ASPECTS OF COLLAPSING CYCLES

B. GREENE, STRINGS 2000

I) QUANTUM VOLUME

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II) ATTRACTOR FLOWS

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III) M-THEORY & TOPOLOGY CHANGE

BRG, G. SHIU, R. SCHALM

I) QUANTUM VOLUME

- BACKGROUND: M = CY 3-fold w/ $J \in H^{1,1}(M)$
  - CLASSICAL VOLUME: $\int_C J$; $\int_S J \wedge J$; $\int_M J \wedge J \wedge J$
  - QUANTUM VOLUME? $Q(C;S;M) \rightarrow \mathcal{Q}(C;S;M)$

- PHYSICAL GUIDE:

  (i) PERTURBATIVE STRING THEORY:
  - WORLD SHEET INSTANTONS

- $\langle \theta_i \theta_j \theta_k \rangle = \int_{M(K)} A_i \wedge A_j \wedge A_k + \sum_I e^{\int_I \Phi^*(K)} \int_I A_i \int_I A_j \int_I A_k$

I) Quantum Volume

- **Background:** M-CY 3-fold w/ $J \in H^{1,1}(M)$
  - Classical Volume: $\int J^3$; $\int J \wedge J \wedge J$; $\int_M J \wedge J \wedge J$
  - Quantum Volume? [illegible] $\longrightarrow$ [illegible]$(C, S; M)$

- **Physical Guide:**

  (i) Perturbative String Theory:
  - World Sheet Instantons
  - $\langle \theta_i \theta_j \theta_k \rangle = \int_{M(x)} A_i \wedge A_j \wedge A_k + \sum_I e^{[illegible]} \int_I \int_I \int_I$
  - Mirror Symmetry

    $= \int$ [illegible]

$\Rightarrow$ K [illegible]

(ii) NONPERTURBATIVE STRING THEORY:

- D-p-BRANES WRAP p-cycles

MASS FOR BPS CASE w/ p=3: (IIB)

$$m(\gamma) = \mathrm{VOL}(\gamma) = |\int_\gamma \Omega| \Big/ \left(\int_M \Omega \wedge \bar{\Omega}\right)^{1/2} = \frac{|q^i \int_{\gamma_i} \Omega|}{\left(\int_M \Omega \wedge \bar{\Omega}\right)^{1/2}}$$

w/ $\gamma = q^i \gamma_i$, $\{\gamma_i\} \in H_3(M, \mathbb{Z})$; $q^i \in \mathbb{Z}$

- P-EVEN: MIRROR SYMMETRY

$C \in \oplus H^{2p}(W, \mathbb{Z}) \rightarrow \gamma \in H^3(M, \mathbb{Z})$

VOL (C) = MASS (C) = M(γ) = AS ABOVE

- NOTE:

$p = 2 \Rightarrow \mathrm{VOL}(C_2)[\mathrm{N.P.}] = |\int \gamma| / \left(\int \Omega \wedge \bar{\Omega}\right)^{1/2}$

$\propto |\mathrm{VOL}(C_2)[\mathrm{Pert}]|$

STRATEGY FOR COMPUTING QUANTUM VOLUMES

(1) CHOOSE BASIS γ_i FOR $H_3(M,\mathbb{Z})$

(2) CALCULATE $\int_{\gamma_i} \Omega^{(z)}$ $\forall$ PTS $z \in \mathcal{M}_{C.S.}(M)$

(3) FIND MIRROR MAP : $\bigoplus_j H_{2j}(W,\mathbb{Z}) \leftrightarrow H_3(M,\mathbb{Z})$

(4) DETERMINE VOL$(C)_W$ $C \in H_j(W,\mathbb{Z})$ FROM D-3-BRANE MASS ON M.

SUBTLE POINTS

- (1), (2) : CALCULATIONALLY INVOLVED
- (3) : IN GENERAL, NOT FULLY WORKED OUT ($j>1$).
- : EXISTENCE OF SUPERSYMMETRIC 3-CYCLES.

CALCULATION

(1) SETTLE FOR "WEAKLY" INTEGRAL CYCLES

- $\{\gamma_i\} \in H_3(M,\mathbb{R})$ w/ $\lambda\gamma_i \in H_3(M,\mathbb{Z})$ $\forall i$.

- PRODUCE "SEEDS" γ_i w/ NONTRIVIAL MONODROMY. USE MONODROMY $\gamma_i \to \sum a^i \gamma_i$ TO PRODUCE OTHER CYCLES.

(3) EASY TO IDENTIFY LARGEST DIMENSION OF $C_\gamma \in \oplus H_j(W,\mathbb{Z})$ MIRROR TO GIVEN $\gamma \in H_3(M)$ OFTEN LEAVE LOWER DIM ADMIXTURES UNSPECIFIED.

(2) USE CLASSICAL MEIJER FUNCTIONS:

- PARTICULAR COMBINATIONS OF GAMMA FUNCTIONS, CLOSELY RELATED TO GENERALIZED HYPERGEOMETRIC FUNCTIONS.

- WELL SET UP FOR EFFICIENTLY PRODUCING SEEDS AND FOR ANALYTIC CONTINUATION.

• MEIJER FUNCTIONS:

• $\Gamma\begin{pmatrix}\sigma_1 \cdots \sigma_n \\ \rho_1 \cdots \rho_m\end{pmatrix} \equiv \dfrac{\Gamma(\sigma_1)\cdots\Gamma(\sigma_n)}{\Gamma(\rho_1)\cdots\Gamma(\rho_m)}$

• $G\begin{pmatrix}\rho_1 \cdots \rho_r & \rho_1' \cdots \rho_{r'}' \\ \sigma_1 \cdots \sigma_s & \sigma_1' \cdots \sigma_{s'}'\end{pmatrix} \equiv \dfrac{1}{2\pi i}\displaystyle\int_{C} dt\, \Gamma\begin{pmatrix}\sigma_1 - t, \ldots, \sigma_s - t, & 1-\rho_1+t, \ldots, 1-\rho_r+t \\ \rho_1' - t, \ldots, \rho_{r'}' - t, & 1-\sigma_1'+t, \ldots, 1-\sigma_{s'}'+t\end{pmatrix} z^t$

$C = \{-i\infty$ to $i\infty$, separating "σ"s & "ρ" poles$\}$
$(|\arg(z)| < \pi)$

• ${}_pF_q\begin{pmatrix}\alpha_1 \cdots \alpha_p \\ \beta_1 \cdots \beta_q\end{pmatrix}(z)$ SOLVES $\left[\delta \prod_{i=1}^{q}(\delta+\beta_i-1) - z\prod_{j=1}^{p}(\delta+\alpha_j)\right]{}_pF_q = 0$; $\delta = z\frac{d}{dz}$

& SATISFIES: ${}_pF_q = \Gamma\begin{pmatrix}\beta_1 \cdots \beta_q \\ \alpha_1 \cdots \alpha_p\end{pmatrix} G\begin{pmatrix}1-\alpha_1 \cdots 1-\alpha_p & \\ 0 \cdots 0 & , 1-\beta_1, \ldots, 1-\beta_q\end{pmatrix}(-z)$

• USEFUL FACT: PARTITIONS ⇒

$G\begin{pmatrix}\rho_1 \cdots \rho_r & \rho_1' \cdots \rho_{r'}' \\ \sigma_1 \cdots \sigma_s & \sigma_1' \cdots \sigma_{s'}'\end{pmatrix}(z)$, $G\begin{pmatrix}\rho_1, \ldots, \hat{\rho}_a', \ldots, \rho_r & \rho_1' \cdots \hat{\rho}_a' \cdots \rho_{r'}' \\ \sigma_1 \cdots \sigma_s & \sigma_1' \cdots \sigma_{s'}'\end{pmatrix}(-z)$

& $G\begin{pmatrix}\rho_1 \cdots \rho_r & \rho_1' \cdots \rho_{r'}' \\ \sigma_1 \cdots \sigma_a' \cdots \sigma_s, & \sigma_1' \cdots \hat{\sigma}_a' \cdots \sigma_{s'}'\end{pmatrix}(-z)$ ALL SATISFY SAME DEQ.

• PROCEDURE FOR PRODUCING "$\log^j z$" SOLUTIONS, COMPUTING MONODROMIES
SEED CYCLES FOR NEARLY INTEGRAL BASES.

Example 1:

- Mirror/Quintic : • $G\left({}^{1/5\ 2/5\ 3/5\ 4/5}_{0 \cdots \quad 0 \cdots}\right)$
  - $h^{11} = 1 \rightarrow J = s\hat{e}$ $\hat{e}$ gen of $H^2(MQ, \mathbb{Z})$, $s \in \mathbb{R}$
  - Classical Volumes :
    - 2-Cycle : s
    - 4-Cycle : $\frac{5}{2}s^2$
    - 6-cycle : $\frac{5}{6}s^3$
  - Quantum Volumes : (Candelas, de la Ossa, Green, Parkes; BRG, Y. Kanter; J. Polchinski, A. Strominger; BRG, C. Lazaroiu)

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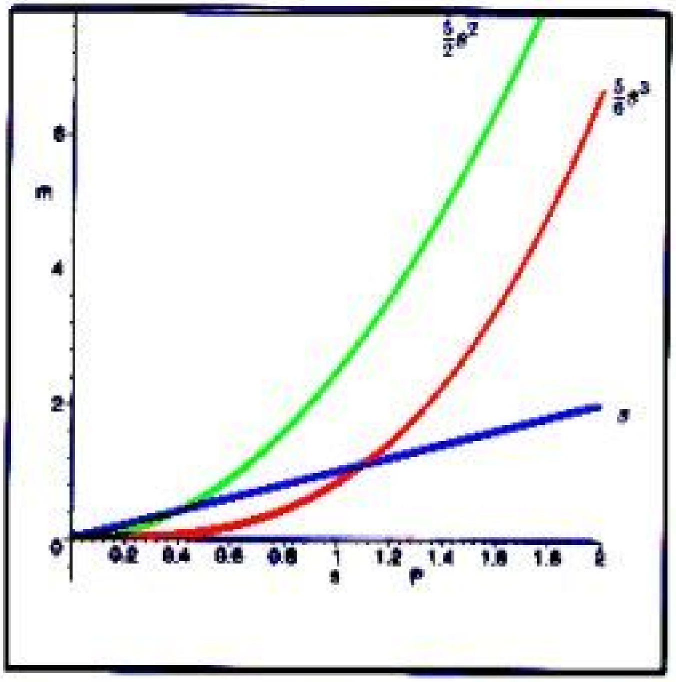
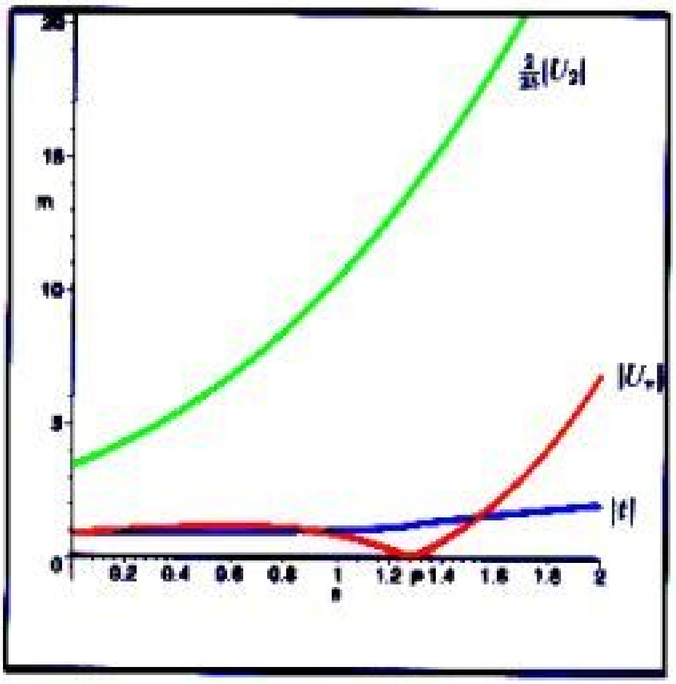


m
0
2
4
6
0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
s
p



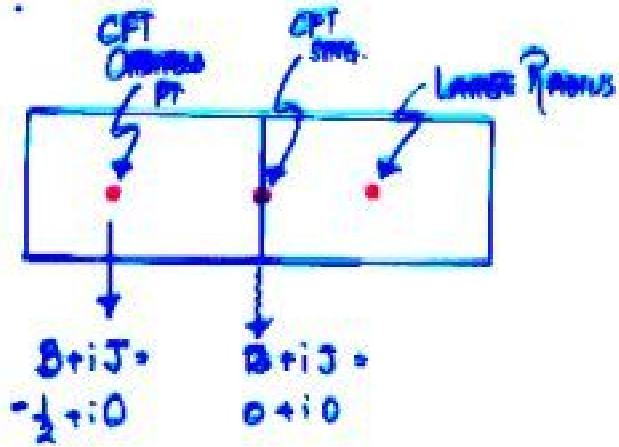
m
0
5
10
15
20
0.2 0.4 0.6 0.8 1 1.2 p 1.4 1.6 1.8 2
s
|t|

- GENERALIZED TO
  - ONE PARAMETER EXAMPLES IN WCP.
  - ONE PARAMETER EXAMPLES IN HIGHER DIM CY'S.
- SAME BASIC BEHAVIOR.

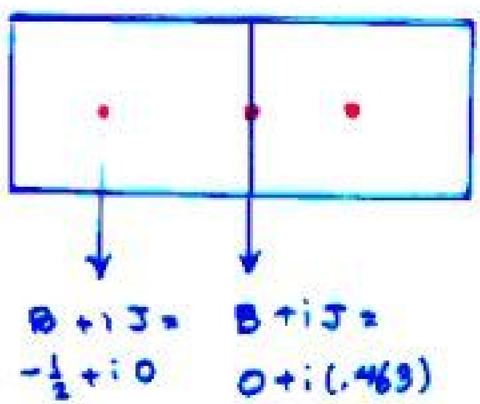
EXAMPLE 2:

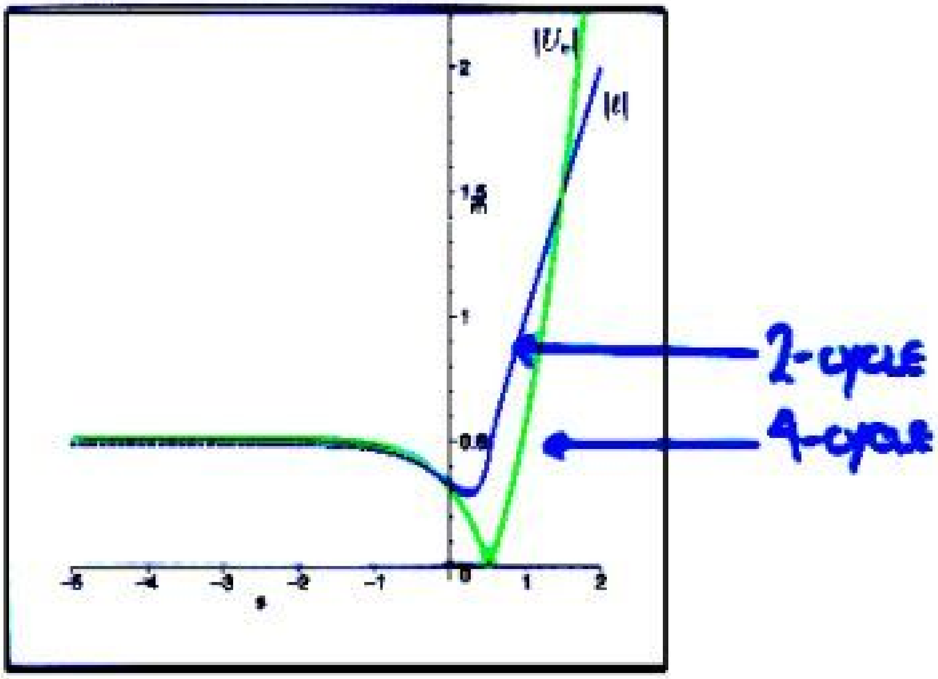
• ORBIFOLDS:

(P. Aspinwall, BRG, D Morrison) • $\mathbb{Z}_2$:



• $\mathbb{Z}_3$:





$\mathbb{Z}_3$ ORBIFOLD EXAMPLE

Example 3:

- Two parameters: $h^{11} = 2$
- X_0 : $x_1^8 + x_2^8 + x_3^4 + x_4^4 + x_5^4 = 0$
- Singularities: Curve Σ : $x_3^4 + x_4^4 + x_5^4 = 0$; Blow-up $\rightarrow E$ = Exceptional Divisor = Ruled surface over Σ.
- Find: $QVol(E) \rightarrow 0$ while 2-cycles $\subset E$ have $QVol > 0$.

II) Attractor Flows (F. Denef, BG, M. Raugas)

- Essential Background { Ferrara, Kallosh, Strominger; Ferrara, Gibbons, Kallosh; Moore; ...

  Link between Special Geometry (N=2 [illegible] Sugra)
  & Symplectic Geometry (D=4 [illegible])

- Concretely: Low E limit of IIB on CY_3

  ↙ ↘ ↑

  $\mathcal{M}_{cs} \leftrightarrow \mathcal{M}_{\text{vector mults}}$ ← Spherically Symmetric Charged Black Holes

  ↓ Flow eqns in modspace ↓

  End on "Attractor Points" ⟷ D-branes on susy cycles at extrema of [illegible] in $\mathcal{M}_{cs}$

ñ

- **EXPLICITLY:**

  WITH METRIC $ds^2 = -e^{2U}dt^2 + e^{-2U}dx_i dx_i$

  & $Z(\Gamma) = \int_\Gamma \Omega \qquad \Gamma \in H_3(CY, \mathbb{Z})$:

  $\dot{U} = -e^U |Z|$, $\dot{z}^a = -2e^U g^{a\bar{b}} \partial_{\bar{b}} |Z|$

  ($\cdot = \partial/\partial\tau$, $\tau = 1/r$; $\{z^a\}$ = [illegible] $\mathcal{M}_{cs}$)

  $\rightarrow \partial_\tau |Z| = -4e^U g^{a\bar{b}} \partial_a |Z| \partial_{\bar{b}} |Z| \leq 0 \rightarrow$ [illegible] FLOWS.

- **RELEVANCE: WIDE. FOR US:**

  - EXISTENCE OF BPS STATES/SUSY CYCLES FOR Γ
    - $\min(|Z(\Gamma)|) \neq 0 \leftrightarrow$ EXPECT BPS STATE
    - $\min(|Z(\Gamma)|) = 0$ AT SING PT IN $\mathcal{M}_{cs}$
    - $\min(|Z(\Gamma)|) = 0$ AT REG PT $\leftrightarrow$ DON'T EXPECT BPS STATE... BUT...

      [illegible] CURVE.

• States @ Large vs Small Radius

(• Classical vs Stringy Geometry)

• **Example:** (Brunner, Douglas, Lazaroiu, Romelsberger; Denef)

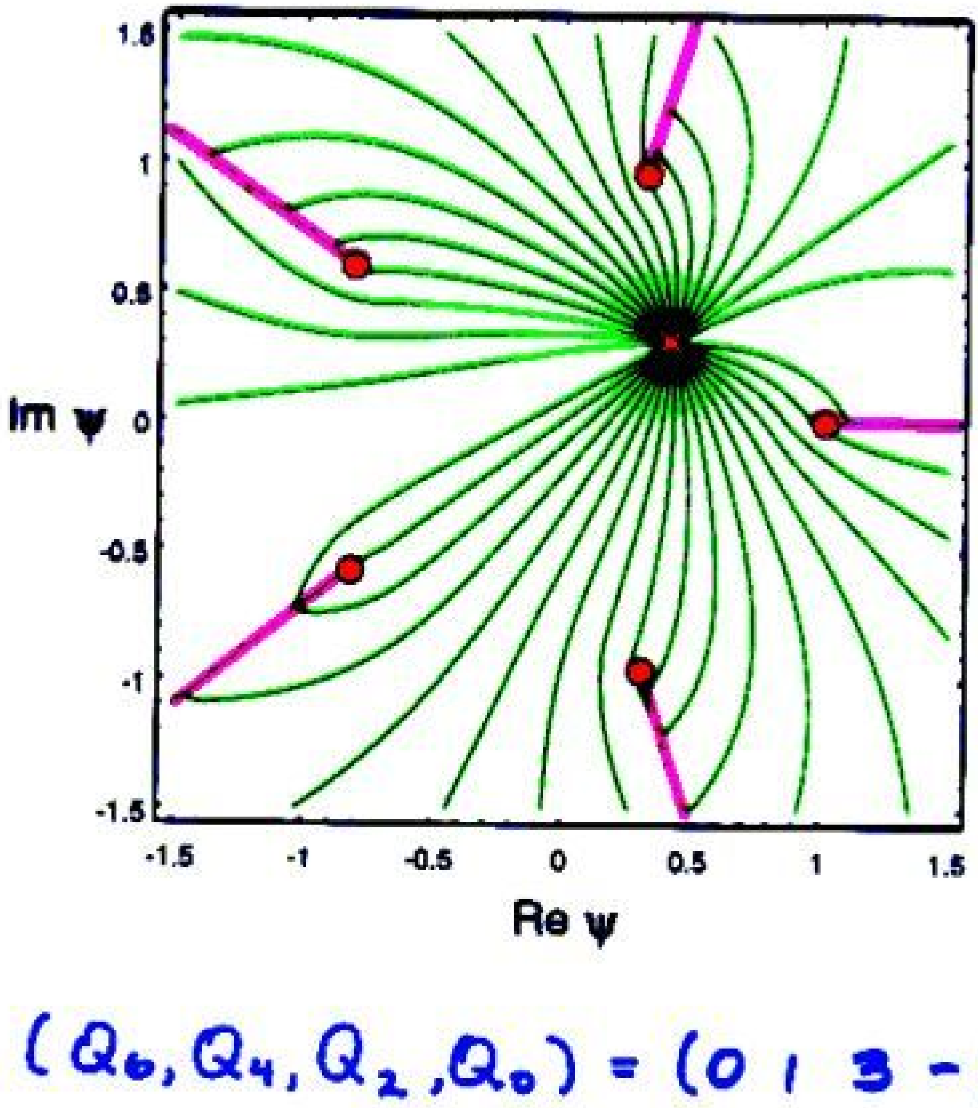
- Start at Gepner point on Quintic (Small Rad.) ($\psi = 0$)
- Consider boundary state $|10000\rangle$.

  At $\psi = -1.46$ $|Z| \to 0$. $\Rightarrow$ Must cross line of marginal stability $\to$ state decays.

- (Denef) Explicitly found splitted attractor flow.

• Complementary Story? Start w/ state at large radius, decays before we get to small radius? Consider, again, Quintic: (Calculate w/ Meijer Fcns)

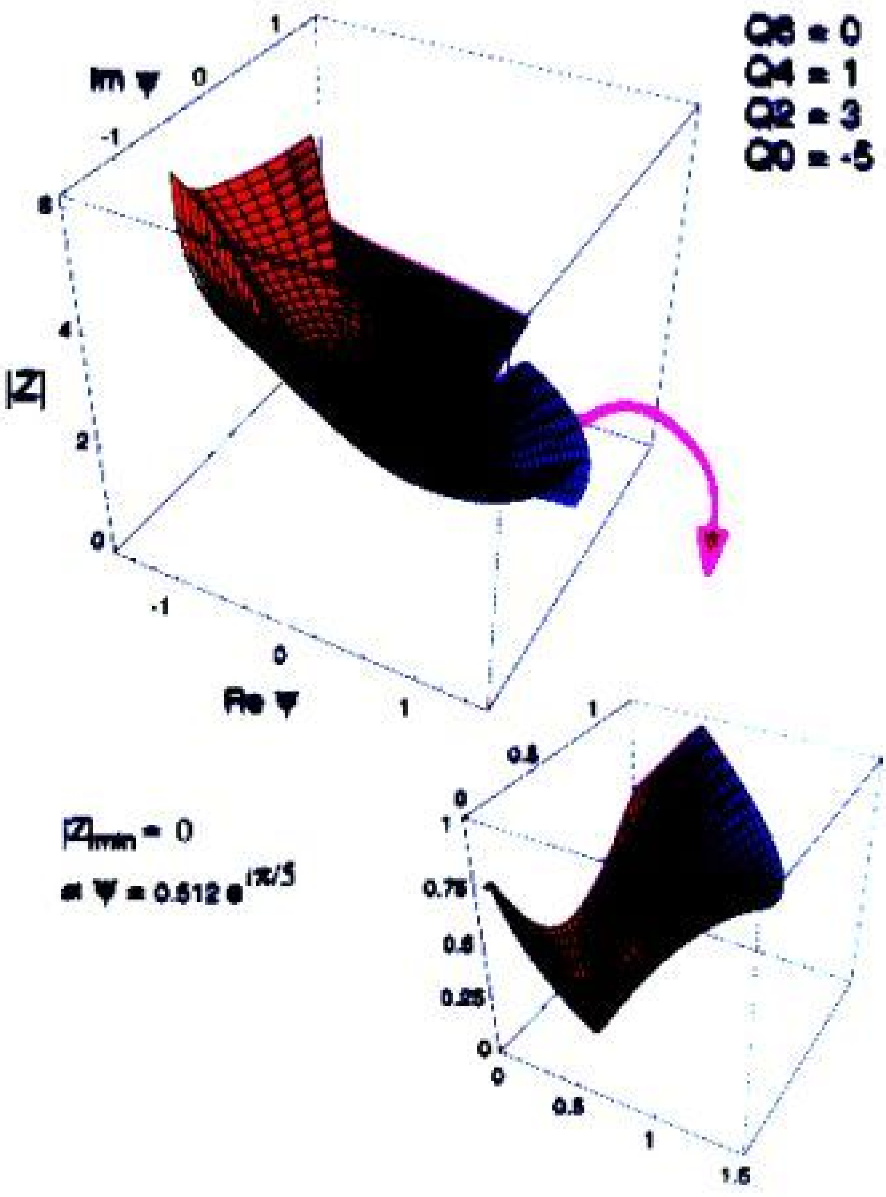
$$\sum x_i^5 - 5\psi x_1 x_2 x_3 x_4 x_5 = 0$$

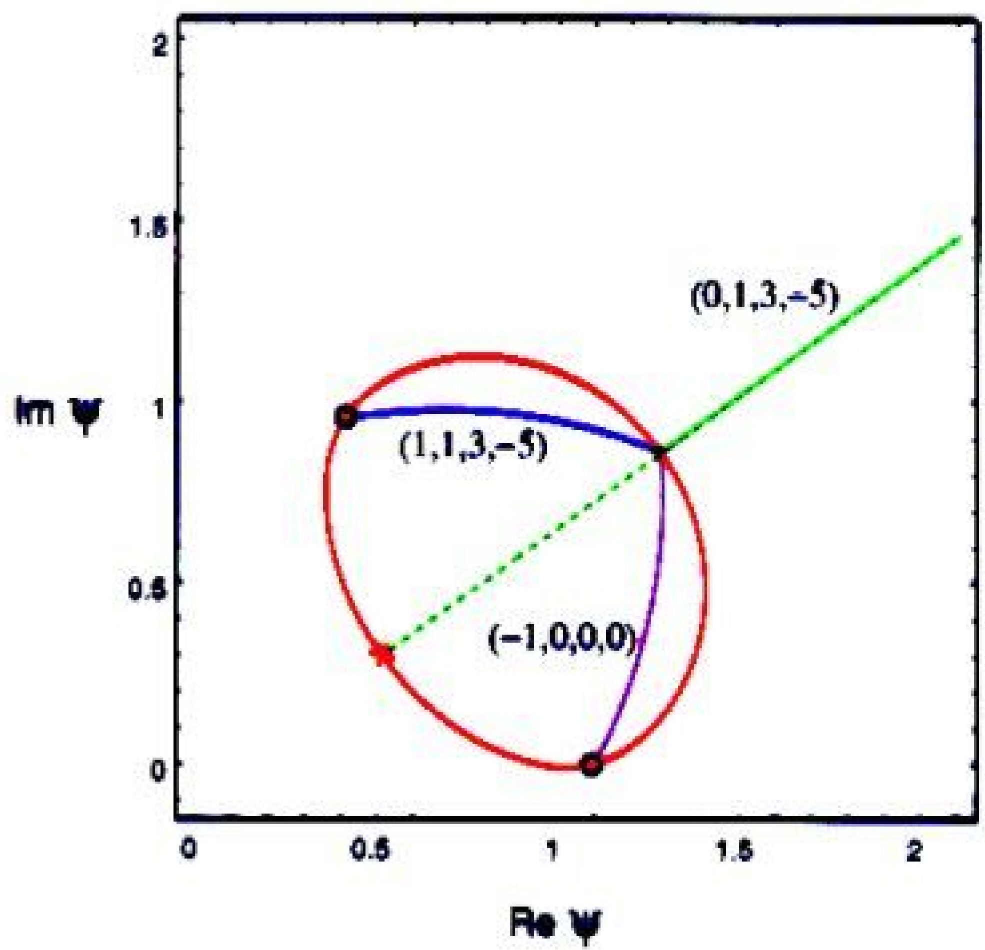


$\Gamma : (Q_6, Q_4, Q_2, Q_0) = (0\ 1\ 3\ -5)$

× = .4146 + .3009i

● = CONIFOLD POINT


Q6 = 0
Q4 = 1
Q2 = 3
Q0 = -5
Im Ψ
Re Ψ
|Z|
|Z|min = 0
at Ψ = 0.512 e^{iπ/5}


2
1.5
1
0.5
0
Im ψ
(0,1,3,−5)
(1,1,3,−5)
(−1,0,0,0)
0
0.5
1
1.5
2
Re ψ

- PRELIMINARY ANALYSIS OF ~7000 CHARGE STATE ATTRACTOR FLOWS → THOSE WITH SENSIBLE SPLIT FLOW HAVE DECAY PRODUCTS WHICH → CONIFOLD POINT.

- MODULI SPACES W/RICHER DISCRIMINANT LOCI ?

III) M-Theory & Topology Change (Greene, Schalm, Shiu)

- IIA/IIB String Theory: → Flops (Witten, Aspinwall, Greene, Morrison) → Conifold Transitions (Greene, Morrison, Strominger)

- M-Theory → Flops (Witten)

- One parameter family of theories w/no obstruction to topology changing transition.

- ¿How about inevitable real spacetime topology changing transitions?
  - Black Hole Attractor Flow Examples (Chou, Kallosh, Rahmfeld, Rey, Shmakova, Wong; Kaida, Mahapatra, Mohaupt, Sabra; Behrndt, Lopes Cardoso, ...)
  - Vacuum Solutions?

• M THEORY ON $S^1/Z_2 \times CY_3$:

(WITTEN; LUKAS, OVRUT, STELLE, WALDRAM; LUKAS, OVRUT, [illegible])

• USUAL $dH = \mathrm{tr}\,F^2 - \mathrm{tr}\,R^2$ OF PERT HET ST →

(HORAVA, WITTEN)

$$dG = \delta(x^{11})\left(\mathrm{tr}\,F_{(1)}^2 - \tfrac{1}{2}\mathrm{tr}\,R^2\right) + \delta(x^{11} - \pi\rho)\left(\mathrm{tr}\,F_{(2)}^2 - \tfrac{1}{2}\mathrm{tr}\,R^2\right)$$

(WITTEN)

• ONE CONSEQUENCE : <u>CY MODULI VARY OVER x^{11}</u>.

• ¿ MIGHT THEY BE FORCED TO VARY THROUGH A TOPOLOGY CHANGE TRANSITION?

• PRELIMINARY RESULT : YES, WITH ONE TWIST :

- $ds_5^2 = e^{2A(x^u)} dx^\mu dx^\nu \eta_{\mu\nu} + e^{2B(x^u)} (dx^u)^2$

  $V = V(x^u)$

  $b^i = b^i(x^u) = V^{-1/6} f^i(x^u)$

  $G = G(x^u)$

- $d_{ijk} f^j f^k = H_i \qquad H_i = c\,\alpha_i |x^u| + k_i$

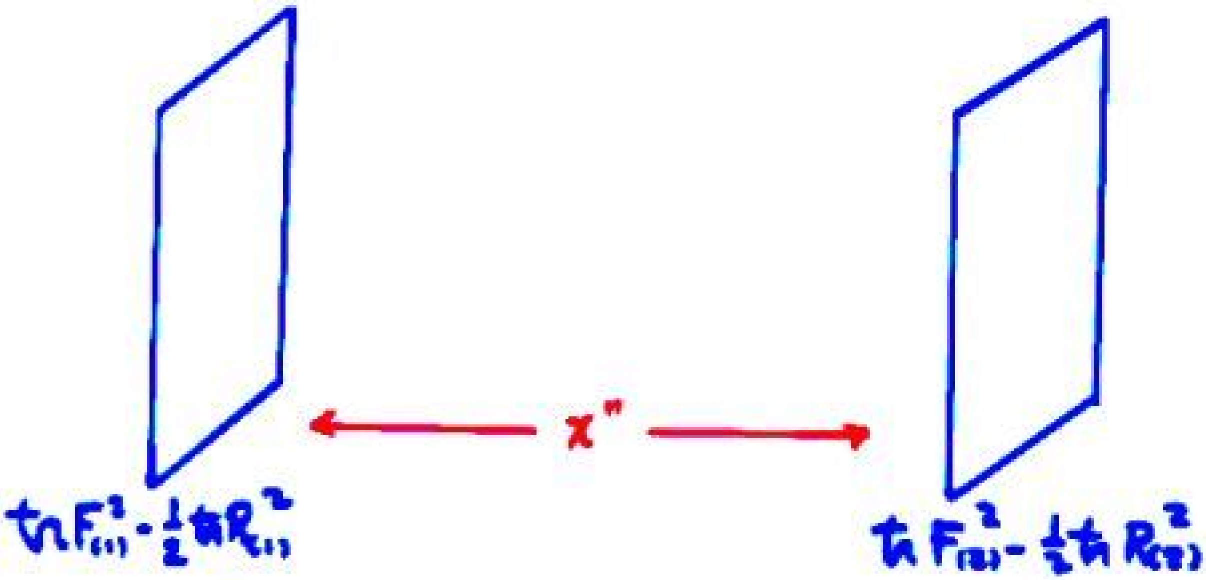
  (d_{ijk}: $D_i \cap D_j \cap D_k$; $c\,\alpha_i$: $C_2(CY) \cdot D_i$)

  $e^A = V^{1/6}$

  $V = (\frac{1}{6} d_{ijk} f^i f^j f^k)^2$

- TWO FACTS:

  - $C_2 \cdot D_i + 2 D_i \cap D_i \cap D_i = 12 \chi^{hol}(D_i)$
  - FLOP: $(D_i \cap D_i \cap D_i)_{FLOP} = D_i \cap D_i \cap D_i - \sum_{C_\beta} (D_i \cap C_\beta)^3$
  - (Tian, Yau) $(C_2 \cdot D_i)_{FLOP} = (C_2 \cdot D_i) - \sum_\beta D_i \cap C_\beta$



- E.g.: STANDARD EMBEDDING: $\mathrm{tr}F_{(1)}^2 = \mathrm{tr}R_{(1)}^2,\ \mathrm{tr}F_{(2)}^2 = 0$
  - If $CY_{(1)} = CY_{(2)} \Rightarrow 0 = \int_{S/\mathbb{Z}_2 \times D_4} dG \Rightarrow \left[\mathrm{tr}F_{(1)}^2 - \frac{1}{2}\mathrm{tr}R_{(1)}^2 - \frac{1}{2}\mathrm{tr}R_{(2)}^2\right] = 0$

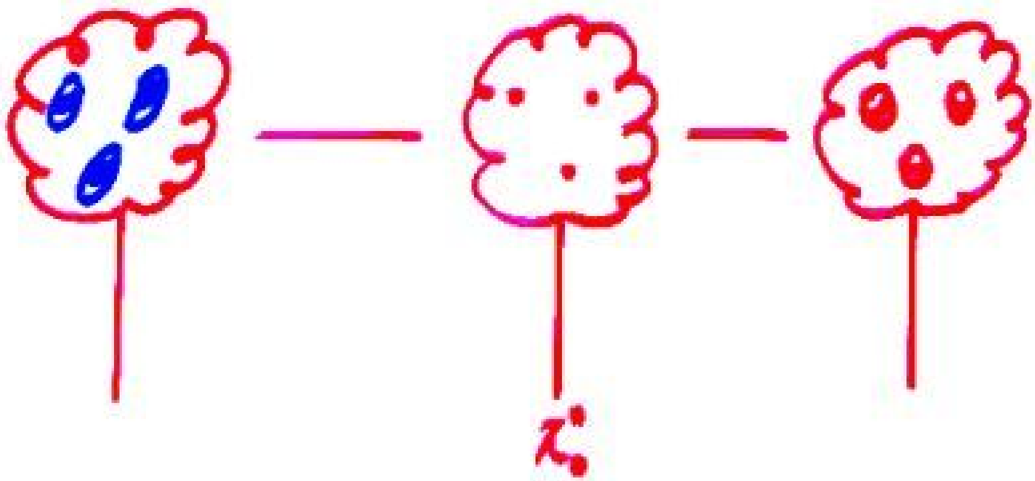
$$\left[\mathrm{tr}F_{(1)}^2 - \mathrm{tr}R_{(1)}^2\right] = 0 \ ✓$$

- BUT: $\int_{D_i} C_2 + 2\, D_i \cap D_i \cap D_i$ = FIXED UNDER A FLOP

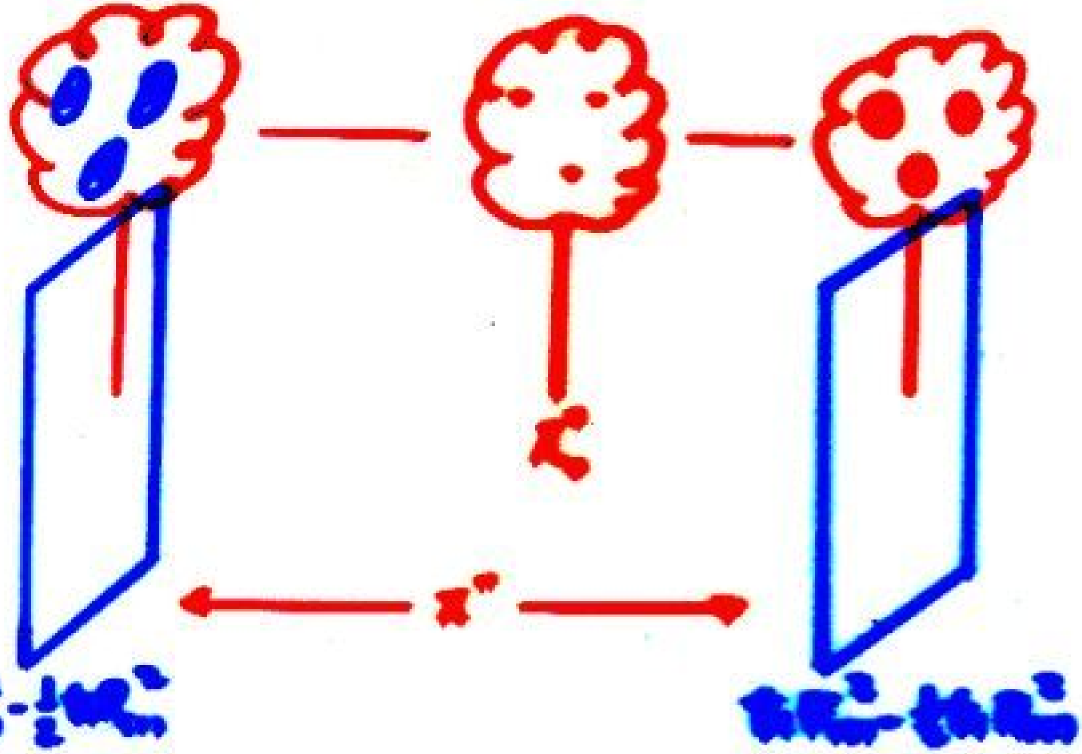
$$\Rightarrow \underline{\Delta(C_2 \cdot D_i) = -2\Delta(D_i^3)}$$

$$\left(\Delta(D_i^3) = -\sum_{C_\beta} (D \cap C_\beta)^3\right)$$

- SO, IF, FOR EX: $\int_{D_i} \mathrm{tr}\, R_{(2)}^2 < \int_{D_i} \mathrm{tr}\, R_{(1)}^2$, FLOP OF CY CONTRIBUTES TO BIANCHI IDENTITY:



$$dG = \delta(X^n)(\mathrm{tr}\,\kappa_1^2 - \tfrac{1}{2}\mathrm{tr}\,\kappa_1^2) + \delta(\text{[illegible]})(\mathrm{tr}\,\kappa_2 - \mathrm{tr}\,\kappa_1) - \sum_{G_p} \text{[illegible]}$$



- E.g.: [illegible] : $\mathrm{tr}\,F_m^2 = \mathrm{tr}\,R_m^2$, $\mathrm{tr}\,F_m^2 = 0$
  - [illegible] $Cl_m = Cl_{m'}$ [illegible] $\int$ [illegible] [illegible]
    [illegible] = ✓
- But: $\int_{D_i} C_2 + 2\, D_i \cap D_i \cap D_i$ = [illegible]

$$\Rightarrow \Delta(C_2 \cdot D_i) = -2\Delta(D_i^3)$$

$$\left(\Delta(D_i^3) = -\sum_{C_p} (D \cap C_p)^3\right)$$

- So [illegible]: $\int$ [illegible] $< \int$ [illegible], [illegible]

$dG =$ [illegible]