Black hole thermodynamics from strongly-coupled gauge theory

strings '00

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Goal: directly study 0-brane quantum mechanics, in the regime where it has a 10-D SUGRA dual.

Motivation: string theory in certain backgrounds can be defined in terms of a dual large-N field theory (AdS/CFT, ...)

Like to "solve" the field theory (in some approximation) to learn about non-perturbative string theory.

Problem: field theory is strongly coupled.

<u>Outline</u>

I. Gravity/gauge duality for 0-branes

II. Mean-field approximation

III. Gauge fields and Gross-Witten

IV. Numerical results

V. Black hole geometry

I. Gravity / gauge duality for 0-branes

Study IIA string theory in the decoupling limit

$g_s \to 0$ weak coupling

$m_s \to \infty$ decouple oscillator excitations

$N \gg 1$ N units of 0-brane charge

Gauge theory / open string description:

0+1 dimensional $U(N)$ gauge theory with $g_{YM}^2 = g_s m_s^3$.

Gravity / closed string description:

Black hole in 10 dimensions with 0-brane charge.

Identify $T_{\text{Hawking}} = T_{\text{gauge}}$ at finite temperature.

In more detail the gauge theory has

$A_0(t)$ — $U(N)$ gauge field

$X_i(t)$ — adjoint scalar fields $(i = 1, \ldots, 9)$

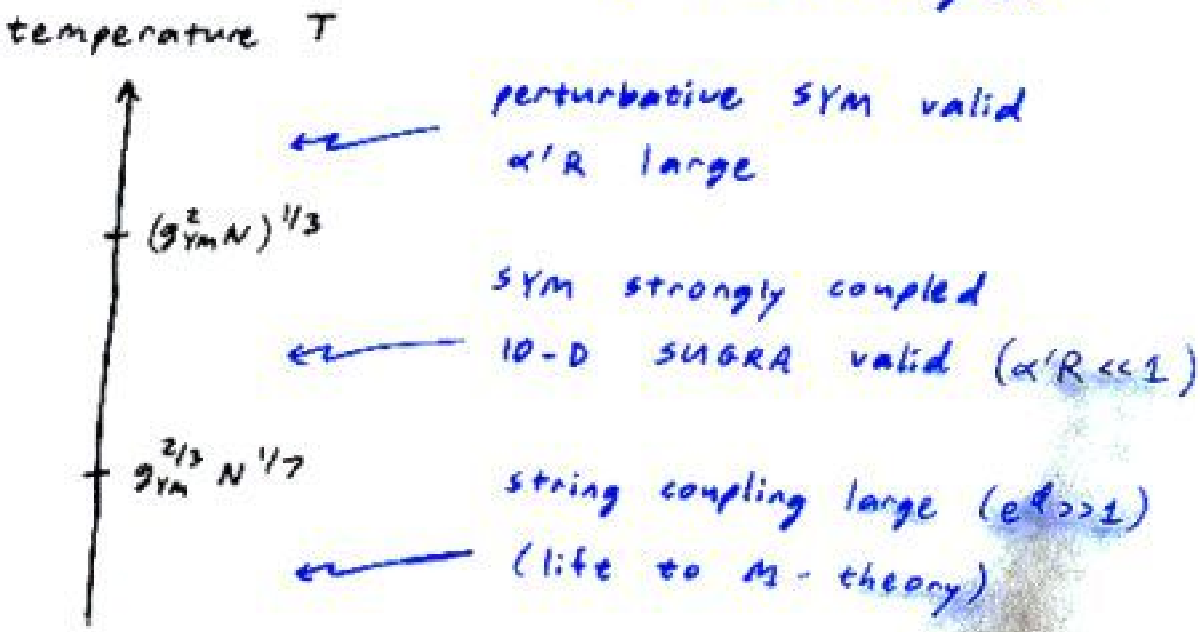
$\Psi_\alpha(t)$ — adjoint fermions $(\alpha = 1, \ldots, 16)$

$$S = \frac{1}{g_{YM}^2} \int dt \; \mathrm{Tr} \left\{ \frac{1}{2}(D_0 X^i)^2 + \frac{1}{4}\left([X^i, X^j]\right)^2 + \text{fermions} \right\}$$

effective coupling $g_{eff}^2 = \dfrac{g_{YM}^2 N}{T^3}$ (dimensionless)

On the gravity side you have to ask where weakly-coupled IIA SUGRA is valid. (IMSY)

This leads to the following "phase diagram"



I'll focus on the 10-D SUGRA regime.

The black hole makes predictions for the thermodynamics of the gauge theory,

$$\text{entropy} \quad S = \text{const. } N^2 \left(\frac{T}{(g^2_{YM} N)^{1/3}} \right)^{9/5}$$

Looks like 't Hooft scaling - can we understand the exponent?

II. Mean-field approximation

Basic idea: treat the $\mathcal{O}(N^2)$ d.o.f. in the matrices X^i, Ψ_α as statistically independent, moving in a mean-field background due to the other N^2-1 d.o.f.

Approx. motivated by the $\mathcal{O}(N^2)$ entropy of the black hole.

To formulate the approx. introduce

$S(A_0, X^i, \Psi_\alpha)$ ← interacting theory

$$S_0(A_0, X^i, \Psi_\alpha) = \sum_{\ell=-\infty}^{\infty} \frac{1}{2\sigma_\ell^2} \mathrm{Tr}(X_\ell^i X_{-\ell}^i) + \ldots$$

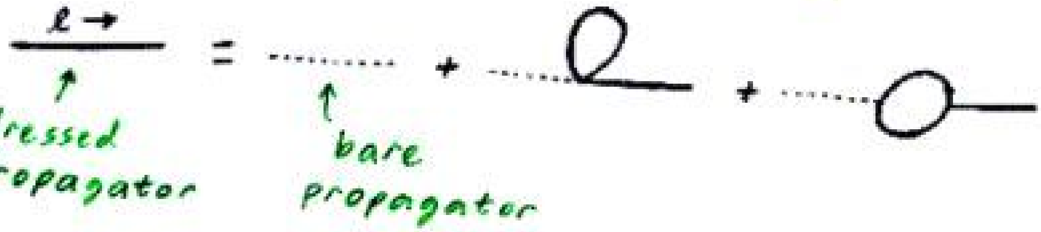
↑ "trial action" - free (Gaussian) theory

($\tau \approx \tau + \beta$, $\ell \in \mathbb{Z}$ Matsubara frequencies)

How to fix σ_ℓ^2?

Write a self-consistent "gap" equation

$\sigma_\ell^2 =$



Resums an infinite number of (planar) Feynman diagrams.

(Mean field theory, variational methods, self-consistent Hartree-Fock, ...)

Can compute βF in an expansion

$$\beta F = \beta F_0 + \langle S - S_0 \rangle_0 + \mathcal{O}\left((S-S_0)^2 \right)$$

Reorganized perturbation series can be well-behaved at strong coupling.

Approximation has some nice features,

1. 't Hooft large-N counting built in

2. Approx. respects all linearly realized symmetries,

$\mathcal{N}=2$ SUSY

$SO(2) \times SO(7)$ rotational symmetry

(out of $\mathcal{N}=16$ and $SO(9)$).

3. Non-perturbative in g^2_{YM}.

To achieve #2 need to use an off-shell superfield formalism,

gauge multiplet $\Gamma = (A_0$, two scalars, fermions)

seven scalar multiplets $\Phi_a \quad a = 1 \dots 7$

III. Gauge field and Gross - Witten

Special d.o.f. : 0+1 dimensional gauge field A_0.

$$\mathcal{U} = \mathcal{P}\, e^{i\oint A_0} = \begin{bmatrix} e^{i\alpha_1} & & 0 \\ & \ddots & \\ 0 & & e^{i\alpha_N} \end{bmatrix}$$

Wilson loop around Euclidean time

Eigenvalues are angular variables; the simplest trial action we can write is

$$S_0 = \frac{1}{\lambda} \, Tr\,(\mathcal{U} + \mathcal{U}^{+}) + \ldots$$

"one - plaquette action"

Use a Schwinger - Dyson equation to fix λ.

T-duality argument $\Rightarrow$ expect transition at $T \approx (g_{YM}^2 N)^{1/3}$

Can think of α_i as twisting the boundary conditions of the other fields.

In general one expects

β small $\Rightarrow$ eigenvalues cluster near zero

λ small

- - - - - - - - - - - - - - - - Gross - Witten transition - - $\lambda = 2$

β large $\Rightarrow$ eigenvalues spread around circle

λ large

IV. Numerical results

set $g_{YM}^2 N = 1$

In general we have to solve the gap equations numerically. At high temperatures ($\beta \ll 1$) we find

$$\beta F \approx 6 \log \beta + \mathcal{O}(1)$$

(expected behavior for a weakly-coupled theory in 0+1).

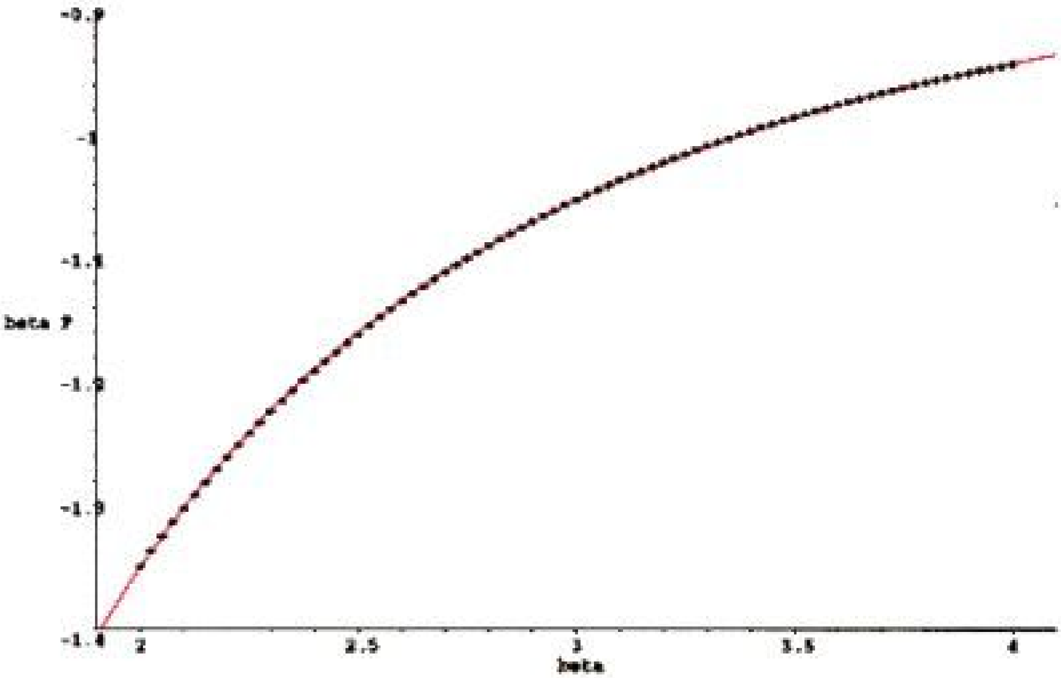
In the range $1 \lesssim \beta \lesssim 4$ the free energy can be well fit by a power law,

$$\beta F \approx \text{const.} - 2.0\, \beta^{-1.8}$$

uncertainty: 1.8 ± 0.1

Black hole prediction:

$$\beta F = -2.52\, \beta^{-1.80}$$



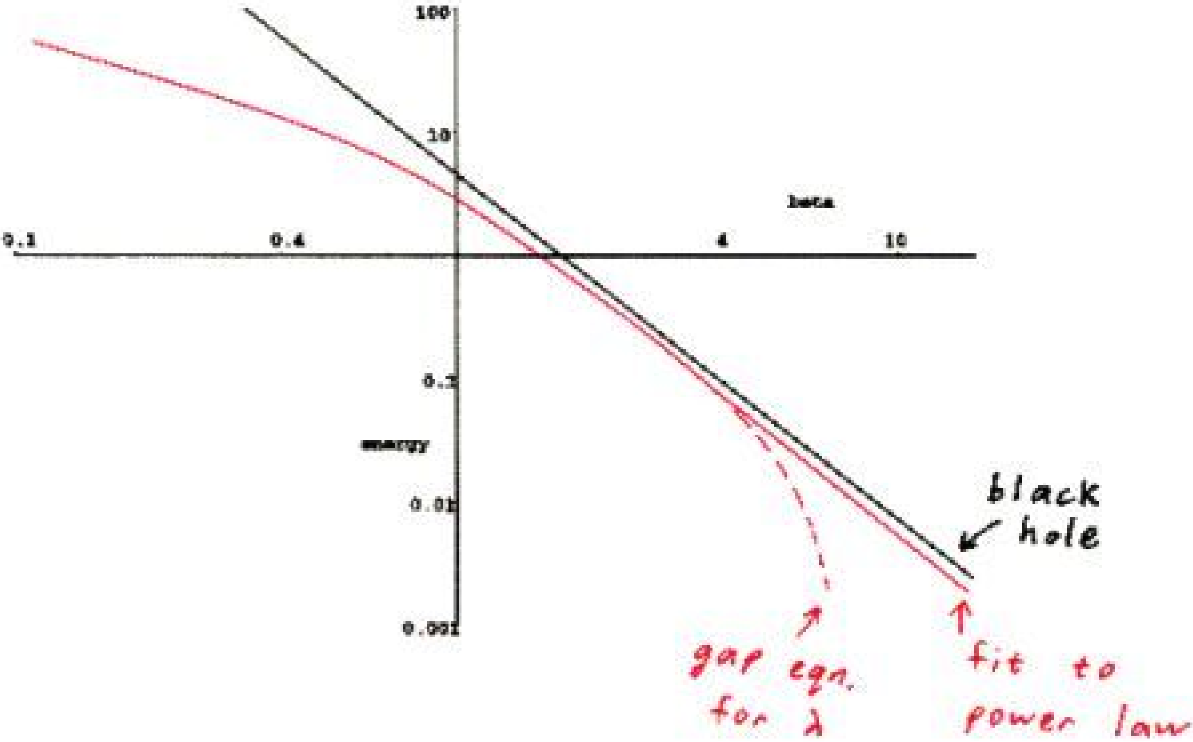
red curve: power-law fit for βF vs. β

black points: numerical calculations

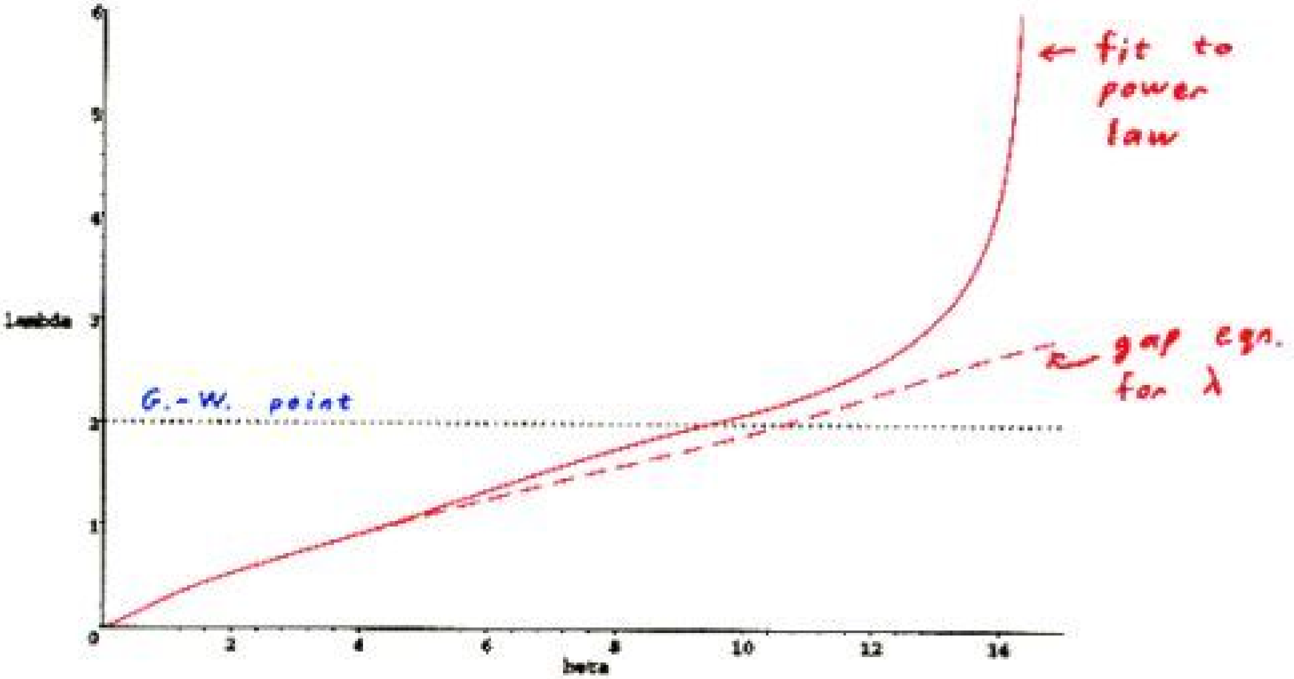
For $\beta \gtrsim 4$ the free energy no longer fits a power law. The problem can be traced to our estimate for the Wilson loop $\langle \mathrm{Tr}\, \mathcal{U} \rangle$.

Prescription: for $\beta > 4$ choose the one-plaquette coupling λ so that the free energy continues to follow the same power law.

(Lets us build a model for the black hole at lower temperatures).



Note 5 orders of magnitude on the vertical axis.



Gross-Witten transition at $\beta = 9.5$ (or 10.6)... the two curves don't differ much for $\beta \leq 13$.

Our prescription breaks down at $\beta \approx 14$, where $\lambda \to \infty$. We're not sure how to continue past this point.

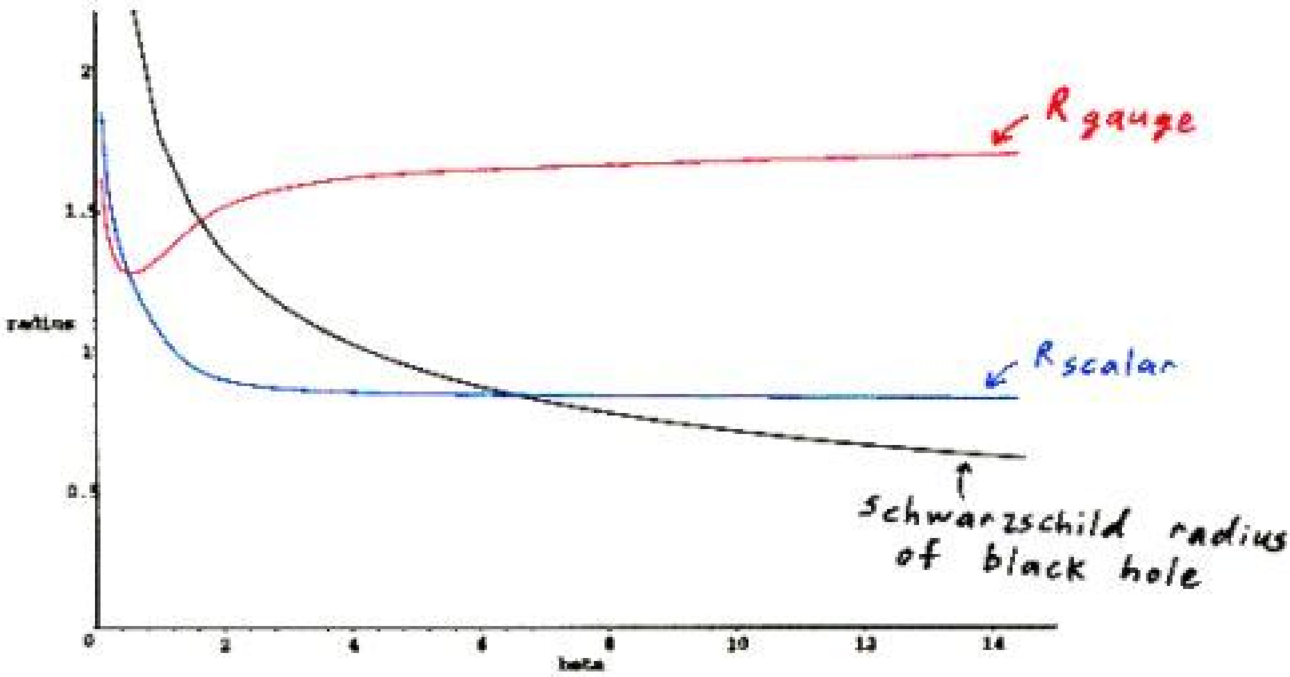
But $\beta = 14$ corresponds to a dimensionless coupling $\frac{g^2_{YM} N}{T^3} \approx 3 \times 10^9$, so maybe we should be satisfied.

This model could be used to study the spacetime structure of the black hole.

For instance, one can define the "size" of the state in the gauge theory

$$R^2_{gauge} = \frac{1}{N} \langle Tr(X^2) \rangle \Big|_{\text{gauge multiplet}}$$

$$R^2_{scalar} = \frac{1}{N} \langle Tr(X^2) \rangle \Big|_{\text{scalar multiplet}}$$


R gauge
R scalar
schwarzschild radius of black hole
ratios
beta
0
2
4
6
8
10
12
14
0.5
1
1.5
2