the G-analyticity

$$D^1 W^1 = 0$$

The tensor multiplet is a linear $\mathcal{D}(2,0,0,0;0)$ multiplet (in this case) so it's not BPS (although it is ultrashort)

F(4) supergravity: (D'Auria, Vaulà
S.F. Lopes Cardoso

Unique supersingleton: hypermultiplet.

$$D^{(i}W^{j)}(x,\theta)=0$$

Harmonic space: $S_2 = \frac{SU(2)}{U(1)}$

$$D^1 W^1 = 0 \qquad \{D^I_\alpha, D^J_\beta\} = i\epsilon^{IJ}\partial_{\alpha\beta}$$

$$D^I_\alpha = D^i_\alpha u^I_i$$

$$W^1(x,\theta) = \phi^1 + \theta^\alpha_2 \psi_\alpha + t.d.$$

$$D^2_\alpha D^{2\alpha} W^1 = 0$$

Analytic superfields $\quad (W^1)^p \qquad p = 2J$

$$\ell = \frac{3}{2}p = 3J$$

intermediate short. $\quad W^1 \bar{W}^1 (W^1)^q$

$$\ell_0 = 3 + 3J \qquad 2J = q$$