D-BR
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NONCOMM
SOLI

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(G.

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Ond E. Martinec Moore)

Executive Summary

- We expect that we can describe D-branes as solitons in weakly coupled open string field theory
- This can be done exactly using NC geometry and a scaling limit.
- The limit is related to, but distinct from the limit giving NC field theory. The simplification is that the solitons, as seen by open strings, are much larger than l_s.
- There is an interesting puzzle in Type II involving light branes
- These constructions give a new perspective on the role of K-theory in classifying D-branes (G. Moore)

References

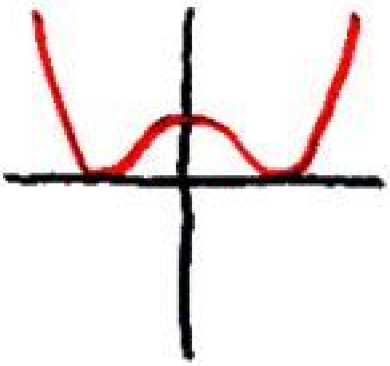
NC Solitons	R. Gopakumar, S. Minwalla, A. Strominger	003160
D-branes + NCS	JH, P. Kraus, F. Larsen, E. Martinec	0005031
	K. Dasgupta, S. Mukhi, G. Rajesh	0005006
	E. Witten	0006071

Unstable D-branes
Tachyon Condensation
NC geometry in String theory

$\Bigg\}$ usual suspects

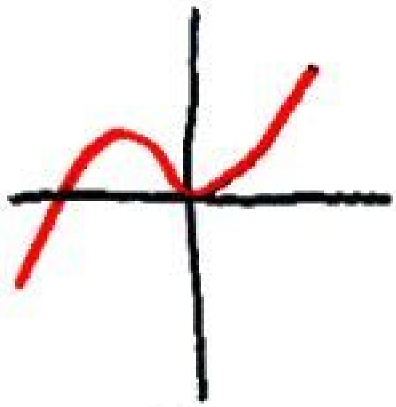
Sen's work on unstable D-brane systems suggests much of our understanding of symmetry breaking and solitons can be extended to string theory:

'Tachyon' (Higgs) potentials



non-BPS-Dp
Type II

Dp-$\bar{D}p$
Type II

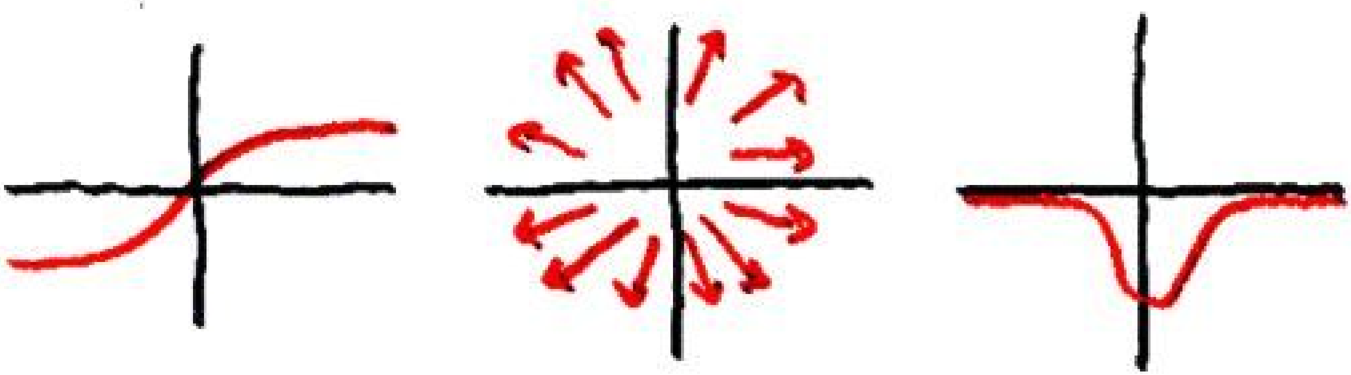


Dp
bosonic

Maximum of V - Unstable D-brane

Minimum of V - Closed string vacuum w/o open strings

'Solitons' - lower-dimensional D-branes



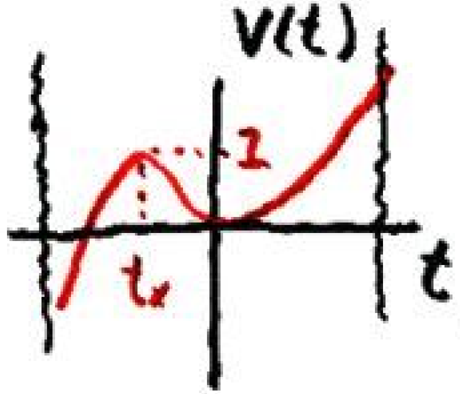
kink vortex lump

D-branes as lumps in bosonic string

Open-bos. string ~ space-filling D25
w/ tachyon t

$$S_{eff} = T_{25}\int d^{26}x\sqrt{g}\left(\tfrac{1}{2}f(t)g^{\mu\nu}\partial_\mu t\,\partial_\nu t + \underbrace{\cdots}_{\text{higher }\partial\text{'s}} - V(t)\right)$$

Truncated OSFT,
RG-flow $\Rightarrow$



e.g. at level 0,

$V(t_*) = 0.68$ $\qquad f(t) = 1$

p-lumps:

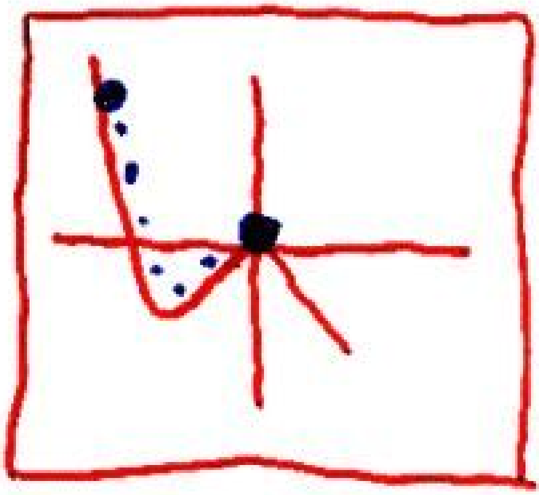
Consider the e.o.m. for a 'p-brane' assuming spherical sym. in transverse dimensions $\rho^2 = (x^{25})^2 + \cdots (x^{p+1})^2$

$$\boxed{\partial_\rho^2 t + \frac{24-p}{\rho}\partial_\rho t - V'(t) = 0}$$

w/ $\rho \sim$ time, $\frac{\partial_\rho t}{\rho} \sim$ damping this is particle motion in potential $-V$

p	T_p^ℓ / T_p
24	.78
23	.71
22	.72
21	.59
20	.30

from P. Kraus + JH



This gives crude qualitative agreement at large p, but improving this requires much hard work, and some features of D-branes, such as the non-Abelian gauge structure, seem out of reach.

These problems can be solved using NC geometry.

As a specific example, consider the construction of a D23 as a lump on D25-brane.

We will introduce $B_{24,25} = B$ and take $\alpha' B \to \infty$.

Then in S_{eff}^{open} we should make the following substitutions:

Fields $AB \to A * B = e^{\frac{i}{2}\Theta^{ij}\partial_i \partial_j'} A(x) B(x')\big|_{x=x'}$

NC param $\Theta^{ij} = (1/B)^{ij}$

Metric $g_{ij} \to G_{ij} \simeq (2\pi\alpha' B^2)_{ij}$

Coupling $g_s \to G_s \simeq g_s\sqrt{G}/(2\pi\alpha' B\sqrt{g})$

Note: In coor. w/ $G_{ij} = \delta_{ij}$, $\alpha' B_{ij} \to 0$, $g^{ee}_{ij} \to 0$

Thus $S_{eff}(t) \simeq \frac{g_s T_{25}}{G_s}\int d^{26}x\sqrt{G}\left(G^{\mu\nu}\partial_\mu t\, \partial_\nu t + \cdots - V(t)\right)$

$$\xrightarrow[\alpha' B \to \infty]{} -\frac{g_s T_{25}}{G_s}\int d^{26}x\,\sqrt{G}\; V(t).$$

e.o.m. is $V'(t) = 0$

Take $V'(t) = t * (t-\lambda_1) * \cdots (t-\lambda_N)$

To find non-trivial soln's apply GMS:

Comm. alg. of fcn's on $\mathbb{R}^2$ $\underset{B\neq 0}{\Longrightarrow}$ NC alg. of fcns $\simeq$ algebra of op's on Hilbert space $\mathcal{H}$

$$A * B \leftrightarrow \hat{A}\hat{B}$$

$$\frac{1}{2\pi\theta}\int d^2x \longleftrightarrow \mathrm{Tr}_{\mathcal{H}}$$

Soln's are $\hat{t} = \sum_{i=1}^{k} \lambda_i \hat{P}_i$

orthog. proj. ops.

In our example w/ one non-trivial stationary pt.

rank k proj. op.

$$\hat{t}_{(k)} = t_* \hat{P}_{(k)}$$

Tension: $S_{eff} = -(2\pi)^2\alpha' T_{25}\int d^{24}x\sqrt{g}\;\underbrace{V(t_*)}_{1}\;\underbrace{\mathrm{Tr}_{\mathcal{H}}\, P_{(k)}}_{k}$

$$\Rightarrow T_{23}^{\text{lump}} = k\, T_{23}^{\text{D-brane}}$$

As a further check that this sol'n represents k - D23-branes, compute the spectrum of fluctuations:

Tachyons on D23 :
$$\delta t = \sum_{n,m=1}^{k} \delta t^{nm}(x^\alpha)\, |n\rangle\langle m|$$

basis of $\hat{P}_{(k)} \mathcal{H}$

Gives k^2 tachyons w/ $(\text{mass})^2 = -1/\alpha'$

Gauge fields: D25 has $U(1) \to NC\ U(1)$ w/ tachyon in adjoint rep.

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i(A_\mu * A_\nu - A_\nu * A_\mu)$$

$$D_\mu t = \partial_\mu t - i(A_\mu * t - t * A_\mu)$$

Choose a basis $|n\rangle$ of $\mathcal{H}$. $\hat{A}_\mu = \sum_{nm} A_\mu^{nm} |n\rangle\langle m|$

$$S_{\text{eff}} \sim T_{23} \int d^{24}x \left(-\tfrac{1}{4} \underbrace{F^{\mu\nu}_{nm} F^{nm}_{\mu\nu}}_{U(\infty)\ \text{gauge sym.}} + \cdots \right)$$

$t = t_* P_{(k)}$ breaks $U(\infty) \to \underset{\text{massless}}{U(k)} \times \underset{\text{Higgsed}}{\cancel{U(\infty - k)}}$

One similarly finds that all massive string states descend to massive modes on the D23.

This can be understood directly in SFT in terms of factorization of the string algebra (Witten)

$$\mathcal{A} = \mathcal{A}_0 \otimes \mathcal{A}_1$$

$\alpha' B \to \infty$

$\mathcal{A}_0$: osc. alg. + fcns in comm directions

$\mathcal{A}_1$: alg. of fcn's in non-comm directions

$\Rightarrow$ Can solve string e.o.m. $QA + A * A = 0$ by

$$A = A_0 \otimes \rho$$

A_0: assumed sol'n describing closed string vacuum

ρ: proj. op. on $\mathcal{H}$

Comments:

- In this limit we get exact results for the lower D-brane tension and spectrum

- In terms of $\alpha' B$, g_{ij}, this limit is the same as the $\alpha' \to 0$ limit giving NC field theory from string theory. However, we keep α' fixed, do <u>not</u> take $\alpha' E^2_{open} \to 0$, and find string theory w/ large NC rather than field theory w/ arbitrary NC.

- After tachyon condensation we are in the closed string vacuum away from the D23, and we can gauge $B \to 0$ - so we are describing D-branes in the usual Lorentz-invariant vacuum

- If we were to consider a general D23 w-volume M^{24}, then tachyon fields varying slowly on M^{24} would be labelled by maps

$$t : M^{24} \to \left\{ \begin{array}{l} \text{space of rank } k \\ \text{proj. ops in } \mathcal{H} \end{array} \right\} \equiv BU(n)$$

If M^{24} has non-trivial topology, there can be non-trivial homotopy classes of such maps which would classify D-branes (G. Moore)

Extension to Type II D-branes

(also Dasgupta, Mukhi + Rajesh)

Non-BPS systems

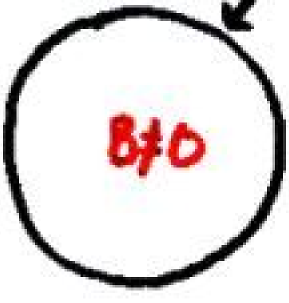
There is a $\mathbb{Z}_2$ (gauge) symmetry and 2 critical pts. at $t \neq 0$

Solutions : $t = t_* P_{(K)}$ K- nBPS D7, D5, ⋯ in IIA

$t = 2t_* P_{(K)}$ p - branes w/ p odd and $T_p \propto V(2t_*) = 0$!

Coupling $\int C \wedge dT \Rightarrow$ semi-classical pic (Martinec, DMR)

loop of BPS D-brane



$B = 0$

The vanishing tension as $\alpha' B \to \infty$ is puzzling as are possible resolutions

- There are $1/\alpha' B$ corrections to tension – but since we can gauge $B \to 0$ at ∞, B would be a continous internal modulus, w/ solutions massless and stable as $\alpha' B \to \infty$

- g_s corrections to the tension would modify string perturbation theory

Dp-$\overline{Dp}$ systems

According to Sen, we can construct $D(p-2)$ BPS branes as vortices on Dp-$\overline{Dp}$ system.

Is this also simplified by large B?

Effective field theory: $U(1) \times U(1)$ gauge

$(1,-1)$ complex tachyon

$$S_{eff} = \int dz\, d\bar{z} \left(\overline{D_\mu t}\, D^\mu t - \tfrac{1}{4} F^+_{\mu\nu} F^{+\mu\nu} - \tfrac{1}{4} F^-_{\mu\nu} F^{-\mu\nu} - V(\bar{t} t) \right)$$

w/ $D_\mu t = \partial_\mu t + i\left(A^+_\mu t - t A^-_\mu\right)$

As before, we introduce B-field in 2 transverse dimensions, take $\alpha' B \to \infty$ and drop deriv's in $z, \bar{z}$ in this limit

This leads to e.o.m.

$$[A^{+\nu},[A^+_\nu,A^+_\mu]] = A^+_\mu t\bar{t} - tA^-_\mu\bar{t} + t\bar{t}A^+_\mu - tA^-_\mu\bar{t}$$

$$[A^{-\nu},[A^-_\nu,A^-_\mu]] = A^-_\mu\bar{t}t - \bar{t}A^+_\mu t + \bar{t}tA^-_\mu - \bar{t}A^+_\mu t$$

$$-A^+_\mu A^{+\mu}t + 2A^{+\mu}tA^-_\mu - tA^{-\mu}A^-_\mu = -tV'$$

<u>Sol'n</u> Let $|n\rangle$ $n = 0, 1, \cdots$ be a basis for $\mathcal{H}$

$$\bar{t} = t_* S, \qquad S|n\rangle = |n+1\rangle$$

$$A^+_z = t\bar{t} = \mathbb{1}$$

$$A^-_z = \bar{t}t = \mathbb{1} - |0\rangle\langle 0|$$

The D-(p-2) charge is $\mathrm{Ind}(t) = 1$.

More generally, using SFT analysis one requires

$$t\bar{t}t = t$$

$$\bar{t}t\bar{t} = \bar{t}$$

This solution has an interesting generalization. In Witten's discussion of K-theory and D-branes the following construction appeared:

To describe BPS p-branes in IIB w/ codimension $p' = 2k$, start w/ $N = 2^{k-1}$ D9 - $\overline{D9}$ pairs and write

$$t = f(r)\,\frac{\vec{\Gamma}\cdot\vec{x}}{|\vec{x}|}$$

$$\Gamma : \underbrace{S_+ \to S_-}_{\text{spinor reps of } SO(2k)}$$

When restricted to S^{2k-1} in transverse space, t generates $\pi_{2k-1}[U(2^{k-1})]$ (ABS) and the tachyon has winding one.

This has a non-commutative version.
Turn on a large B-field in $\mathbb{R}^{2k}$ and skew-diagonalize:

$$\Theta^{ij} = \begin{pmatrix} 0 & \theta_i & & \\ -\theta_i & 0 & & \\ & & \ddots & \\ & & & \ddots \end{pmatrix} \quad \text{so} \quad [x^{2i-1}, x^{2i}] = -i\,\theta_i$$

Write the Clifford algebra as

$$\gamma_i = \begin{pmatrix} 0 & \Gamma_i \\ \bar{\Gamma}_i & 0 \end{pmatrix}$$

and take $t : H \otimes S^- \to H \otimes S^+$ to be

$$t = f\,\Gamma_i x^i$$

One can solve

$$t\bar{t}t = t$$
$$\bar{t}t\bar{t} = \bar{t}$$

for f to find

$$\bar{t} = \bar{\Gamma}_i x^i \frac{1}{\underbrace{\sqrt{\Gamma_i x^i \bar{\Gamma}_j x^j}}_{\ker = \emptyset}}$$

Note that this reduces to the previous sol'n for $2k = 2$ w/

$$\gamma^1 = \sigma^1, \quad \gamma^2 = \sigma^2, \quad \Gamma^1 = 1, \quad \Gamma^2 = -i$$

$$\bar{\Gamma}_i x^i = a^\dagger$$

and one can check that again $\mathrm{ind}(t) = 1$.

The connection between the index of t and the winding number of the ABS configuration involves non-trivial mathematics (to be explained by Greg)