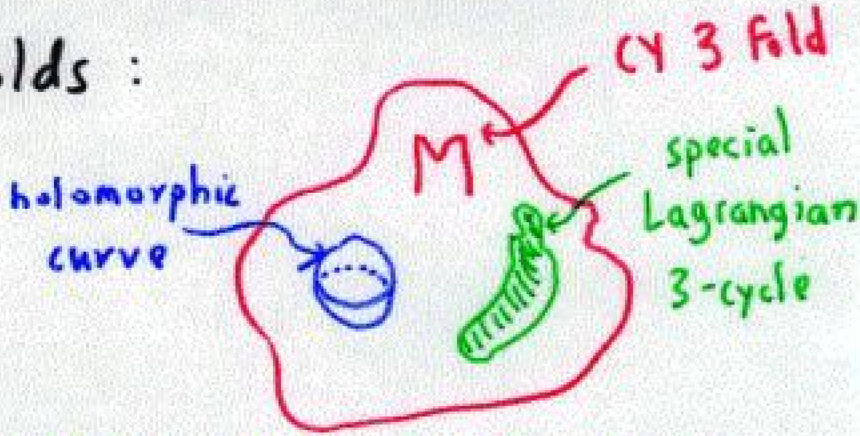
Stringy Geometry of D-branes

- Work in progress w/ A. Lawrence, J. McGreevy
- hep-th/9912151, 0006047 w/ Katz, Lawrence and McGreevy

Introduction

There are several motivations for studying D-branes wrapped on (SUSY) cycles of Calabi-Yau manifolds:



1) Building blocks of 4d, $\mathcal{N}=1$ SUSY string compactifications (stringy "brane worlds").

2) Their physics in regimes where sizes are $\lesssim l_s$ is a source of info. about stringy modifications of geometry.

My focus in this talk will be on 2).

I will (briefly) describe two different aspects of the subject:

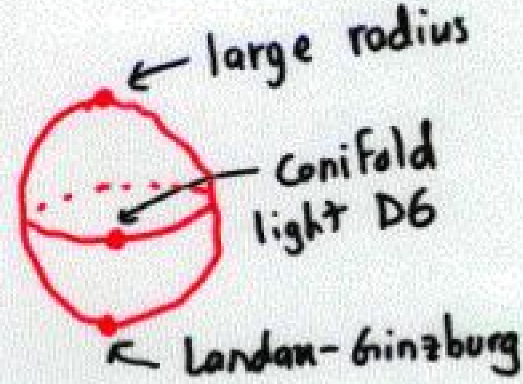
- A world-sheet boundary CFT approach to learning about branes outside the supergravity regime, based on gauged linear sigma models.
- An example of the implications of mirror symmetry for the brane world-volume field theories (in which classical geometry maps to stringy disc instanton effects).

To start, lets dispense with the branes and ask what closed strings see as you shrink a Calabi-Yau space.

For concreteness, we describe IIA strings on M = quintic in $\mathbb{CP}^4$.

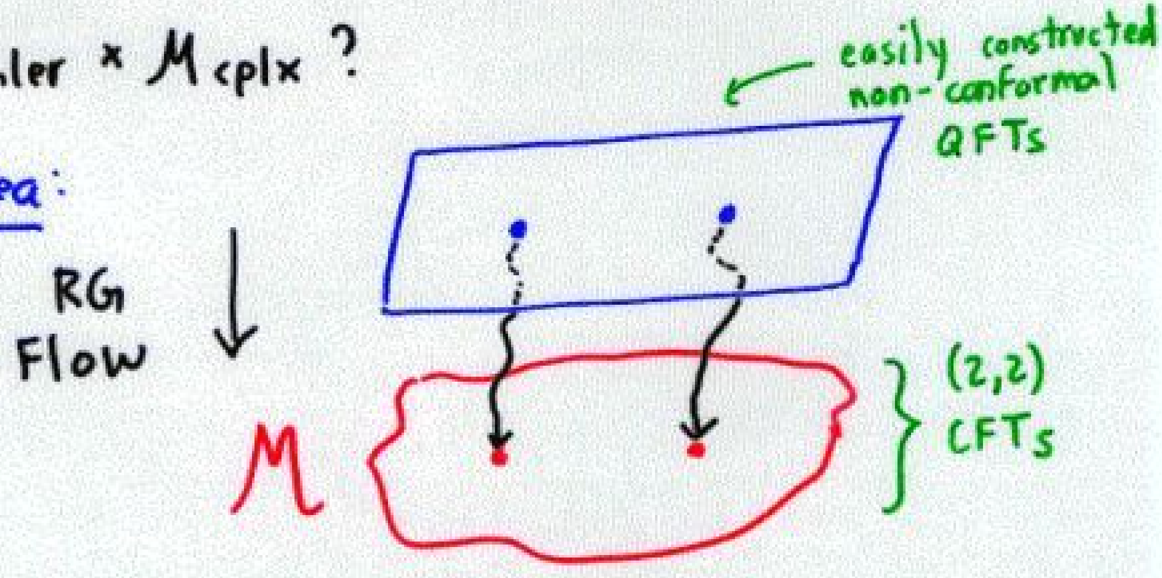
- Size controlled by a single kähler modulus
- $p = \sum_{i=1}^{5} z_i^5 + \cdots = 0$ has 101 deformations $\rightarrow$ 101 cplx structure moduli

Schematic view of kähler moduli space (Candelas et al) :



How to describe string theory at an arbitrary pt in $\mathcal{M}_{\text{kähler}} \times \mathcal{M}_{\text{cplx}}$?

Basic Idea:



- $\exists$ RG invariant quantities $\to$ can compute them in such a description
- Parameters of massive QFTs which multiply eventual marginal ops. represent moduli

Gauged Linear Sigma Models

Witten '93

Work in a superspace with 4 supercharges

$Q^\pm$, $\bar{Q}^\pm$ chiral mults $\Phi = \phi + \theta^+\psi_+ + \theta^-\psi_- + \cdots$

In such a superspace, we consider the $U(1)$ gauge theory with following field content:

Field	$U(1)$ Charge
$\Phi^{i=1\cdots 5}$	1
P	-5

In addition to kinetic terms, we include two special terms in the 2d action:

1) Fayet – Iliopoulos D-term

$$S_D = r \int d^2z\, D - \frac{i\theta}{2\pi} \int d^2z\, f$$

auxiliary field $\in$ gauge mult.

2) Superpotential

$$S_W = \int d^2z\, d^2\theta \quad P W(\Phi)$$

$U(1)$ gauge invariance $\Rightarrow$ W is quintic in Φ^i

With a Lagrangian

$$\mathcal{L} = \mathcal{L}_{kinetic} + \mathcal{L}_W + \mathcal{L}_D$$

integrating out the auxiliary fields $\Rightarrow$

a potential:

$$U(p,\phi,\sigma) = \sum_{i=1}^{5} \left|\frac{\partial W}{\partial \phi_i}\right|^2 |p|^2 + |W(\phi)|^2$$

$$+ \frac{e^2}{2}\left(\sum_i |\phi_i|^2 - 5|p|^2 - r\right)^2 +$$

$$|\sigma|^2 \left(\sum_i |\phi_i|^2 + 25|p|^2\right)$$

where σ is a scalar in the SUSY gauge mult.

Some regions of interest:

I) $r \to +\infty$

- W such that $W = dW = 0$ only if $\phi_i = 0 \Rightarrow$

must have:

$$\sum |\phi_i|^2 = r \to \mathbb{CP}^4$$

$$p = 0$$

$$W = 0 \to \text{quintic CY} \subset \mathbb{CP}^4$$

This is "Calabi-Yau" phase.

II) $r \to -\infty$

- $p \neq 0 \to$ must have $\phi_i = 0$ in vacuum,
  $\langle p \rangle = \left(-\frac{r}{5}\right)^{1/2} \to U(1)$ broken to $\mathbb{Z}_5$
- The massless ϕs whose fluctuations are governed by $W(\phi)$, orbifolded by the $\mathbb{Z}_5 \to$ "Landau-Ginzburg orbifold" phase. Vafa Martinec

III) $r = 0$ (and set $\theta = 0$ too)

Then can set $\phi_i = p = 0 \to \sigma$ has no potential $\to$ a non-compact "σ-branch" arises.

The CFT is singular; this is the (mirror) conifold.

This was all brane-less. Can we get a description of branes in similar spirit?

GLSMs for boundary CFTs

other approaches: Hori, Iqbal, Vafa; Govindarajan + Jayaraman

Lets still work on quintic in $\mathbb{CP}^4$. Say we want to describe a brane wrapping the divisor

$$Q(\phi) = 0 \subset \text{quintic}$$

where Q is of degree d.

- We also want to describe a gauge bundle V (stable + holomorphic) on $Q=0$. One way to try & manufacture this is by defining V as the kernel of an exact sequence:

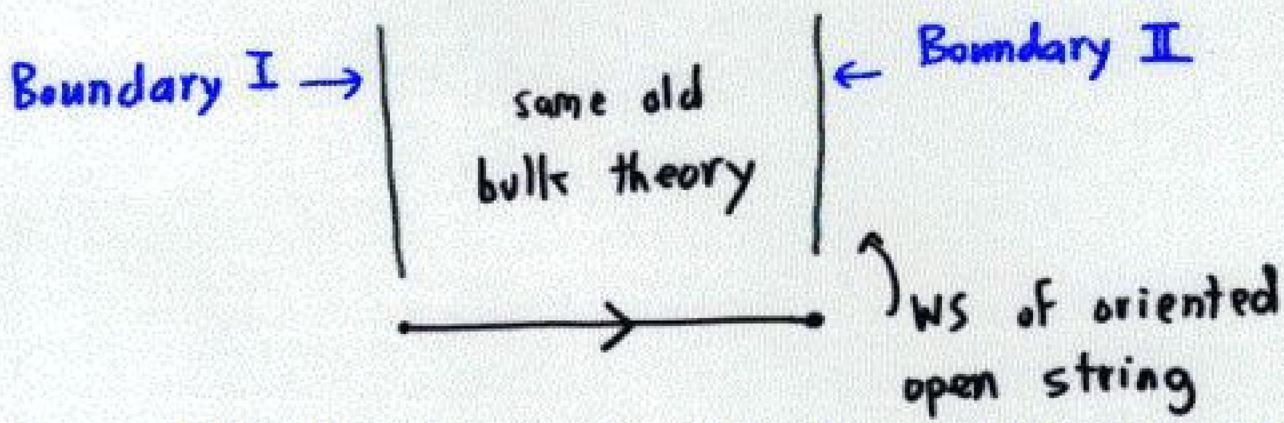
$$0 \to V \to \bigoplus \mathcal{O}(n_a) \xrightarrow{\otimes F_a(\phi)} \mathcal{O}(m) \to 0$$

Need $c_1(V) = 0$ in case at hand $\to$ $m = \sum n_a$.

Can we find a GLSM that describes this brane at large radius, and allows us to continue to small radius (where the SUGRA/classical math language is inapplicable).

Yes. Consider now a 2d QFT on the strip.

- In bulk, take the same old GLSM



Now, add boundary d.o.f. & couplings as follows (consistent with $\frac{1}{2}$ of bulk SUSY of worldsheet).

see e.g. Warner '95

On Boundary I:

Work in $N=2$ superspace $\to$ 2 relevant multiplets:

$$\text{Chiral} \quad \mathcal{P} = p + \theta\pi + \cdots$$

$$\text{Fermi} \quad \Lambda = \lambda + \theta l + \cdots$$

We choose to add:

	Field	U(1) charge
Chiral $\to$	$\mathcal{P}$	$-m$
Fermi	Λ^a	n_a
Fermi	Σ	$-d$

Also add boundary kinetic terms, and

$$W_I = \int d\theta \; \Sigma Q(\Phi) + \mathcal{P}\Lambda^a F_a(\Phi)$$

Now, what will happen in GLSM?

I) At large radius $r \to +\infty$:

Bulk $\to$ must live in quintic CY as before

Boundary I : New contributions to U at ∂

$$U|_\partial = |Q(\phi)|^2 + |\rho|^2 \sum_a |F_a(\phi)|^2$$

+ boundary values of bulk terms

So:

- The end of the string must live on the submanifold $Q(\phi) = 0 \subset$ quintic.
- Lets assume the $F_a(\phi)$ are such that F_a never simultaneously vanish on $Q = W = 0$. Then, we see $\rho = 0$ too.

The boundary superpotential also $\to$ mass terms for fermion $\in \Sigma$, and the fermion in γ pairs up with the linear combination $\lambda^a F_a(\phi)$.

What remains:

- WS boundary must lie on $Q=0$
- Fermions λ transforming as sections of V defined by

$$0 \to V \to \oplus\, \mathcal{O}(n_a) \to \mathcal{O}(m) \to 0$$

This is exactly right for an open string ending on a D-brane wrapping $Q=0$, with bundle V. Simple changes $\to$ boundary II has fermions which are a section of V^*.

What happens at $r \to -\infty$?

- Bulk analysis $\to$ LG phase as before
- Boundary ?

- vacuum at $\phi = 0$, $p = 0$

[$p = 0$ due to coupling to D field]

- fluctuations governed by

$$\int d\theta \; p \Lambda^a F_a(\phi) + \Sigma Q(\phi)$$

So the boundary CFT has an LG phase which should lend itself to explicit calculations.

For instance,

$$\{\bar{Q}, Q\} = L_0$$

Very similar to: S.K. + Witten; Distler + S.K.

$\to$ on Ramond ground states ($L_0 = 0$), spectrum computable via $\overline{Q}$ cohomology. $\exists$ an explicit construction of a suitable $\overline{Q}$ at LG, so the spectrum on brane @ LG is computable.

Should be able to: ✓ c.f. Douglas et al.

- compare to Gepner model results on boundary states
- learn about behavior of worldvolume theory on brane at singular pts in its moduli space (the parameter space of the boundary d.o.f.).

Another useful window on stringy effects has been provided, for closed strings, by mirror symmetry.

e.g. compactify IIA on CY M $\rightarrow$

kähler moduli space of M $\approx$ vector multiplet moduli space of 4d $\mathcal{N}=2$ effective thy

The metric on moduli space, gauge coupling function,... are determined by a <u>prepotential</u> $\mathcal{F}$, and it is a familiar fact that:

$$\mathcal{F}_{exact} = \mathcal{F}_{\substack{\text{tree lvl} \\ \text{in } \alpha'}} + \sum (\text{worldsheet instantons})$$

Effects suppressed by $e^{-A(C)/\alpha'}$, where

C is a rational curve in M

Mirror symmetry then relates:

$$\mathcal{F}_{exact}\begin{pmatrix} \text{IIA on} \\ M \end{pmatrix} = \mathcal{F}_{tree}\begin{pmatrix} \text{IIB on} \\ W \end{pmatrix}$$

Analogous results for open strings?

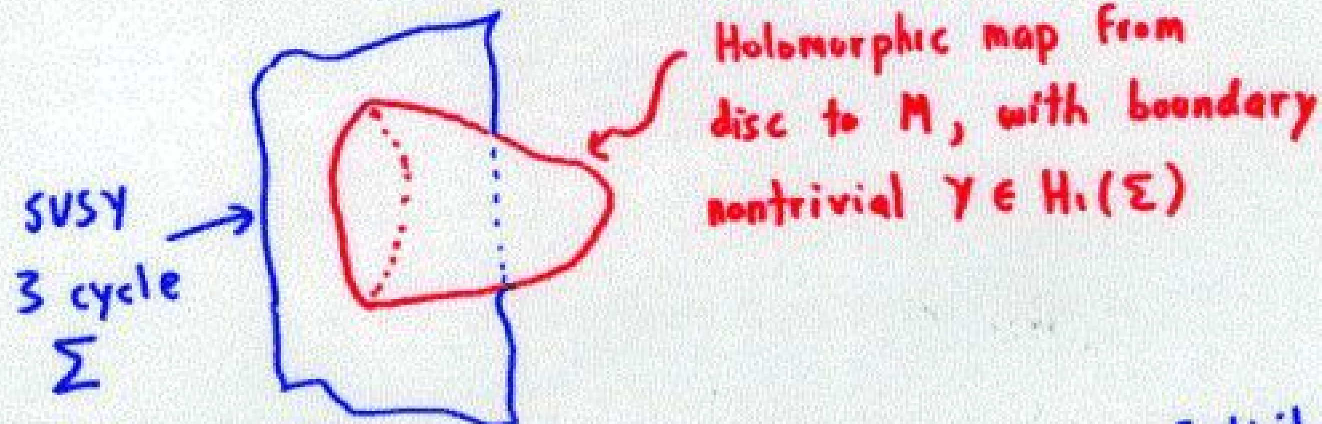
$\exists$ two natural classes of SUSY brane configs:

- B type branes: wrap hol. cycle in M + stable, hol. vector bundle
- A type branes: wrap special Lagrangian three cycle in M + flat connection on 3-cycle

At string tree lvl, at generic pts in open string moduli space:

- Superpot. terms in WV theory of B type branes is tree lvl in α' (computable via classical geometry) BDLR; KKLM

- Superpot. for A type branes is entirely non-pert in α', from "disc instanton" effects:



Mirror symmetry takes: (for instance) Explicit examples $\in$ KKLM

5 brane on curve in IIB $\longleftrightarrow$ 6 brane on 3-cycle in IIA

tree lvl superpot. $\longleftrightarrow$ superpot. generated by disc instanton sum

Much remains to be done in:

- Fleshing out the use of mirror symmetry as a tool in "solving" 4d, $\mathcal{N}=1$ brane worlds.
- Understanding stringy spectrum of branes on CY manifolds ($\rightarrow$ partial classification of 4d, $\mathcal{N}=1$ string vacua).
- Exploring phase transitions, dualities, ... between these (and other) constructions of string vacua w/ 4 supercharges.