Enhançons,

Fuzzy Spheres

and

Multi-Monopoles

By

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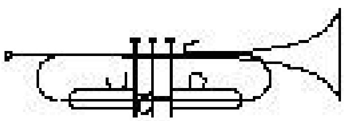


based on

hep-th/9911161 (c.v.j., Peet and Polchinski)
hep-th/0004068 (c.v.j.)
also c.v.j., unpublished

Outline

- Motivation
- Enhançon Recap.
- Closer Look: Smearing of Branes and Fuzzy Geometry
- Closer Look: Multi-monopoles and Nahm Data
- Seasoning to Taste*
- More Probing*
- Closing Remarks: Searching for dual descriptions....

****probably won't get to it in time!***

Motivation

Lots of overlapping motivations:

- Search for supergravity/stringy duals of pure gauge theory

  .... fewer susy's
  .... what of the Seiberg-Witten story
  .... what about fundamental matter?
  .... etc...

- The study of certain types of singularities

- Search for other duals of large N gauge theory

  (non-gravitational?)

- Better detailed understanding of the "enhançon"

- Etc...

Enhançon Recapitulation

- Place Q D(p+4)-branes on a K3 of volume V (say in x^6, x^7, x^8, x^9).

- Wrapping induces the negative charge of Q Dp-branes in worldvolume (due to $R \wedge R$ term)

- *SU(Q)* p+1 dimensional N=2 gauge theory on world volume

- Write down naïve supergravity solution in search of large Q dual

- Sugra gets asymptotic behaviour right…fails in interior. Naked *"repulson"* singularity.

- There is stringy behaviour in inside – new massless degrees of freedom. *6 dim. U(1) enhanced to SU(2) at small enough radius:*

  where $V_{K3} = (2\pi)^4 \alpha'^2 = V_*$

- This physics switches on in neighbourhood of *enhançon locus*.

- Probing with wrapped D-brane reveals how to repair interior geometry.

  Take "decoupling" limit $\alpha' \to 0$, keep $U = r/\alpha'$ and g_{YM} fixed….

e.g., take $p=2$: wrap D6-branes on K3, resulting in membrane in 6D, a point in x^3, x^4, x^5

Moduli space metric as seen by membrane probe:

$$ds^2 = \frac{1}{8\pi^2 g_{YM}^2}\left(1-\frac{\lambda}{U}\right)\left(dU^2 + U^2 d\Omega_2^2\right) + 8\pi^2 g_{YM}^2\left(1-\frac{\lambda}{U}\right)^{-1}\left(d\sigma - \frac{Q}{8\pi^2}A_\phi d\phi\right)^2$$

$\lambda = Q g_{YM}^2;\ A_\phi = \pm 1 - \cos\theta$ ('t Hooft coupling and Dirac monopole potential)

- *This is tree level + one loop result expected from gauge theory.*

Enhançon is $U=\lambda$, where probe becomes massless.

Moduli space metric is singular…

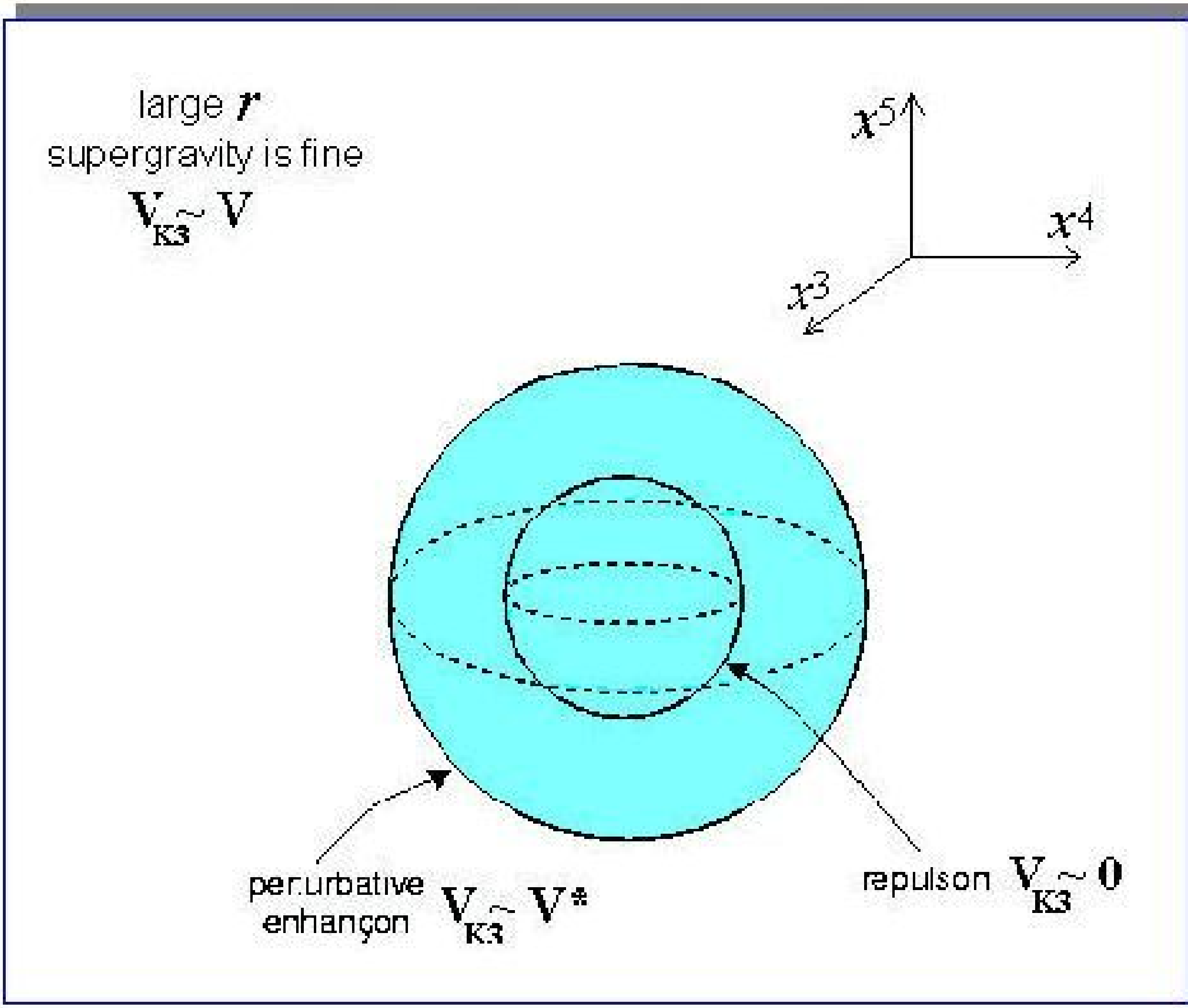
…Non-perturbative instanton corrections…

The supergravity?

- *Sugra solution is good far away from enhançon, and then matches onto approximately flat geometry inside.*
- *Details of smoothing to be worked out…*

Note: full spacetime geometry does not clean up nicely in decoupling limit! *(One reason why there is no pure sugra dual.)*

Enhançon Summary



- At enhançon locus, constituent branes become massless.
- *There is an enhanced gauge symmetry there, since e.g., wrapped D4's play the role of W-bosons.*
- *The probe is a monopole and so become delocalised as they get to the enhançon. Whole geometry is made of lots of these monopoles.*
- *Geometry inside is flat, since sources smeared out at* enhançon

Closer Look

The picture is not complete

- Want more data on how exterior glues smoothly onto approximately flat interior.

- For p=2, effective wrapped brane is a BPS monopole of the $SU(2)\rightarrow U(1)$ problem

Other p will have different details…e.g. $p=3$ should involve vortices….

- We have a large, heavy charge Q monopole.

- Near enhancon radius, monopole probes cease to be pointlike

Smeared brane: Expect non-commutative geometrical description to take over

Do we have such a description available?

YES! It is *via* Nahm's Equations! *(later)*

Multi-Monopoles

So (at least part of) our system ($p=2$) has a description as the Q BPS monopoles of $SU(2) \rightarrow U(1)$

Reminder of the Bogomol'nyi system:

$$B_i \equiv \frac{1}{2}\varepsilon_{ijk}F_{jk} = D_i\mathrm{H}, \quad \text{with}$$

$$F_{ij} = \partial_i A_j - \partial_j A_i + [A_i, A_j]; \quad D_i\mathrm{H} = \partial_i \mathrm{H} + [A_i, \mathrm{H}]$$

there is a gauge invariance ($g(\mathbf{x}) \in SU(2)$)

$$A_i \rightarrow \mathbf{g}^{-1}A_i\mathbf{g} + \mathbf{g}^{-1}\partial_i\mathbf{g}; \quad \mathrm{H} \rightarrow \mathbf{g}^{-1}\mathrm{H}\mathbf{g}$$

and an asymptotic condition on the Higgs field $\mathrm{H}(\mathbf{x})$:

$$\|\mathrm{H}(x)\| \equiv \frac{1}{2}\mathrm{Tr}[\mathrm{H}^*\mathrm{H}] \rightarrow H \quad \text{as} \quad r \rightarrow \infty$$

Example: One monopole

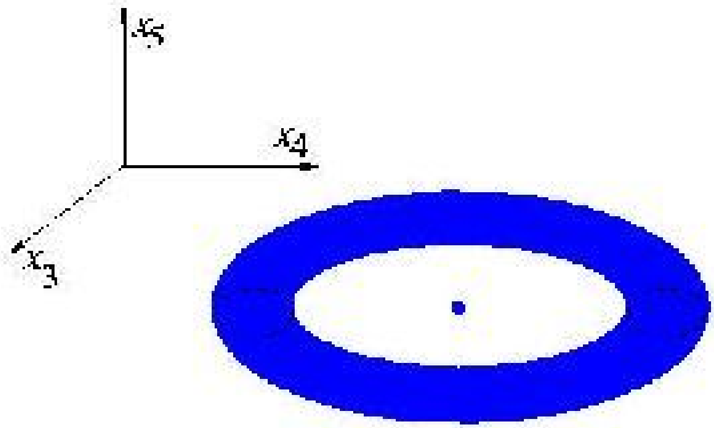
$$\mathrm{H}(r) = \frac{1}{r}\left(\coth r - \frac{1}{r}\right)i\sigma_i x_i; \quad A_i(r) = \frac{1}{r}\left(\frac{1}{\sinh r} - \frac{1}{r}\right)i\varepsilon_{ijk}\sigma_j x_k$$

$$H \rightarrow 1 - r^{-1} + \ldots \qquad \mathbf{x} = (0,0,r)$$

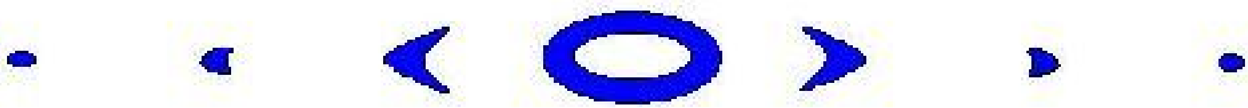
Example: Two monopoles

The two-monopole system is well studied and understood

- It is not spherically symmetric
- Most symmetric solution is coincident case (axisymm)



- Dot is two coalesced Higgs zeros
- For large separations, looks like two 1-monopoles, then they deform as they approach.....



Familiar?

- Moduli space of configurations is Atiyah-Hitchin Space

Atiyah-Hitchin Space

The *relative* moduli space of two monopoles is four dimensional: a position and a phase.

The metric on this space is:

$$ds_{\mathrm{AH}}^2 = f^2 d\rho^2 + a^2\sigma_1^2 + b^2\sigma_2^2 + c^2\sigma_3^2 \,,$$

$$\sigma_1 = -\sin\psi d\theta + \cos\psi \sin\theta d\phi;$$

$$\sigma_2 = \cos\psi d\theta + \sin\psi \sin\theta d\phi;$$

$$\sigma_3 = d\psi + \cos\theta d\phi;$$

$$\frac{2bc}{f}\frac{da}{d\rho} = (b-c)^2 - a^2, \ \text{(cyclic)}; \ \rho = 2K\left(\sin\frac{\beta}{2}\right)$$

$$K(k) = \int_0^{\frac{\pi}{2}} (1 - k^2 \sin^2\tau)^{\frac{1}{2}} d\tau$$

Note: $\pi \le \rho \le \infty$

Special Cases:

ρ large: Then the metric becomes:

$$ds_{\mathrm{TN}}^2 = \left(1-\frac{2}{\rho}\right)\left(d\rho^2+\rho^2 d\Omega_2^2\right)+4\left(1-\frac{2}{\rho}\right)^{-1}\left(d\psi+\cos\theta d\phi\right)^2$$

ρ then is the relative separation. (not when small)

The singularity at ρ = 2 is an artifact, and has no meaning.

There are exponential corrections to the large ρ limit before this.

$\rho = \pi$: Near this, we get

$$a = 2(\rho-\pi)+O\left((\rho-\pi)^2\right)+\cdots,$$
$$b = \pi+O\left((\rho-\pi)^2\right)+\cdots,\ \ c = -\pi+O\left((\rho-\pi)^2\right)+\cdots$$

So there is an apparent singularity , since the S^3 collapses to S^2.

In fact, this is a coordinate ("bolt") singularity, and the space becomes $R^2 \times S^2$, where the S^2 has radius π.

The bolt represents the coincident two monopole case.

So What?

Returning to our large Q $SU(Q)$ gauge theory/probe computation, a rescaling and gauge transformation:

$$\rho = 2U/\lambda$$

$$\psi = \sigma \frac{8\pi^2}{Q}$$

$$A_\phi = -\cos\theta$$

gives for our moduli space metric:

$$ds^2 = \frac{g_{YM}^2 Q^2}{32\pi^2} ds_{IN}^2$$

Therefore the unique family of exponential corrections we are looking for are those of Atiyah-Hitchin.

So in fact, we learn immediately what the corrections to our enhançon geometry are:

The enhançon is slightly outside where it was perturbatively, it is at the bolt.

How can Atiyah-Hitchin be telling us about spacetime? Well, ρ is still a relative coordinate, but one monopole is extremely heavy.

So ρ is accurately the (pulled back) spacetime radial coordinate!

Spherical Symmetry?

While the Q-monopole is not spherically symmetric, it is asymptotically:

$$\mathrm{Tr}(\mathrm{H}^*\mathrm{H})/2 \rightarrow H - Q\mathbf{e}_m/r + \ldots$$

This fits with the fact that our supergravity solution is spherical. In fact, expanding:

$$\frac{V(r)}{V^*} - 1 = \left(\frac{V}{V^*} - 1\right) - \left(\frac{V}{V^*} + 1\right)\frac{g\alpha'^{1/2}Q}{r} + \cdots$$

we can identify:

$$\mathbf{e}_m = \left(\frac{V}{V^*} + 1\right)\frac{g\alpha'^{1/2}}{2}$$

$$H = \left(\frac{V}{V^*} - 1\right)$$

This will be useful later on.

The idea here is that deep into the region where supergravity is not valid, we will see the structure of the monopoles.

The Enhançon as a Fuzzy Sphere

The shell can only a smooth sphere for Q large.

In fact, it has a description in variables very similar to those used to describe a "fuzzy sphere":

Multi-monopoles are well described using the system:

$$\frac{d\Phi^i}{d\sigma} + [\Phi_0, \Phi_i] = \frac{1}{2}\varepsilon_{ijk}[\Phi^j, \Phi^k] \qquad -H \leq \sigma \leq H$$

The Φ's are anti-Hermitian $SU(Q)$ matrices. (swop $i = 3,4,5$ for 1,2,3)

There is a gauge invariance ($G(\sigma) \in SU(Q)$):

$$\Phi_0 \to G\Phi_0 G^{-1} - \frac{dG}{d\sigma}G^{-1}\,, \qquad \Phi_i \to G\Phi_i G^{-1}$$

This can be used to set Φ_0 to 0, giving "Nahm's Equations"

Boundary condition:

$$\Phi_i(\sigma) \to \frac{\Sigma_i}{\sigma \mp H}\,, \qquad \sigma \to \pm H$$

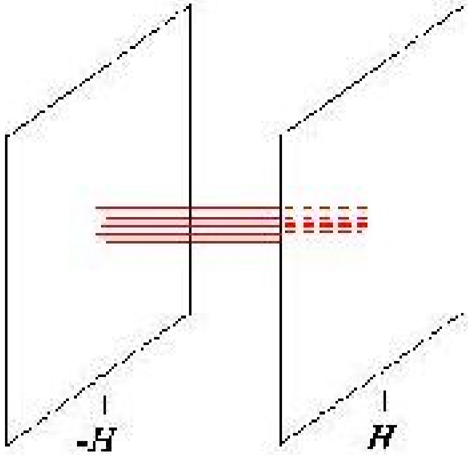
Σ_i is the Q dimensional irrep. of $SU(Q)$.

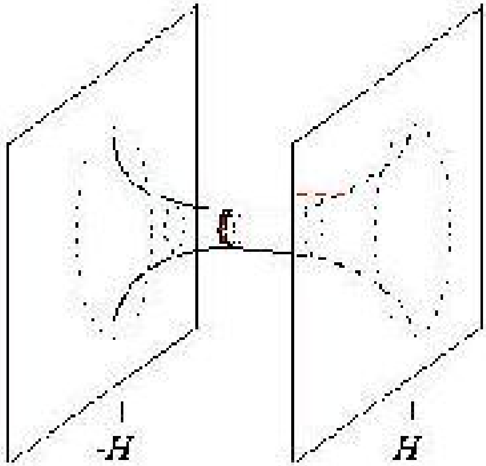
$$[\Sigma_i, \Sigma_j] = 2i\varepsilon_{ijk}\Sigma_k\ , \ \text{with} \quad \sum_i (\Sigma_i)^2 = 3(Q^2 - 1)\mathrm{I}_{Q\times Q}$$

Amusingly, this system of equations is adapted to one of our dual descriptions of the enhançon!

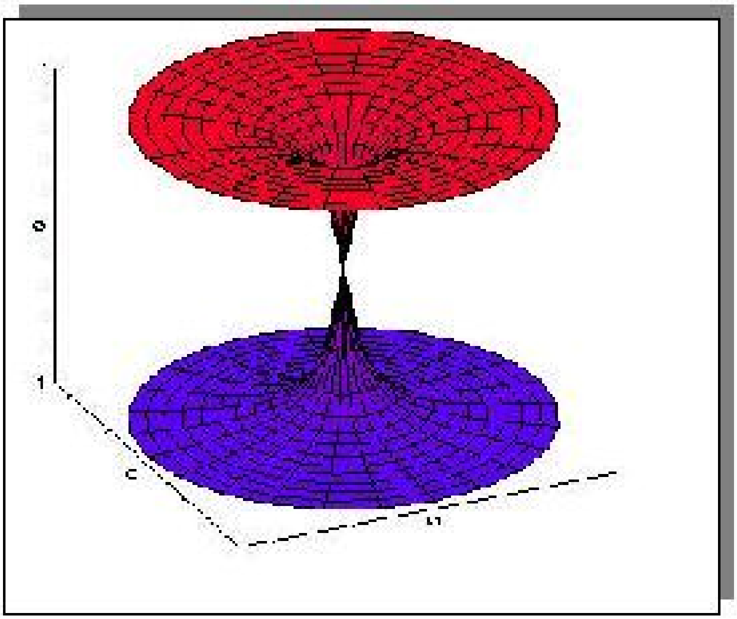
scale by $2\pi\sqrt{\alpha'}$ and restore g.

Φ_i 's are D-brane positions within the NS5-branes





For the spherically symmetric one-monopole, the Nahm data is simply the three constants (ix_1, ix_2, ix_3) representing position, and looks like:



(H=1 here.)

Since for large Q our system is approximately spherical, guess that solution is of form:

$$\Phi_i(\sigma) = -if(\sigma)\Sigma_i$$

and get

$$\frac{df}{d\sigma} = \frac{f^2}{g}, \quad f(\sigma) = \frac{g}{\sigma \mp 1}, \quad \text{for } \sigma \to \pm 1$$

So, putting in the dimensions, the radius at σ is:

$$R^2 = \frac{4\pi^2\alpha'}{Q}\sum_i \mathrm{Tr}(\Phi_i^2) \;=\; 12\pi^2\alpha'(Q^2-1)f^2(\sigma)$$

by symmetry, f will have some minimum in the centre, f_e, giving:

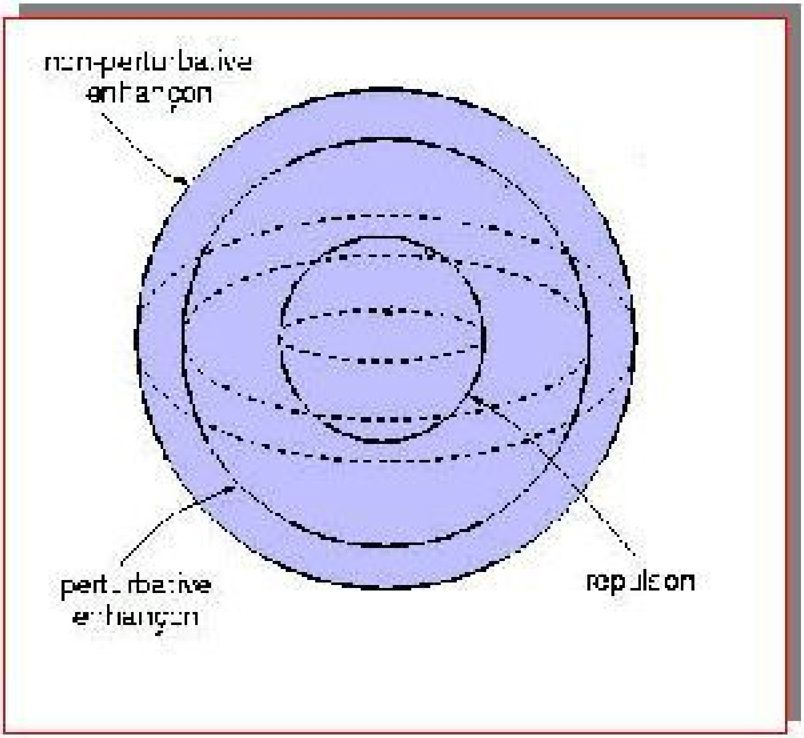
$$R_Q = 2\sqrt{3}\pi g\sqrt{\alpha'}\, f_e\sqrt{Q^2-1} \approx 2\sqrt{3}\pi g\sqrt{\alpha'}\, f_e Q$$

This compares well with the supergravity enhancon radius:

$$r_e = gQ\sqrt{\alpha'}\left(\frac{V}{V^*}-1\right)^{-1} \approx \frac{Q}{He}$$

and is the correct expression for the size of a Q-monopole.

Summary of Non-Perturbative Story



Comment:

This (fuzzy) geometry is only smooth and spherical because Q is large.

Note:

Still need to reverse pullback to turn into full spacetime details

Concluding Remarks

- We have learned a lot about our system ($p=2$) *via* BPS monopoles

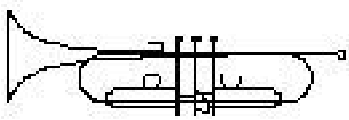
- Other p?… hence other gauge theories?

  Can vortices, etc, help for $p=3$?

- Capture large Q gauge theory dynamics with large Q Nahm equations?

  …Or perhaps decoupled spacetime monopole solution ($\mathbf{A}$,H) itself?

Suggestive of a dual (string?) theory.



Seasoning to Taste

⇒It is easy to add hypermultiplets to the system in order to make fundamental flavours.

⇒Just add more Dp's to the D($p+4$)–Dp system…

$$N_H = 2(Q_{p+4}^2 + Q_{p+4}Q_p)$$

- Q_p is net number of Dp-branes
- Number of flavours is $N_F = 2(Q_p + Q_{p+4})$
- Generally, write $Q_p = Q' - Q_{p+4}$

Previously $Q_p = -Q_{p+4}$, so pure gauge

Note: 4D conformal case is $Q_3 = 0$; quite simple!

(Problem: gauge branes are roughly same size as flavour branes.)

Supergravity?

Simply modify Z_p harmonic function.

$$e.g.\ Z_2 = 1 + \frac{V^*}{2V_r} Q_2 \alpha'^{1/2}; \quad Q_2 = Q' - Q_6$$

Probing?

Choices:

- Probe with a wrapped D($p+4$)
- Probe with a pure Dp (can do this with no flavours too)

Results of Probing

Unscaled:

Wrapped D6 probe:

$$ds^2 = F(r)\left(dr^2 + r^2 d\Omega_2^2\right) + F(r)^{-1}\left(ds/2 - \mu_2 C_\phi d\phi/2\right)^2 ;$$

$$F(r) = \frac{1}{2g}\left(\mu_6 V Z_2 - \mu_2 Z_6\right)$$

D2 probe:

It's easy to see what the result is: send $\mu_6 \rightarrow 0$ and $\mu_2 \rightarrow -\mu_2$

$$F(r) = \frac{\mu_2}{2g} Z_6$$

Note: Result completely ignores the D2 component !

Scaled: i.e., take $\alpha' \rightarrow 0$, keep $U = r/\alpha'$ and g_{YM} fixed

Wrapped D6 probe result is basically what we saw before....

More interesting is the result for D2 probe:

$$g_{YM}^2 = (2\pi)^4 g\alpha'^{3/2} V^{-1}$$

$$g_{YM}^2 = g\alpha'^{-1/2}$$

This is the gauge coupling on the wrapped D6-brane

This is the gauge coupling on the pure D2-brane

Depending upon which one holds fixed, the resulting metric is different:

$$ds^2 = f(U)\left(dU^2 + U^2 d\Omega_2^2\right) + f(U)\left(d\sigma - \frac{Q}{8\pi^2} A_\phi d\phi\right)^2$$

$$f(U) = \frac{Q}{16\pi^2 U}$$

$$f(U) = \frac{1}{8\pi^2 g_{YM}^2}\left(1 + \frac{Q g_{YM}^2}{2U}\right)$$

Ordinary Taub-NUT

The probe brane is a flavour in the $SU(Q)$ gauge theory.

Focussing on the probe brane's $U(1)$ gauge theory with $Q/2$ hypermultiplets.

Difference between the two is that the "1" does not survive the limit in the pure brane case. This is more like AdS/CFT…