MONOPOLES AND STRINGS IN NONCOMMUTATIVE GAUGE THEORY

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Strings 2000

With N. Nekrasov

hep-th/0005204
hep-th/0007 NEXT WEEK

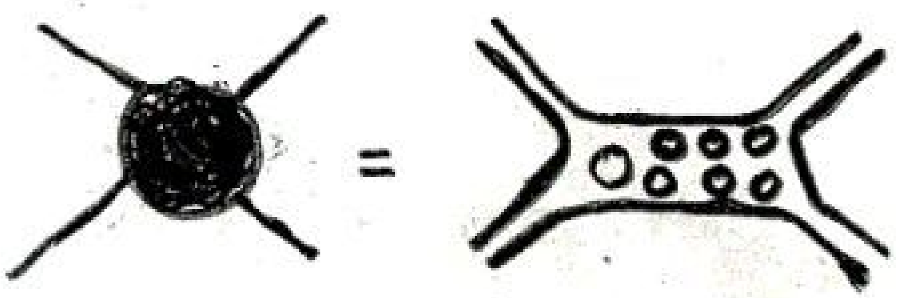
NONCOMMUTATIVE GAUGE THEORIES

- APPEAR AS LIMITS OF STRING THEORY IN A BACKGROUND B FIELD — DOUGLAS ET. AL. SEIBERG, WITTEN

- EXHIBIT INTERESTING FEATURES ~ STRING THEORY

  NON LOCALITY
  UV-IR CONNECTION
  CONTAIN STRINGS.

THEY PROVIDE SIMPLE MODELS OF STRING THEORY THAT COULD TEACH US ABOUT NON-LOCALITY IN STRING THEORY
&
THEY MIGHT BE USEFUL IN THE STUDY OF LARGE N GAUGE THEORY:



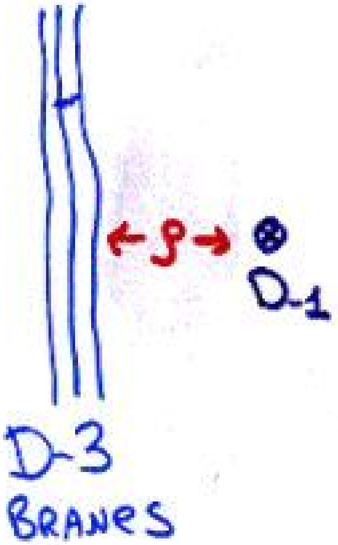
$$+\frac{1}{\theta N}$$

PLANAR x PHASES

$$G^{U(1)}[g^2,\theta] = \prod_{i<j} e^{i p_i \circ p_j}\, G^{SU(N=\infty)}[g^2 = g^2 N] + \frac{1}{\theta}$$

INSTANTONS

Nekrasov, Schwarz

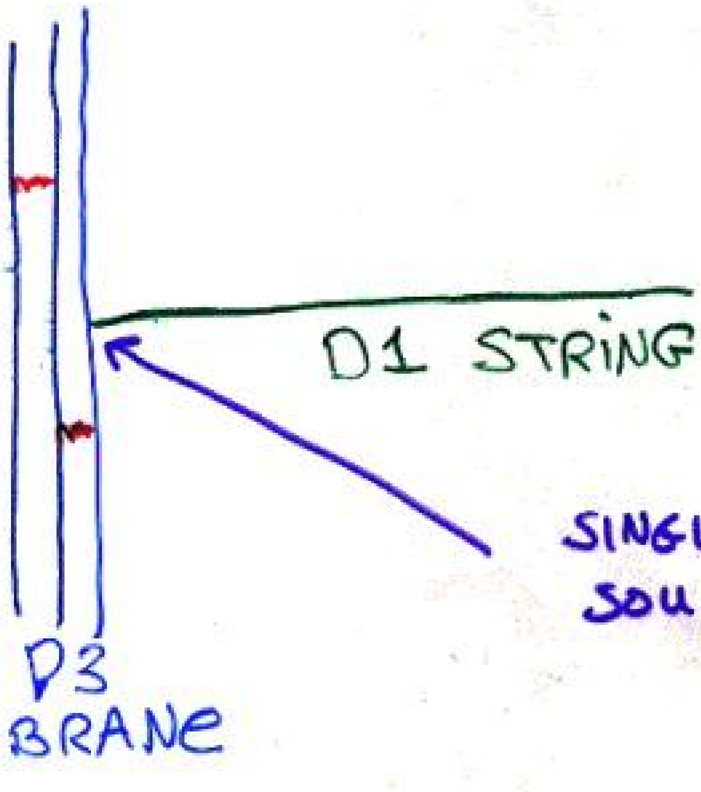


When D-1 approaches the branes → 'zero size instanton'

$B \neq 0$ RESOLVES INSTANTON MODULI SPACE

U(1) INSTANTON

MONOPOLES



— DUAL TO **ELECTRIC** FUNDAMENTAL STRING.

SINGULAR MAGNETIC SOURCE – DIRAC MONOPOLE

Non Commutative Field Theory

Fields are functions of x_3, t +

$$[X_1, X_2] = i\theta$$

- Use ordinary functions, $f(x)$, but

$$f \cdot g \longrightarrow f \ast g(x) \equiv \exp\left[\frac{i\theta}{2}\left(\frac{\partial}{\partial x_1}\frac{\partial}{\partial y_2} - \frac{\partial}{\partial x_2}\frac{\partial}{\partial y_1}\right)\right] \cdot$$
$$\cdot f(x) g(y) \Big|_{x=y}$$

$$f \ast (g \ast h) = (f \ast g) \ast h$$
$$\partial_i (f \ast g) = \partial_i f \ast g + f \ast \partial_i g$$

- $f(x) \rightarrow \hat{f}(\hat{x}_1, \hat{x}_2)$ Operator in Fock Space

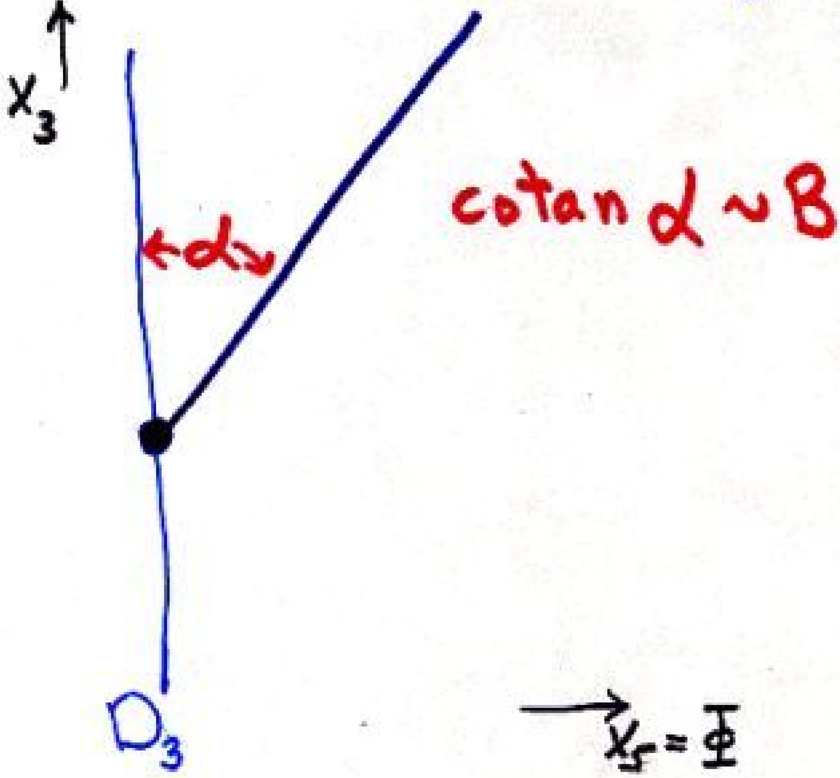
of $\alpha^+ = \frac{1}{\sqrt{2\theta}}(x_1 + i x_2) \qquad \alpha = \frac{1}{\sqrt{2\theta}}(x_1 - i x_2)$

$$\underset{\text{Wely Ordering}}{\hat{f}(\hat{x}) \equiv} \int \frac{d^2 p}{(2\pi)^2} \cdot e^{i p \cdot \hat{x}} \int d^2 y \, e^{-i p \cdot y} f(y)$$

If $\begin{matrix} f \rightarrow \hat{f} \\ g \rightarrow \hat{g} \end{matrix} \quad \Rightarrow \quad f \ast g \rightarrow \hat{f} \cdot \hat{g}$

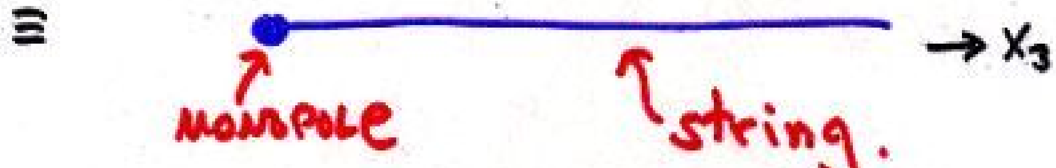
TURN ON $\frac{1}{2} B\, dx_1 \wedge dx_2$

Hashimoto[2]



Use BI approx.

FROM POINT OF VIEW OF D-3 brane



WE FIND EXACT, NON-SINGULAR, SOLUTION OF BOGOMOLNY EQUATIONS

FOR NON-COMMUTATIVE $U(1)$.

$\therefore$ D1 STRINGS DESCRIBABLE BY N.C. WEAKLY COUPLED G.T.

TRANSLATIONS = GAUGE transformations.

$$\phi(x) \to \phi(x+a)$$

$$\downarrow \qquad\qquad \downarrow$$

$$\hat{\phi} \to \hat{U}(a)\,\hat{\phi}\,\hat{U}^{+}(a)$$

$$U(a) = e^{a_i \hat{\partial}} = e^{i a_i \theta^{-1}_{ij} \hat{x}_j}$$

$$A \xrightarrow[\text{transl.}]{} U(a) A U(a)^{+} = \left[U(A-\alpha^{+})U^{+} + \alpha^{+}\right] + U\left[\alpha^{+}, U^{+}\right]$$

$$= U D U^{+} + \alpha^{+} \qquad = \text{GAUGE TRANSF.}$$

$$-(a_1 + i a_2) =$$

$\therefore$ TRANSLATION = GAUGE TRANSFORMATION + SHIFT OF A_μ BY CONSTANT

- NO LOCAL GAUGE INV. OBSERVABLES.

- COMPLETE SET OF GAUGE INVARIANT OBSERVABLES $\sim \left\langle \mathrm{Tr}\left(\Pi e^{i D \cdot a_i}\right)\right\rangle$ = OPEN WILSON LOOPS WITH DEFINITE MOMENTA

HASHIMOTO, ITZHAKI, D.G (2000)

NON-COMMUTATIVE SOLITONS: $\theta=1$

$$S_{NCYM} = \int dt\, dx_3\, \mathrm{Tr}\left(F_{\mu\nu}^2 + (\nabla \Phi)^2 \right)$$

Adjoint Scalar

$$F_{\mu\nu} = [\nabla_\mu, \nabla_\nu]$$

where $\nabla = \nabla_1 + i\nabla_2 \rightarrow \hat{\alpha} + \hat{A}$

$\nabla \Phi \rightarrow [\hat{\alpha} + \hat{A}, \hat{\Phi}]$

$\Phi_a = \hat{n}_a \Phi$

$\hat{n} \in S^5$

$$\int dx_1 dx_2\, f(x) \rightarrow \mathrm{Tr}(\hat{f}(\hat{x}))$$

NO GAUGE INVARIANT LOCAL OBSERVABLES.

$$\mathcal{E}_{NERGY} = \frac{1}{2g^2} \int dx_3\, \mathrm{Tr}\left[B_i^2 + [\nabla_i, \phi]^2 \right]$$

for time independent $- A_0 = 0 = E_i -$ configurations.

BPS EQUATIONS

$$[\nabla_i, \phi] = \pm B_i = \pm \frac{1}{2} \epsilon_{ijk} [\nabla_j, \nabla_k]$$

Easily Generalize NAHM eq's

Diaconescu

The Monopole-String Solution.

$$\Phi = \sum_N \phi_N |N\rangle\langle N|$$

$$\phi_n(x_3) = (\hat{n}-1)\eta_{n-2} - \hat{n}\,\eta_{n-1} \qquad n>0$$

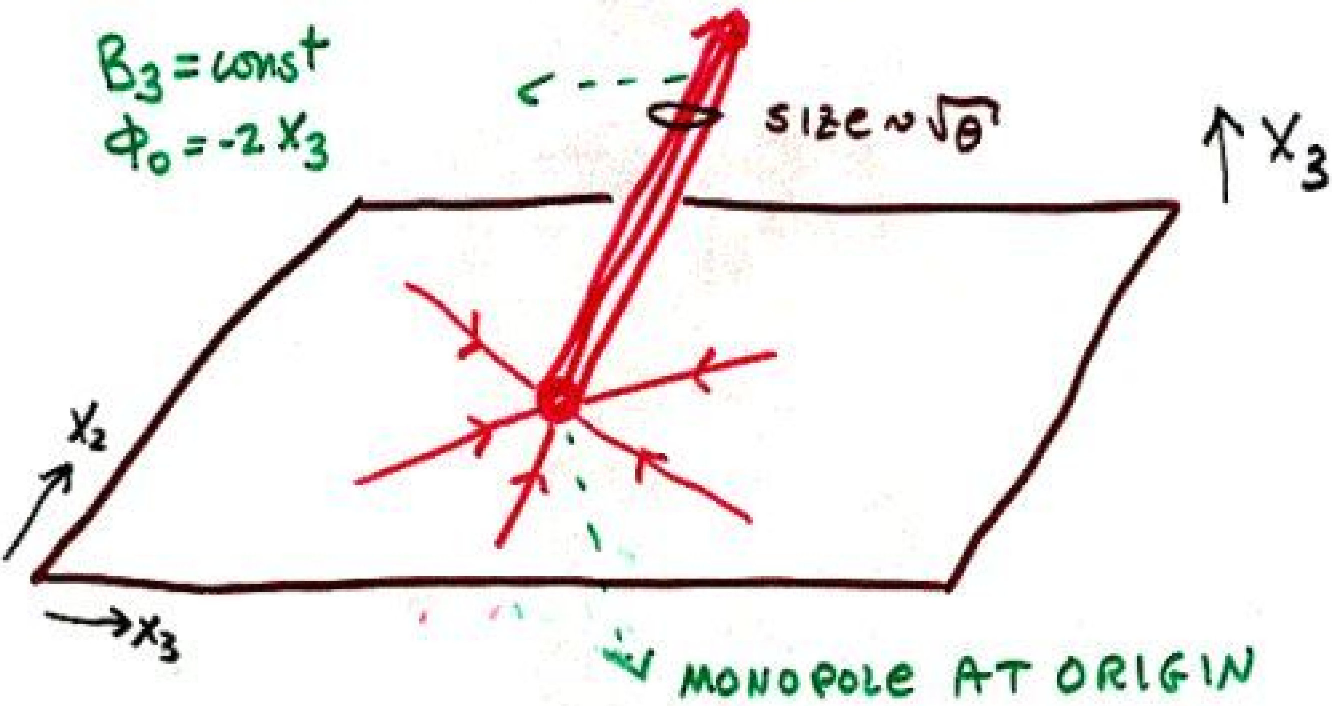
$$\phi_0(x_3) = -2x_3 - \frac{1}{\zeta_0} \qquad n=0$$

$$A = A_1 + iA_2 = \alpha^+\left(1 - \frac{\xi(n)}{\xi(n+1)}\right)$$

FINITE, NON SINGULAR.

$$\zeta_n = \int_0^\infty d\rho\, \rho^n e^{-\rho^2/2 + 2\rho x_3}, \quad \eta_n = \frac{\zeta_n}{\zeta_{n+1}}, \ldots$$

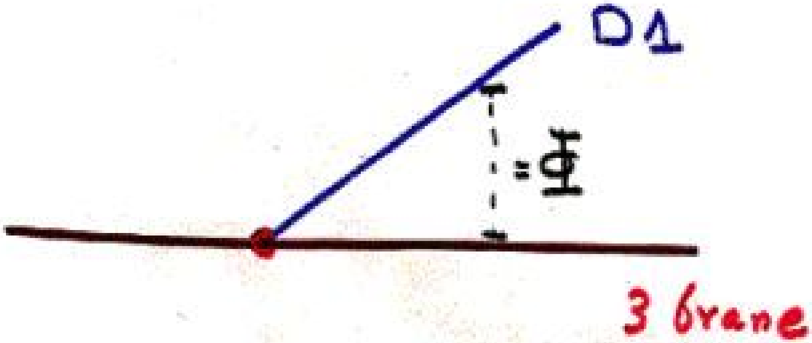
$$\phi_n(x_3=0) = \sqrt{2}\left(\frac{\left(\frac{n-1}{2}\right)!}{\left(\frac{n-2}{2}\right)!} - \frac{\left(\frac{n}{2}\right)!}{\left(\frac{n-1}{2}\right)!}\right)$$



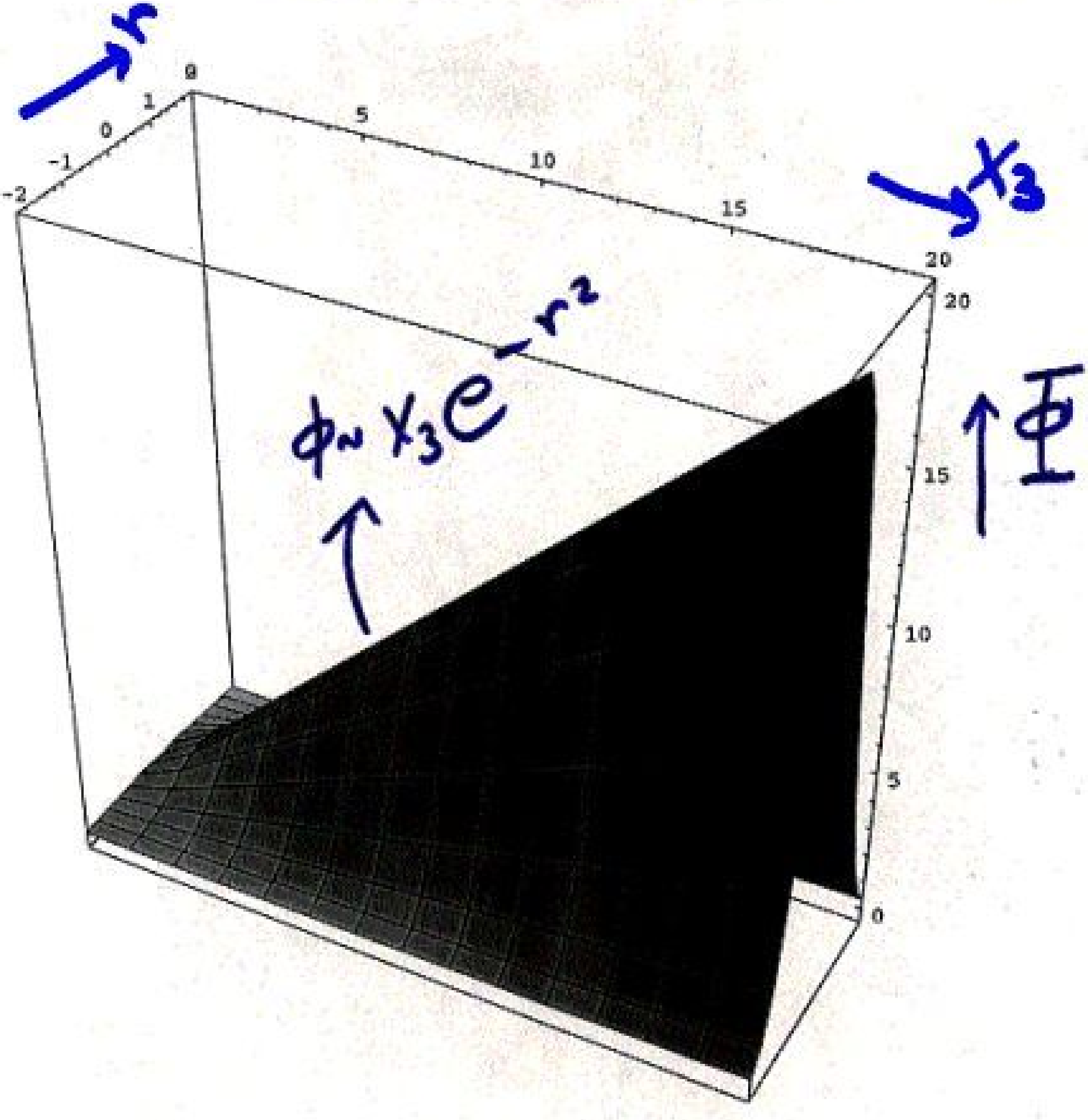
$$\phi \sim -\frac{2X_3}{\theta}\,\theta(X_3)\,e^{-(X_1^2+X_2^2)/2\theta} - \frac{1}{2|X|}$$

$$\vec{B} \sim \frac{+2}{\theta}\hat{X}_3\, e^{-\frac{x_1^2+x_2^2}{\theta}} - \frac{\vec{X}}{2|X|^3} \qquad |X| \gg \sqrt{\theta}$$

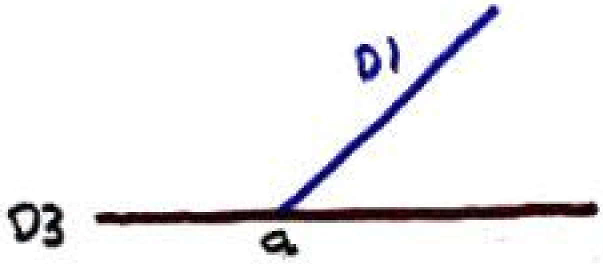
Looks like:



$\Phi(r=\sqrt{x_1^2+x_2^2},\ x_3)$



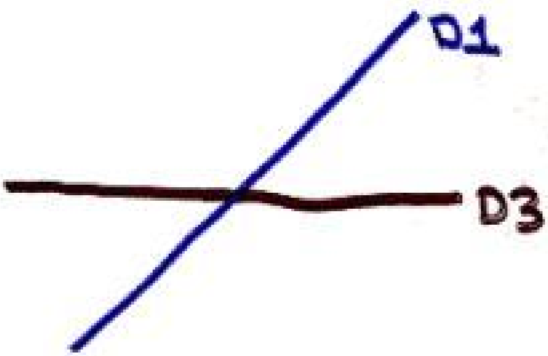
The Fluxon



$$\hat{\Phi}(x_3) = -2(x_3 - a)\hat{P}_0 + O\left(\frac{1}{a}\right)$$

$$\hat{B}_3(x_3) = +2\hat{P}_0 + O\left(\frac{1}{a^2}\right)$$

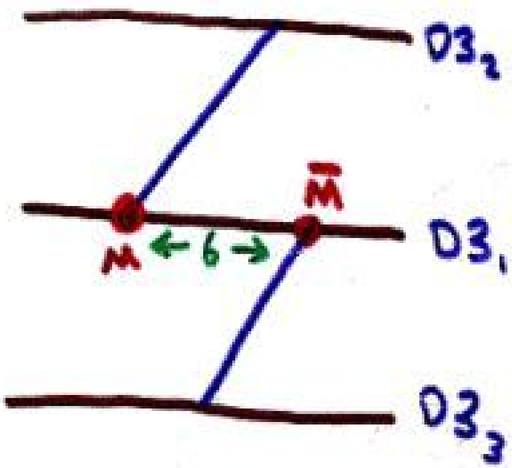
↓ Take: $a \to \infty$, $\hat{\Phi} \to \hat{\Phi} - 2a\hat{P}_0$



$$\hat{\Phi} = -2x_3\hat{P}_0$$

$$\hat{B}_3 = 2\hat{P}_0$$

More Generally:



DESCRIBES A MONOPOLE -ANTI MONOPOLE PAIR in $U_3 \xrightarrow[\text{HIGGSED}]{} U(1)$

FLUXON = PAIR WHEN $\sigma = 0$.

Properties of the Fluxon.

$$\Phi = -2x_3 P_0 = -2x_3 |0\rangle\langle 0|$$

$$A = A_1 + iA_2 = \alpha^+\left(1 - \sqrt{\frac{N}{N+1}}\right) \quad A_3 = 0$$

$$D = -\alpha^+\sqrt{\frac{N}{N+1}} \quad \bar{D} = \sqrt{\frac{N}{N+1}}\,\alpha$$

$$B_3 = 2\left([D, \bar{D}] + 1\right) = 2P_0$$

<u>BPS EQS</u>

$$B_3 = -\partial_3 \Phi \quad \checkmark$$

$$\left\{\begin{array}{l} [D, \phi] = [\bar{D}, \phi] = 0 \\ B_i = \epsilon_{ij}\partial_3 A_j = 0 \end{array}\right\} \checkmark$$

USING $\quad DP_0 = P_0 D = \bar{D}P_0 = P_0\bar{D} = 0.$

$$\bar{D} = S\alpha S^+ \qquad S = \alpha^+\frac{1}{\sqrt{N+1}} = \text{Shift}$$

$$S^+S = 1 \qquad S|N\rangle = |N+1\rangle$$

$$SS^+ = 1 - P_0.$$

Almost pure GAUGE.

Tension:

$$\mathcal{E}\text{nergy}/_{\text{per unit } x_3} = \frac{\Omega}{4g^2\theta}\,\mathrm{Tr}\left(B^2 + [D,\phi]^2\right)$$

$$T = \frac{2\pi}{g^2\theta}$$

Com. Limit $\theta \to 0$.

$$P_0 \longmapsto e^{-(x_1^2+x_2^2)/\theta} \to \theta\,\delta(x_1)\,\delta(x_2)$$

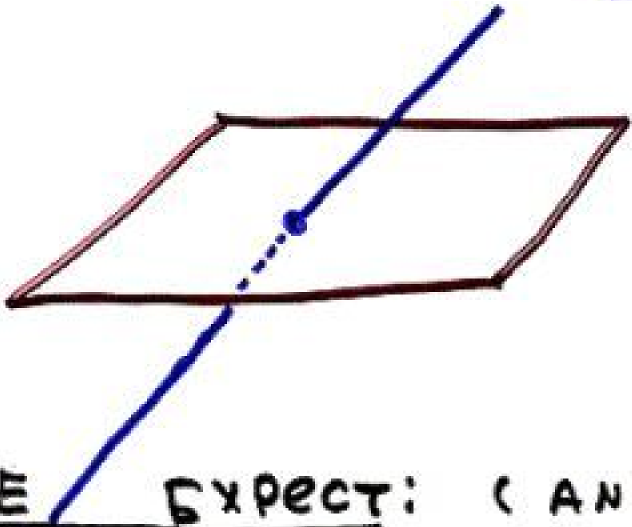
$$\Phi = -x_3\,\delta(x_1)\,\delta(x_2)$$

$$A = \frac{x_1\,dx_2 - x_2\,dx_1}{2\pi\,(x_1^2+x_2^2)}$$

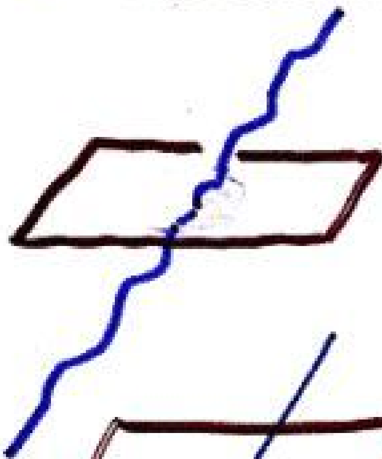
$$B = dA = \delta(x_1)\,\delta(x_2)$$

SINGULAR SOLENOID ALONG $\hat{x}_3$

Fluctuations About the Fluxon

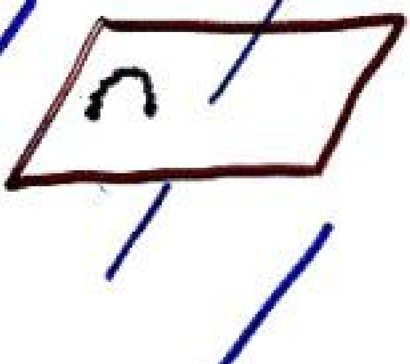


WE EXPECT: (AND WE FIND)



Fluctuations of D1 susy string - in 10 DIM 11

- Stable to emission of closed strings or to breaking.
- NO UV CUTOFF ON MODES



Spectrum of photons + scalars + fermions

33

Spectrum of fundamental strings attached to D1 + to D3

- localized, bound, at $x_3=0$. 13
- DISCRETE ENERGIES.

Fluctuations

EXPAND A_a, $\phi_i = i A_{3+i}$
$a = 1..10$, $i = 1..6$

About: $A_a^0 = (A_0 = 0, A_1, A_2, A_3 = 0, A_4 = -i\Phi, 0, 0, 0, 0, 0)$

$A_a = A_a^0 + g\, a_a$, $[D_a^0, a_a] = 0$

$$\mathcal{L} = \frac{2\pi}{g^2\theta}\, \underline{D}_0$$
$$-\tfrac{1}{2}\{ [D_a^0, a_b]^2 + 2F_{ab}^0 [a_a, a_b] + \bar{\lambda} \not{D}^0 \lambda \}$$
$$-g\{ [D_a^0, a_b][a_a, a_b] + \bar{\lambda}\Gamma_a [a_a, \lambda] \}$$
$$-\tfrac{g^2}{4} [a_a, a_b]^2$$

SPECTRUM

33 $a_a(t, x_3, x_1 x_2) \sim \zeta_A(\omega, \bar{k})\, e^{i(\omega \cdot t - k \cdot x)}$

$\omega^2 = k^2$
$\omega \zeta_0 + k \cdot \zeta = 0.$

11 $a_a \sim \zeta_a(\omega, k)\, e^{i(\omega t - k x_3)}$

$\omega \zeta_0 + k \zeta_3 = 0$

ON SHELL $\omega = \pm k$ GAUGE AWAY ζ_0 or ζ_1 $\Rightarrow$ 8 Physical MODES.

13:

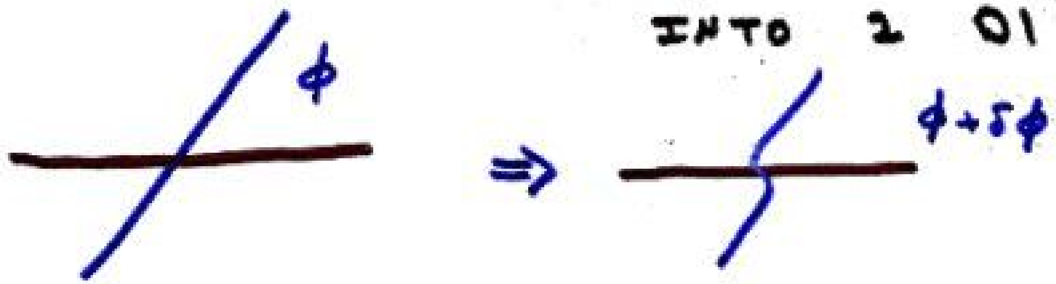
$$a_1 + i a_2 \sim \overset{\text{Hermite poly.}}{H_n(x_3)} e^{i\omega t - x_3^2} |0\rangle\langle m|$$

$$\omega^2 = 4(n+m+1) \qquad m = 1 \dots \infty,\ n = 0 \dots \infty$$

$$\underset{\delta\phi}{a_3 + i a_4} \sim H_n(x_3) e^{i\omega t - x_3^2} |0\rangle\langle m|$$

$$\omega^2 = 4(n+m-1). \qquad m = 1 \dots \infty,\ n = 0 \dots \infty$$

$m = 1$, $n = 0$: zero mode that splits fluxon ~~into~~ into 2 D1 strings



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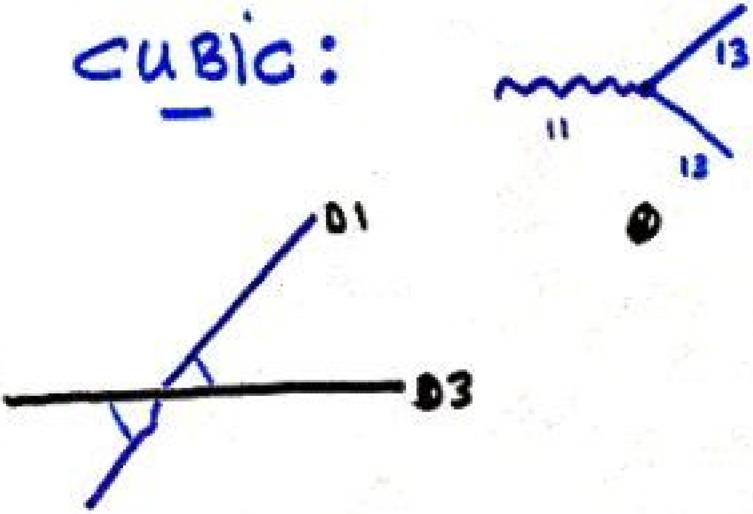
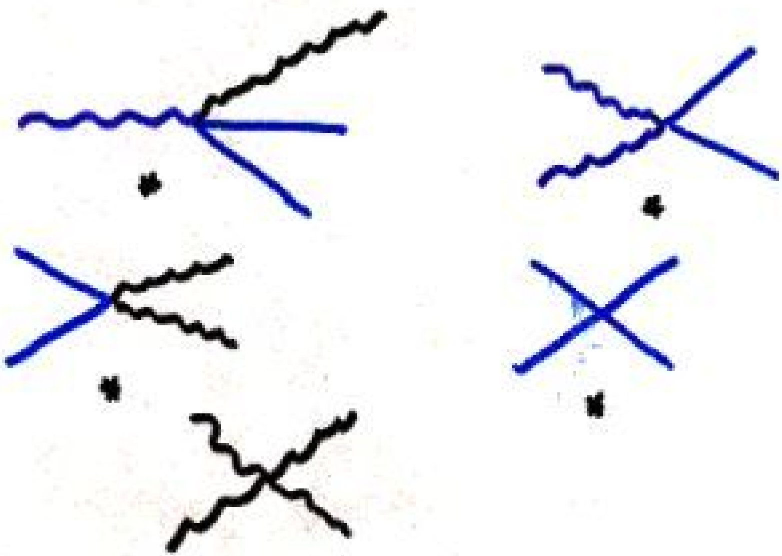
INTERACTIONS

CUBIC:



USUAL CUBIC INTERACTIONS OF $\mathcal{N}=4$ NC SUSYM.

QUARTIC:



* All these interactions are localized at $X_3 \sim 0$.

Dyons

Energy of U(1) NC YM $\propto \int dx_3 \, \mathrm{Tr}\left\{ E^2 + B^2 + [D, \phi]^2 \right\}$

$$\geq \int dx_3 \, \mathrm{Tr}\left\{ (E + \sin\alpha [D, \phi])^2 + (B + \cos\alpha [D, \phi])^2 \right\}$$

BPS: $\cos\alpha [D_i, \Phi] = B_i$

$\sin\alpha [D_i, \Phi] = E_i$

$\Rightarrow \quad A_i = A_i^{MON} \qquad \phi = \dfrac{\Phi^{MON}}{\cos\alpha}$

$A_t = -i \tan\alpha \, \Phi^{MON}$

$Q_e = \tan\alpha \, Q_m =$ integer upon QUANTIZATION

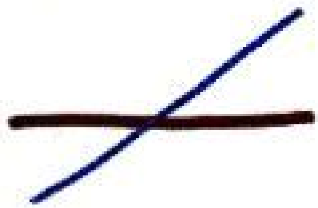
$\sim$ (1, q) strings.

CONCLUSIONS

- U(1) NCGT HAS BPS, NONSINGULAR MONOPOLE (AND DYON) STRING SOLUTIONS THAT DESCRIBE

1. SEMI-INFINITE D1 STRINGS AND

2. INFINITE, PIERCING D1 STRINGS



- THE FLUCTUATIONS OF THE NCGT IN THE SOLITON BACKGROUND ARE THOSE OF

1. PHOTONS AND HIGGS AND FERMIONS OF *N=4 SUSYM*

2. A 10 DIMENSIONAL SUPERSTRING

3. FUNDAMENTAL STRINGS LOCALIZED AT THE INTERSECTION OF THE D1 STRING AND THE D3 BRANE.

EXTENSIONS

- $U(N)$

  FINITE STRINGS

- $\theta \to \infty$

  $$T = \frac{2\pi}{g^2\theta} \to 0$$

  What are the Implications for LARGE N CYM?

-
-
-